TOWARD A TAXONOMY OF RESEARCH ON STATISTICAL KNOWLEDGE FOR TEACHING

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ABSTRACT

The knowledge needed to teach statistics overlaps with, but is not limited to, the knowledge needed to do statistics. Hence, research on statistical knowledge for teaching should not be limited to the study of teachers' subject matter knowledge. This article outlines a taxonomy describing multiple foci for research on statistical knowledge for teaching. The theoretical structure for the taxonomy is sketched and then stress-tested using a collection of articles from the Statistics Education Research Journal. Challenges of using the taxonomy to categorize research are made explicit, and ideas for navigating them are provided. It is shown how the taxonomy can serve as a framework to track the prevalence of various research foci in the field and plan future studies. Directions for future scholarship to refine the taxonomy itself are also proposed.

Keywords: *Statistics education research; statistical knowledge for teaching; pedagogical content knowledge; teacher education*

1. INTRODUCTION

The need for research in statistics teacher education has grown greatly during the past few decades. Statistics subject matter, once mainly reserved for late secondary and tertiary education, began to appear across all grade levels in prominent curriculum documents near the end of the 20th century (Jones & Tarr, 2010). As a result, teachers were at times asked to teach subject matter they themselves had never encountered as students (Conference Board of the Mathematical Sciences, 2001). At present, statistics curricula continue to evolve rapidly in response to developments in data science (National Academies of Sciences, Engineering, & Medicine, 2023). Given the frenetic pace of disciplinary change and the emergence of new curricula, understanding and fostering development of the knowledge needed to teach statistics continues to become an ever-greater challenge for researchers.

As research on the knowledge needed to teach statistics continues to accumulate, it is useful to assess the overall state of the field in terms of patterns and trends across studies. Doing so can provide perspective on the types of research that have been done and areas in need of more attention. Toward that end, the purpose of this article is to sketch a taxonomy that can be used to characterize research on statistical knowledge for teaching. The proposed taxonomy is meant to provide a framework to enhance communication among researchers about overall needs in the field. Such communication is useful for facilitating collaboration within and across research teams, aggregating findings across studies, and identifying the highest-priority areas for further research.

Any method of organizing and interpreting information has a theoretical structure, whether the structure is explicitly stated or not. This article begins by explicitly describing the theoretical assumptions underlying its approaches to characterizing statistical knowledge for teaching and constructing a taxonomy. Then, a tentative taxonomy to classify studies of statistical knowledge for teaching is proposed. The tentative taxonomy is compared against a collection of existing research articles on statistical knowledge for teaching to help further explore and delineate the nature of each category of the taxonomy. Issues and challenges in defining taxonomy categories and their boundaries are considered. To conclude, implications for future research on statistical knowledge for teaching and for further iterative development of the taxonomy itself are discussed.

Statistics Education Research Journal (early access), https://doi.org/10.52041/serj.931

2. THEORETICAL ASSUMPTIONS ABOUT STATISTICAL KNOWLEDGE FOR TEACHING

The taxonomy presented in this article is based on the premise that statistical knowledge for teaching consists of both subject matter knowledge and pedagogical content knowledge (Groth, 2013). That is, teachers need to know statistics and also how to make statistics comprehensible to others. Although research on teachers' statistics subject matter knowledge is important and necessary, it is not sufficient. Research on pedagogical content knowledge is also needed so we can understand the dynamics of fusing knowledge of content with knowledge of pedagogy to teach statistics. This section describes guiding theoretical assumptions about the components of subject matter knowledge and pedagogical content knowledge (Ball et al., 2008; Hill et al., 2008) and relationships between them (Groth, 2013).

Subject matter knowledge and pedagogical content knowledge are complementary aspects of statistical knowledge for teaching. Although subject matter knowledge is not sufficient for teachers on its own, it provides a foundation for the development of pedagogical content knowledge. Key developmental understandings (Simon, 2006) of statistical ideas provide a basis for constructing pedagogically powerful ideas (Silverman & Thompson, 2008) for teaching statistics (Groth, 2013). Key developmental understandings seed the development of pedagogically powerful ideas as teachers go through the process of de-centering (Silverman & Thompson, 2008), which requires attending to how others might go about learning the content they themselves have learned. Under this theoretical process, developing statistical knowledge for teaching involves constructing knowledge of statistics content and then expanding one's perspective to learn ways to make that content accessible to a wide range of learners. Given this process, subject matter knowledge can be considered a precursor to the development of pedagogical content knowledge.

The Learning Mathematics for Teaching (LMT) framework (Ball et al., 2008; Hill et al., 2008) described teachers' subject matter knowledge as consisting of three primary components: common content knowledge, specialized content knowledge, and horizon knowledge. Groth's (2013) statistical knowledge for teaching framework used the same three subject matter knowledge components while pointing out that doing statistics requires some content knowledge that is not inherently mathematical in nature (Cobb & Moore, 1997). Common content knowledge is that which is needed by both teachers and other professionals who use statistics in their work; it includes statistics content that is both used in practice by statisticians and also included in school curricula. Specialized knowledge, in contrast, is specifically needed for the task of teaching statistics. For example, teachers benefit from knowing how to represent data and distributions in TinkerPlots and CODAP and diagnose students' work within such platforms, but other professionals can focus solely on using the most advanced statistical tools and platforms available to support their own work. Knowledge of advanced platforms and statistical ideas not included in a teacher's specific grade-level curriculum are, however, examples of horizon knowledge, which allows teachers to know where the content of their curriculum eventually leads later in school and beyond (Ball & Bass, 2009).

Like subject matter knowledge, pedagogical content knowledge can be characterized as consisting of three components: knowledge of content and students, knowledge of content and teaching, and curriculum knowledge (Hill et al., 2008; Groth, 2013). Knowledge of content and students allows teachers to anticipate students' learning of specific statistical ideas and where students might struggle or excel with the content. Knowledge of content and teaching encompasses teaching strategies that help students understand specific ideas in the curriculum. Some pedagogical strategies specific to statistics, for example, are the Random Rectangles activity (Scheaffer et al., 1996) to teach students the difference between subjective and random samples, the growing samples approach (Ben-Zvi et al., 2015) to teach students to reason about effects of sample size, and model-eliciting activities that draw students into statistical investigations (Garfield & Zieffler, 2012). Curriculum knowledge helps teachers see how the big ideas within their curriculum fit together, and this third component of pedagogical content knowledge also entails knowing the types of curricula available, the methods and tools associated with them, and the circumstances and populations for which they are best suited (Shulman, 1986).

To illustrate how components of subject matter knowledge and pedagogical content knowledge may appear in specific research questions, consider the case of researchers interested in studying teachers' knowledge related to implementation of the Pre-K-12 *Guidelines for Assessment and Instruction in Statistics Education II* (GAISE II, Bargagliotti et al., 2020). Suppose the researchers want to focus on teachers' knowledge related to the following Pre-K-12 GAISE II Level B student learning goals: "Understand that a sample can be used to answer statistical investigative questions about a population. Recognize the limitations and scope of the data collected by describing the group or population from which the data are collected" (p. 17). Some sample research questions they might ask are shown in Table 1. Common content knowledge questions would ask about teachers' ability to do the statistics described in the student learning goal (e.g., What strategies do Level B teachers use to draw representative samples from a population?). Specialized content knowledge questions could ask about teachers' ability to represent and unpack the content in multiple ways (e.g., How do Level B teachers use CODAP and TinkerPlots to represent and describe distributions of sample statistics?). Horizon knowledge questions ask what teachers know of statistical ideas related to but beyond Level B curricula (e.g., How do Level B teachers use R and Python to simulate distributions of sample statistics and subsequently interpret p-values?).

Table 1. Components of statistical knowledge for teaching (SKT) and sample research questions

SKT Type	SKT Component	Description	Hypothetical research questions
Subject Matter Knowledge (SMK)	Common Content Knowledge (CCK)	Statistical knowledge in Pre-K-12 curricula needed by teachers, students, and other professionals.	What strategies do Level B teachers use to draw representative samples from a population?
	Specialized Content Knowledge (SCK)	Knowledge of how to unpack and represent statistics content in multiple ways.	How do Level B teachers use CODAP and TinkerPlots to represent and describe distributions of sample statistics?
	Horizon Knowledge (HK)	Knowledge of statistical ideas related to but beyond the immediate curriculum.	How do Level B teachers use R and Python to simulate distributions of sample statistics and subsequently interpret *p*-values?
Pedagogical Content Knowledge (PCK)	Knowledge of Content and Students (KCS)	Knowledge of how students tend to approach and think about specific statistical tasks.	To what extent are teachers able to anticipate their students' thinking about the concept of statistical sample?
	Knowledge of Content and Teaching (KCT)	Knowledge of strategies for teaching specific statistical ideas.	What instructional strategies do Level B teachers use to help students understand how results of random sampling tend to differ from subjective sampling?
	Curriculum Knowledge (CK)	Knowledge of types of statistics curricula, their suitability for students, and viable learning progressions.	What curricular sequences do teachers use to support students' progression from drawing random samples to describing distributions of sample statistics?

Pedagogical content knowledge questions would contain elements of statistics content knowledge, but would also contain complementary pedagogical aspects. Specifically, pedagogical content knowledge questions would ask about the use of statistical knowledge for tasks related to teaching rather than teachers just using the knowledge to perform statistical tasks themselves. One example of a pedagogical content knowledge question, about knowledge of content and students, would be, "To what extent are teachers able to anticipate their students' thinking about the concept of a representative statistical sample?" To perform this task related to teaching, teachers need to know not only the concept of representative statistical sample but also how students tend to think about the concept (e.g., Jacobs, 1999). To study teachers' knowledge of content and teaching, researchers could ask, "What instructional strategies do Level B teachers use to help students understand how results of random sampling can differ from subjective sampling?" Researchers gathering data on this question would learn not only about teachers' knowledge of the two types of samples but also the nature of the lessons they design to help students learn about them (e.g., they might observe the use of the *Random Rectangles* lesson script mentioned earlier, or other lesson scripts). Researchers could also be interested in how teachers combine statistical knowledge with pedagogical knowledge to design longer sequences of lessons rather than a single lesson. In such a case, researchers could ask a question like, "What curricular sequences do teachers use to support students' progression from drawing random samples to describing distributions of sample statistics?" Teachers' curriculum knowledge could be inferred from the data gathered in such a study.

It is important to note that simply developing several components of statistical knowledge for teaching does not guarantee that the components will be put to good use (de Vetten et al., 2023). It is possible for various types of knowledge to remain inert (Renkl et al., 1996). Regarding subject matter knowledge, it is possible to know of statistical tools and techniques but not be able to put them to good use to conduct statistical investigations; "cookbook" approaches to teaching statistics exacerbate this problem (Singer & Willett, 1990). The problem of inertness has also been observed in the domain of pedagogical content knowledge, as just holding this type of knowledge does not guarantee it can be used in classroom practice (de Vetten et al., 2023). So, the context in which teachers' knowledge is studied is important to consider in deciding what can be learned from the research. The relevance of study context to taxonomic classification of research is described in further detail in the next section.

3. THEORETICAL ASSUMPTIONS ABOUT TAXONOMY STRUCTURE

The Structure of the Observed Learning Outcome (SOLO) taxonomy was designed to characterize different levels of response to academic tasks (Biggs & Collis, 1982; 1991). It has been applied in many statistics education research studies (Langrall et al., 2017; Shaughnessy, 2007). In this article, it is used in a slightly different way to characterize the work of researchers rather than research participants. Specifically, SOLO will be used to describe how researchers have responded to the academic task of doing research on statistical knowledge for teaching. This section explains SOLO's structure and how it provides a scheme to help categorize statistical knowledge for teaching research.

Given the theoretical discussion in the previous section, research on statistical knowledge for teaching can be defined as that which studies one or more components of prospective or practicing teachers' subject matter knowledge or pedagogical content knowledge (in the remainder of this article, for conciseness, the word "teacher" will frequently be used to encompass both prospective and practicing teacher research participants). As noted in the previous section, subject matter knowledge development theoretically precedes pedagogical content knowledge development. In terms of the SOLO taxonomy, subject matter knowledge and pedagogical content knowledge development could be considered as belonging to separate learning cycles (Pegg & Davey, 1998), with the former preceding the latter (Groth, 2013). Studies of statistical knowledge for teaching might focus on either cycle.

In the SOLO model, cycles each consist of three levels: unistructural, multistructural, and relational (Pegg & Davey, 1998). Unistructural responses to academic tasks focus on one aspect of relevance, multistructural responses contain several parallel relevant aspects, and relational responses synthesize and integrate aspects into a unified whole (Biggs, 1999). Given the theoretical assumption that subject matter knowledge is a precursor to pedagogical content knowledge (Groth, 2013), the research taxonomy proposed in this article positions research on components of subject matter knowledge in its first SOLO-based cycle and research on components of pedagogical content knowledge in its second.

The hypothetical nature of the levels within each cycle is summarized in Table 2 and described in detail in the remainder of this section.

Table 2. Categories of research on statistical knowledge for teaching

Cycle	Level	Research Foci
Cycle 1: Subject Matter Knowledge	Unistructural	Study of one of the following components of teachers' subject matter knowledge: common content knowledge, specialized content knowledge, or horizon knowledge
	Multistructural	Study of two or more components of teachers' subject matter knowledge
	Relational	Study of teachers' subject matter knowledge as used in the context of conducting statistical investigations
Cycle 2: Pedagogical Content Knowledge	Unistructural	Study of one of the following components of teachers' pedagogical content knowledge: Knowledge of content and students, knowledge of content and teaching, or curriculum knowledge
	Multistructural	Study of two or more components of teachers' pedagogical content knowledge
	Relational	Study of teachers' pedagogical content knowledge as used in the context of teaching statistics

The first SOLO cycle shown in Table 2 deals with subject matter knowledge. Research at the unistructural level of the subject matter knowledge cycle can be described as examining teachers' development of one component of subject matter knowledge: common content knowledge, specialized content knowledge, or horizon knowledge. Multistructural responses differ quantitatively in that they include more than one relevant element (Biggs, 1999). So, research at the multistructural level of the subject matter knowledge cycle is that which examines more than one of the three components of subject matter knowledge. Relational level SOLO responses differ qualitatively, rather than quantitatively, from other levels in that the relational level involves application of knowledge to its intended purpose rather than just demonstration of one or more knowledge components in parallel (Biggs, 1999). In Pre-K-12 statistics, this culminating synthesis and application is generally considered to be the ability to use subject matter knowledge in the service of doing statistical investigations (Bargagliotti et al., 2020; Wild & Pfannkuch, 1999). Synthesizing and drawing upon one's statistical knowledge to conduct investigations indicates that subject matter knowledge is put to its intended use and does not remain inert, so research on how teachers do this is positioned at the relational level of Cycle 1.

Pedagogical content knowledge is put to its intended use when applied during the act of teaching. So, research on how teachers integrate various components of pedagogical content knowledge while teaching can be considered as belonging to the relational level in the second cycle of the taxonomy shown in Table 2. The unistructural and multistructural levels of the second cycle are defined in terms of the components of pedagogical content knowledge: knowledge of content and students, knowledge of content and teaching, and curriculum knowledge. Research can be categorized at the unistructural level of this cycle if it focuses on teachers' development of just one of the three, or at the multistructural level if it examines the development of more than one. Relational level studies, as noted above, would be those that examine teachers' pedagogical content knowledge being used in the context of teaching practice. As with subject matter knowledge, the difference between unistructural and multistructural is quantitative, and the difference between multistructural and relational is qualitative (Biggs, 1999).

Although the cycles and levels shown in Table 2 are ordered using the SOLO framework, these levels are not necessarily indicative of the quality of research focused on different parts of the development of statistical knowledge for teaching. For instance, research that focuses solely on teachers' development of one component of statistics subject matter knowledge is not necessarily of lower quality than research that focuses on several components or studies examining teachers' abilities to conduct statistical investigations. A well-designed study at the unistructural level is more valuable than a poorly constructed one at the relational level. Unistructural Cycle 1 studies can provide insight about the earliest stages of the development of statistical knowledge for teaching, given the premise that content knowledge development precedes pedagogical content knowledge development. Saying that such a study is automatically less valuable than a relational Cycle 2 study would be somewhat akin to saying that the work of a pediatrician is automatically less valuable than the work of a gerontologist. So, although Table 2 may initially appear to suggest a research hierarchy, it is not a research quality ranking system. It does, however, provide a potentially useful tool to take inventory of the types of statistical knowledge for teaching research that have been done and foci needing more attention.

Figure 1 summarizes the decision-making process involved in categorizing a research article using the taxonomy. First, one would decide if the article reported data on teachers' subject matter knowledge or pedagogical content knowledge. Both types involve statistical knowledge, but statistical knowledge is used purely for statistical tasks in the former and has pedagogical elements in the latter. If teachers' subject matter knowledge was studied in the context of them doing a statistical investigation, the article would be categorized as relational in Cycle 1. If teachers' subject matter knowledge was studied in a context other than doing a statistical investigation (e.g., solely responding to a researcher-posed statistical task, test, survey, interview item, etc.), one would identify which components of subject matter knowledge (common content knowledge, specialized content knowledge, and/or horizon knowledge) were studied. An article reporting data on one component of subject matter knowledge would be categorized as unistructural in Cycle 1, and an article reporting on multiple components as multistructural in Cycle 1. Similarly, research on pedagogical content knowledge would be categorized as relational in Cycle 2 if the article reported on teachers' knowledge as observed while teaching students, and either unistructural or multistructural in Cycle 2 otherwise. Choosing between a unistructural or multistructural categorization would be done by deciding how many components of pedagogical content knowledge (knowledge of content and students, knowledge of content and teaching, and/or curriculum knowledge) were studied.

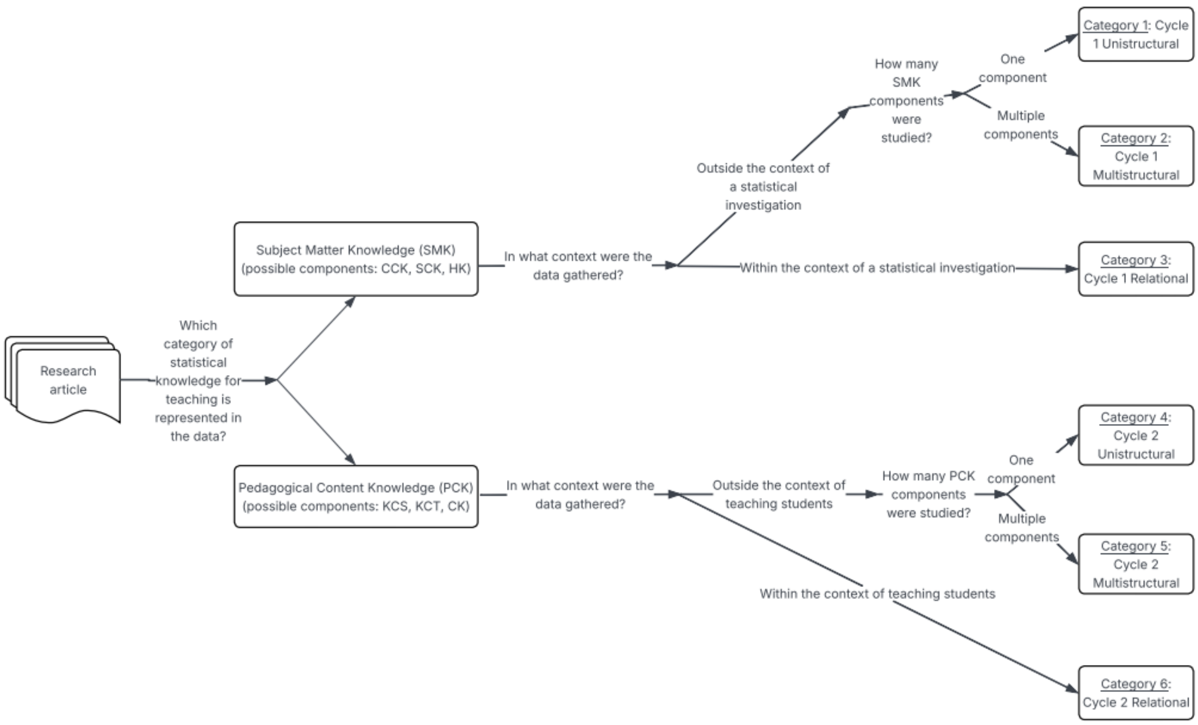

Figure 1. A decision tree to guide taxonomic classification of research articles on statistical knowledge for teaching. The abbreviations that appear are defined in Table 1.

4. COMPARING THE PROPOSED TAXONOMY TO EXISTING RESEARCH ON STATISTICAL KNOWLEDGE FOR TEACHING

Although the decision tree in Figure 1 illustrates how the SKT and SOLO frameworks can be used together to categorize research, it likely oversimplifies the ease with which such categorization can be done. With any given taxonomy, some things will be more difficult to categorize than others. It is useful for taxonomy users to know some of these difficulties in advance and how they might be mitigated. Hence, this section puts the theoretical decision-making structure shown in Figure 1 to a stress test by using it to categorize existing research articles. The purpose of the stress test is to help potential taxonomy users anticipate and negotiate some of the difficulties they might encounter in categorizing research articles using the process shown in Figure 1.

The stress test described in this section was done using statistical knowledge for teaching research published in *Statistics Education Research Journal* (SERJ) from the first issue of the journal (in 2002) through Volume 24, Issue 1 (2025). In this article, statistical knowledge for teaching research is defined as that which gathers and reports on quantitative or qualitative data pertaining to one or more aspects of teachers' subject matter knowledge or pedagogical content knowledge. This is not inclusive of all research in SERJ that names teachers as participants. It excludes studies that focus only on teachers' affective traits (e.g., attitudes toward statistics) but includes studies with data about both their statistical cognition and affect. Using these criteria, 59 SERJ articles on statistical knowledge for teaching were selected for examination (listed in Appendix A and marked with an asterisk in the reference list). SERJ was used because of the journal's exclusive focus on statistics education research, and, because all SERJ articles are freely available online, by using each doi in the reference list, readers can retrace and check the reasoning described in this section by reading the original reports archived on the journal's website.

Table 3 summarizes the approximate number of SERJ articles on statistical knowledge for teaching that fit each of the six categories introduced in Table 2. The numbers presented in Table 3 are approximate because, in some cases, an article's categorization can be reasonably disputed. Categorizing information with a taxonomic system will unavoidably involve difficult cases, so it is helpful for researchers to anticipate them. Hence, rather than privately resolving disputable categorizations internally among a research team and presenting an agreed-upon definitive set of numbers in Table 3, this article describes some of the more difficult categorization decisions in detail, using selected SERJ articles from Appendix A to explain the nature of each categorization challenge. Revealing some of the challenges of categorizing research by using the proposed taxonomy can help define the boundaries of its categories and raise awareness of nuances to know when using it. Despite some of the categorization challenges to be discussed, it will be argued that the taxonomy is nonetheless useful for identifying high-priority areas for research on statistical knowledge for teaching and thus providing directions for future studies. Next, the categories of the taxonomy (Table 3) are described along with SERJ articles related to each one.

Table 3. Taxonomy categories and the approximate number of SERJ articles (through Volume 24, Issue 1) that fit each one

Taxonomy Category	Number of Articles
Category 1: One Component of Subject Matter Knowledge	14
Category 2: Multiple Components of Subject Matter Knowledge	9
Category 3: Subject Matter Knowledge in the Context of Statistical Investigation	8
Category 4: One Component of Pedagogical Content Knowledge	7
Category 5: Multiple Components of Pedagogical Content Knowledge	6
Category 6: Pedagogical Content Knowledge in the Context of Teaching Practice	14

4.1. CATEGORY 1: RESEARCH ON A SINGLE COMPONENT OF SUBJECT MATTER KNOWLEDGE

Approximately 14 of the SERJ articles listed in Appendix A focused on a single component of teachers' subject matter knowledge: common content knowledge, horizon knowledge, or specialized content knowledge. Studies that examined teachers' abilities to do the statistics in the curricula they taught were considered to be focused on common content knowledge. These studies at times analyzed teachers' performance on tasks intended for their students, such as National Assessment of Educational Progress items (Isoda et al., 2018), tasks designed for Level A of the Pre-K-12 GAISE document (Dollard, 2011), or probability games matching school curricula they would teach (Kazak & Pratt, 2017; Malaspina & Malaspina, 2020). Others used survey items and interview questions based on the teachers' grade-level statistics content (Canada, 2006). Some used other means for gathering information about teachers' common content knowledge, such as listening to teachers' talk about variation (Makar & Confrey, 2005), discerning teachers' interpretations of the median while reading about the HIV/AIDS epidemic (Hobden, 2014), or investigating teachers' reinvention of a test statistic for categorical data (Dolor & Noll, 2015). A shared thread running through all these Category 1 studies, however, was that they focused on studying teachers' knowledge of the statistics their students would be expected to know.

In some cases, it can be difficult to tell if a study pertains to teachers' understanding of statistical ideas in the specific curriculum they teach (common content knowledge) or their understanding of content beyond the curriculum (horizon knowledge). For example, Huber et al.'s (2024) study was placed in Category 1 in Appendix A because it reported data gathered via test items about descriptive statistics often taught in early secondary education. However, the types of schools in which participants taught were not recorded. If there were any primary level teachers in the study, the test items used would be assessments of horizon knowledge rather than common knowledge for those participants. Even if the grade levels taught by study participants are carefully reported, it can still be difficult to disentangle common content knowledge and horizon knowledge. For example, the concepts teachers bring up during statistical discourse (e.g., Makar & Confrey, 2005) or while solving a rich, open-ended task (Malaspina & Malaspina, 2020) with one another may occasionally go beyond those included in the curricula they teach. If that occurs, then both horizon knowledge and common knowledge are in play, making such a study fit Category 2 more closely than Category 1. Knowing the curricula that teachers participating in a study are to teach is a key piece of information needed to understand which teacher knowledge components are at stake.

At times, researchers have explicitly stated that their research focused on the single subject matter knowledge component of horizon knowledge, making placement of such studies into Category 1 relatively straightforward. For example, Bansilal (2014) specifically stated that her study of teachers' understanding of the normal distribution consisted of content beyond the curricula participants taught and justified this focus by referring to the LMT framework component of horizon knowledge. Similarly, Legacy et al. (2025) stated that their intent was to develop teachers' horizon knowledge of multivariate reasoning through professional development. In other cases, a focus on horizon knowledge may not be explicitly stated but is reasonable to infer. For example, Magalhães and Magalhães (2021) studied responses to prompts about concepts of correlation and independence that did not appear likely to be embedded in the curricula their participants would teach.

Two of the Category 1 SERJ articles in Appendix A focused on specialized content knowledge, which is subject matter knowledge that is useful to teachers but not necessarily needed by professionals who use statistics for other purposes. Lee et al. (2016) examined how teachers drew diagrams that could be used to represent the process of repeated sampling for inference. Biehler et al. (2017) asked their participants to set up and evaluate a model using TinkerPlots. Both studies (Lee et al., 2016; Biehler et al., 2017) dealt with aspects of subject matter knowledge potentially useful for unpacking and representing content but not commonly needed for other professional purposes. Hence, the two studies were placed in Category 1 due to their unique focus on the specialized content knowledge component of subject matter knowledge.

4.2. CATEGORY 2: RESEARCH ON MULTIPLE COMPONENTS OF SUBJECT MATTER KNOWLEDGE

Category 2 research articles report data on more than one component of teachers' subject matter knowledge. In some cases, researchers have referred to theoretical models of statistical knowledge for teaching to explicitly identify multiple components of subject matter knowledge of interest. Noll (2011), for example, used the LMT model to specify a focus on graduate teaching assistants' common and specialized content knowledge of sampling. Casey and Wasserman (2015) described their findings about teachers' knowledge of informal line of best fit as being indicative of participants' common content knowledge and specialized content knowledge in the LMT model. González (2021) examined teachers' common content knowledge of variability with an item that required matching histograms to variables and a specialized content knowledge (Ball et al., 2008) item that required assessing students' interpretation of variability as represented in histograms.

In some Category 2 studies listed in Appendix A, the theoretical framing differed slightly from the LMT model, yet researchers used similar models to identify multiple components of subject matter knowledge they studied. In one such case, Ruz et al. (2021) characterized some of their items as assessments of teachers' common content knowledge and others as assessments of extended content knowledge (Pino-Fan & Godino, 2015), with extended content knowledge being a slightly reformulated version of horizon knowledge (Ball et al., 2008). Similarly, Gómez-Torres et al. (2016) used a reformulated concept of horizon knowledge, advanced content knowledge, to describe some of the items on a questionnaire to assess probability content knowledge; other items on the questionnaire were identified as assessing common content knowledge and specialized content knowledge.

At times, studies involve research on multiple components of subject matter knowledge but do not explicitly name the components using a theoretical model of statistical knowledge for teaching. Lesser and Winsor (2009), for example, studied participants' interpretation and use of words like bias, random, causal, correlation, significant, parameter, nominal, range, independent, mode, median, and mean. The participants would be responsible for teaching some, but not all, of these ideas directly to students, so it could be inferred that the research related both to participants' common content knowledge and horizon knowledge. Frischemeier and Biehler (2018) developed two frameworks to analyze teachers' performance on a group comparison task; one framework assessed TinkerPlots competence, and the other assessed statistical reasoning when comparing groups. Given that TinkerPlots is primarily used to generate accessible statistical representations, the first framework could be considered pertinent to specialized knowledge, and the second to common content knowledge or horizon knowledge. Fergusson and Pfannkuch (2022) investigated teachers' knowledge of the curricular topic of linear regression as well as knowledge of ideas not explicitly included in the school curriculum, so their research could be construed as addressing both common content knowledge and horizon knowledge. Peters (2011) analyzed teachers' responses to some items about topics they were responsible for teaching, as well as some that asked them to unpack and diagnose a student-generated statistical representation; these items might be considered indicative of teachers' common and specialized content knowledge, respectively.

As with Category 1, it can sometimes be difficult to discern which knowledge components are addressed in a study and to decide if it fits Category 2. Although some articles explicitly named multiple subject matter knowledge components that were studied (e.g., Casey & Wasserman, 2015; Noll, 2011), readers may not universally agree with how knowledge components are characterized by the authors of an article. For example, Casey and Wasserman (2015) stated that their research dealt with common content knowledge and horizon knowledge. However, in their study, one of the tasks was for participants to evaluate line of best fit representations that students had drawn. The ability to diagnose student-produced representations has been characterized as specialized content knowledge in some research (Ball et al., 2008). Specialized content knowledge is still an aspect of subject matter knowledge, so despite this potential disagreement, the study could still be classified as Category 2. A discrepancy in categorization could occur, however, with readers who believed that this sort of task assessed a component of pedagogical content knowledge. Such discrepancies need not be viewed in a negative light, however, as they can catalyze productive disagreements (Leslie, 2021) that prompt researchers to examine theoretical distinctions from prior studies and spark conversations about how to design items that assess various knowledge components.

4.3. CATEGORY 3: RESEARCH ON SUBJECT MATTER KNOWLEDGE USED WHILE CONDUCTING STATISTICAL INVESTIGATIONS

Eight of the SERJ articles in Appendix A studied how teachers used subject matter knowledge to conduct statistical investigations. In these Category 3 studies, teachers engaged in investigative cycles like those described in the Pre-K-12 GAISE II document (Bargagliotti et al., 2020) in that they formulated questions, collected/considered data, analyzed the data, and interpreted the results. In some of these articles, teachers did statistical investigations set in school-related contexts. Gómez-Torres (2021) described how teachers navigated their way through all stages of a statistical investigation to examine perceptions of a proposal prohibiting the use of mobile devices in schools. Green et al. (2018) examined how teachers conducted action research in their own classrooms, explaining how they planned their studies, collected quantitative data, analyzed the data, and interpreted their results. Bilgin et al. (2017) asked teachers to formulate a testable question for an education or science topic, identify variables and a sampling strategy, download relevant data, analyze the data, and present their results.

In some studies, teachers engaged in complete cycles of investigation that required exploring contexts other than schools and education. Lee et al. (2014) asked teachers to examine a pre-existing vehicle data set. The data set contained several qualitative and numerical attributes such as model, engine type, gas mileage, and weight. Teachers were to pose their own questions about the data and investigate them. Yilmaz et al. (2023) examined how teachers posed and investigated questions about data sets from the historical contexts of World War I, World War II, and a regional development project. Gould et al. (2017) used a model-eliciting activity to prompt teachers to pose and investigate statistical questions about a complex data set related to landfill use. Ubilla et al. (2021) had teachers gather, analyze, and interpret data on statistical questions they had formulated about topics such as the prevalence of smoking in a given population, amounts of time spent on leisure activities, gender differences in sports participation, and relationships between musical preferences and study strategies.

It can at times be difficult to decide if a study fits best in Category 2 or 3 of the proposed taxonomy. In some studies, teachers were involved in a substantial number of phases of an investigative cycle but were given questions to investigate rather than formulating their own. In the Lee et al. (2014) study, for example, one of the tasks was for teachers to explore a data set containing school data such as expenditures per student, teacher salaries, and student-teacher ratios for different regions of a country to answer the researcher-posed question (rather than a participant-posed question) of where they would prefer to teach. In another study, Leavy (2006) directed participants toward specific questions to investigate as they gathered data from experiments. One might consider the statistical investigation process to be truncated in such studies, especially in regard to examining how teachers form guiding questions. However, because many phases of statistical investigation were still involved, and teachers' knowledge was examined in that setting, such cases are listed in Category 3 rather than Category 2 in Appendix A. As more research of this nature accumulates, it may be worth revisiting this categorization issue and debating it further to help bring clarity to the types of teacher knowledge that are being elicited in such cases.

4.4. CATEGORY 4: RESEARCH ON A SINGLE COMPONENT OF PEDAGOGICAL CONTENT KNOWLEDGE

Category 4 consists of research articles that address a single component of teachers' pedagogical content knowledge for statistics. Unlike research on general pedagogy, research on pedagogical content knowledge is tied to teaching specific statistics subject matter. Although such research is grounded in statistics content knowledge, it goes beyond studying teachers' subject matter knowledge to explore their knowledge of content and students, knowledge of content and teaching, or curriculum knowledge. Notably, there are no articles in Appendix A that focused solely on teachers' knowledge of content and students. Hence, the examples for Category 4 reported here are limited to those that focused either on knowledge of content and teaching or on curriculum knowledge.

Leavy and Frischemeier (2022) studied how teachers worked with peers and instructors to generate statistical questions suitable for use in their future classrooms. Their study could be considered an example of one that focuses on the development of knowledge of content and teaching, as participants had to design classroom-appropriate questions rather than just those they would investigate for themselves. Groth and Bergner (2005) sought to investigate knowledge of content and teaching through the lens of teacher-created metaphors. In their study, participants wrote and explained metaphors for the statistical concept of sample. Metaphors were elicited in this study because they provide a window into how teachers may go about representing content to students and tying new ideas to students' prior knowledge.

In some studies shown in Appendix A, findings about teachers' knowledge of content and teaching for statistics were mixed in with findings about their general pedagogical knowledge. Carey and Dunn (2018) studied how to help teachers implement jigsaw and think-pair-share cooperative learning. Though these are general pedagogical techniques that can be applied to virtually any subject area, Carey and Dunn specifically studied their application to teaching students how to analyze statistical studies. So, the Carey and Dunn study could be considered reflective of Category 4 because it dealt with teachers' knowledge of content and teaching for that specific statistical task. Umugiraneza et al. (2018) posed an open-ended prompt to elicit teachers' ideas for improving statistics learning. In response, participants offered some ideas of general pedagogical strategies, but some also included teaching strategies specific to statistics. Similarly, Ferguson et al. (2020) elicited participants' perceptions related to both general and statistics-specific teaching strategies. Such studies were considered to be categorizable using the proposed taxonomy because they dealt with an aspect of pedagogical content knowledge for statistics, even though some findings about teachers' knowledge of general pedagogy were included. Studies that deal with general pedagogy alone would fall outside the domain of the taxonomy.

In addition to studies that focused on the component of knowledge of content and teaching, two articles fit Category 4 because they reported on teachers' curriculum knowledge. Groth (2008) studied participants' knowledge and perceptions of the Pre-K-12 GAISE document (Franklin et al., 2007). The study provided information about teachers' preparedness to implement statistics curricula reflecting the vision outlined in the document. Legacy et al. (2022) designed a survey to assess teachers' inclusion and emphasis of data, simulation, and coding practices in their curricula.

4.5. CATEGORY 5: RESEARCH ON MULTIPLE COMPONENTS OF PEDAGOGICAL CONTENT KNOWLEDGE

Category 5 research probes multiple components of teachers' pedagogical content knowledge. Surveys were used to carry out this type of research in some of the Category 5 studies shown in Appendix A. Gómez-Blancarte et al. (2021) and Justice et al. (2017) surveyed teachers about what should be emphasized in their statistics curricula and how specific statistical ideas should be taught. The survey questions thus teased out both curriculum knowledge and knowledge of content and teaching. Xu et al. (2020) surveyed similar knowledge components for university statistics instructors.

In some cases, Category 5 studies went beyond surveys to include other methods of data collection. Findley (2022) supplemented data from a survey about curriculum and instruction in statistics with data from interviews and artifacts participants produced during a workshop on teaching statistics. Watson and Nathan (2010) conducted follow-up interviews to further probe the thinking of middle school teachers who had completed a survey designed to assess their pedagogical content knowledge. The subsequent corpus of data yielded insights about the teachers' abilities to identify big ideas in statistics curricula (curriculum knowledge), anticipate students' statistical thinking (knowledge of content and students), and develop teaching strategies to address specific student learning needs (knowledge of content and teaching).

Although teacher surveys were used as part of almost every Category 5 study shown in Appendix A, one of the studies relied upon teachers' written lesson plans to explore multiple components of their pedagogical content knowledge. Huey et al. (2018) studied teachers' lesson plans about standard deviation. They examined the extent to which the lesson plans aligned with Pre-K-12 GAISE recommendations, providing information about teachers' curriculum knowledge. They also analyzed specific teaching moves included in the plans, providing insights related to knowledge of content and teaching. Participants were interviewed about the lesson plans they had produced, but the researchers did not observe them as they implemented the plans in a classroom. Hence, the study was placed in Category 5 rather than Category 6. Although lesson plans can be somewhat authentic approximations of practice, there can still be gaps between a teacher's intended and implemented curriculum (Tatto & Bankov, 2018). The separation between Categories 5 and 6 in the proposed taxonomy helps emphasize this distinction.

4.6. CATEGORY 6: RESEARCH ON PEDAGOGICAL CONTENT KNOWLEDGE USED WHILE TEACHING

Category 6 contains research that investigates teachers' pedagogical content knowledge as they enact it while teaching their students. This is an important distinction from Category 5, because, as de Vetten et al. (2023) noted, it cannot be assumed that teachers automatically use their pedagogical content knowledge in teaching; studying participants as they teach reveals how much they are able to mobilize their knowledge in practice. For example, de Vetten et al. found that prospective teachers struggled at times to interpret students' thinking about informal statistical inference and help students understand how to generalize from a sample. These findings led to recommendations for teacher education practice. Similarly, de Souza et al. (2014) studied how early childhood teachers implemented statistical projects they designed and offered observations to improve teacher education. Savard and Manuel's (2016) study of three middle school teachers found that participants took mainly procedural approaches to teaching pie charts, emphasizing the need to help teachers develop knowledge of content and teaching that leads to fostering students' conceptual understanding of statistical representations. Makar and Rubin (2009) used data from primary school classroom teaching episodes to illustrate the importance of helping teachers foster authentic statistical inquiry in their classrooms. Studies of this nature can help teacher educators make well-grounded decisions about what to emphasize and prioritize when seeking to support the enactment of statistical knowledge for teaching.

Classroom observations of teachers' knowledge put into practice are often conducted to assess teachers' implementation of ideas from professional development. In Jacobbe and Horton's (2010) study, teachers were observed in their classrooms after attending eight hours of professional development on the implementation of a new curriculum. Hammerman and Rubin (2004) introduced new software tools for data analysis to middle and high school teachers during professional development and then later gathered data on how the teachers used the new tools with their own students. Strayer et al. (2019) studied the extent to which university instructors facilitated active learning in statistics while using researcher-designed instructional modules. De Oliveira Souza et al. (2014) did two case studies of teachers' lessons on probability simulations after the teachers had received professional development on the topic. Verbisck et al. (2024) also used case study methods, following a sixth-grade teacher into her classroom to see how she taught data visualization, graphs, and statistical inquiry after completing professional development. Along with revealing aspects of teachers' pedagogical content knowledge, this type of Category 6 research can help those who design and deliver professional development assess the extent to which they have met their teacher education goals.

At times, researchers study statistical knowledge for teaching as it is developed during Pre-K-12 classroom-based professional development. The purpose of such professional development is to build statistical knowledge while teaching, rather than having stand-alone professional development occur mainly before teachers implement ideas in practice. Lesson study is one example; its core activities are collaborative lesson design followed by lesson implementation and group reflection (Lewis, 2002). Leavy (2010) used lesson study to investigate how pre-service teachers taught informal inference in primary school classrooms. De Oliveira Souza et al. (2015) studied statistics teachers' learning in the context of a teacher professional development cycle model, which was structurally similar to lesson study in encouraging collaborative planning, lesson implementation, and subsequent reflection on future improvements to be made. Groth (2017) included these elements in a program for prospective teachers but also included elements of design-based research (Bakker & van Eerde, 2015) to engage the prospective teachers in designing and teaching a sequence of lessons to develop middle school students' understanding of measures of center and distributions. Data from the design-based research project provided specific information about participants' knowledge of content and students and knowledge of content and teaching as they built and taught their lesson sequence.

In some cases, researchers have mainly aimed to study teachers' subject matter knowledge during classroom observations, but also captured elements of their pedagogical content knowledge, as it is difficult to study subject matter knowledge in isolation in the context of teaching. Pfannkuch (2006) studied a teacher's reasoning about boxplots as revealed while teaching, but inevitably had to document elements of pedagogical content knowledge as well, explaining, "her (the teacher's) reasoning...is linked to how she teaches and therefore consideration is given to instructional methods in the analysis" (p. 33). Casey (2010) sought to study three teachers' subject matter knowledge of statistical association, but along the way also described actions linked to pedagogical content knowledge, such as how the teachers selected homework problems and data sets that would be suitable for their students. Such actions draw upon not only content knowledge, but also some knowledge of how students think about the content at hand (i.e., knowledge of content and students). Jacobbe and Horton (2010) also had the main objective of studying teachers' statistical subject matter knowledge, specifically focusing on elementary teachers' comprehension of data displays. As they reported their observations of teachers' classrooms, however, they included extended examples of how teachers facilitated statistical discourse among students and helped them construct age-appropriate statistical representations with sticky notes. Such instances unavoidably reveal aspects of teachers' knowledge of content and teaching and knowledge of content and students. Such studies were placed in Category 6 because they described not only teachers' statistical knowledge but also how they helped make the content understandable to students. In other words, the studies revealed how teachers used the "amalgam of content and pedagogy" (Shulman, 1987, p. 8) that constitutes pedagogical content knowledge within their teaching practice.

5. IMPLICATIONS FOR FUTURE RESEARCH AND DEVELOPMENT

The taxonomy proposed in this article can help advance scholarship surrounding statistical knowledge for teaching in at least three ways. First, it provides a framework to inventory the types of statistical knowledge for teaching research that have been done and identify the types in need of further attention. Second, it can enhance communication within research teams by providing a catalyst for discourse about the specific components of statistical knowledge for teaching that are at stake in a given study. Such discourse can help a research team justify and clarify the focus and framing of their work. To facilitate this type of discourse, the difficult decision points encountered during the preceding sections describing the stress test of the taxonomy categorization process (Figure 1) are summarized in Table 4 along with advice for navigating them. Third, additional scholarship can be done on the taxonomy itself to further enhance its ability to support productive discourse across and within research teams. As part of that process, researchers might investigate the applicability of the taxonomy beyond

SERJ, which provided the focus for this article. Next, these potential directions for future scholarship involving the taxonomy are discussed in further detail.

Table 4. Difficult decision points for researchers to anticipate when using the proposed taxonomy and advice for working through each decision point; abbreviations used in the table are defined in Table 1

Article section	**Difficult decision point**	**Advice for navigating the decision point**
Category 1	Distinguishing between CCK and HK	Gather information about the curriculum participants are responsible for teaching; CCK is that within the curriculum and HK is beyond it.
Category 2	Identifying knowledge components from studies with different theoretical models or different interpretations of SKT components.	Discuss definitions of components as defined in prior literature among research team, debate interpretations, and work toward consensus among team members while allowing for and honoring interpretive disagreements.
Category 3	Deciding if a study context engages teachers in doing statistical investigations.	Look for evidence of teachers engaging in the Pre-K-12 GAISE II investigative cycle, at a minimum touching its four stages of formulate questions, collect/consider data, analyze the data, and interpret the results.
Category 4	Deciding if a study includes information about teachers' statistical PCK or just their general pedagogical content knowledge.	Ask if any of the teaching strategies that were studied apply to the teaching of specific statistics/data science content or if they all apply in general to teaching virtually any subject area. Studies are categorized within the taxonomy in the former case and outside it in the latter.
Category 5	Deciding between a Category 5 and Category 6 designation for a study that includes teacher-created pedagogical materials.	Consider whether the study gathered data on teachers' implementation of curricular plans/materials with students or just the design of them. Place the study in Category 5 in the latter case and Category 6 in the former.
Category 6	Categorizing studies that claim to examine only teachers' subject matter knowledge as they teach statistics.	Examine the study for instances of information about the strategies/methods participating teachers used to help students learn statistics. If such information is provided, place the study in Category 6 rather than Categories 1, 2, or 3.

5.1. USING THE TAXONOMY TO TRACK THE PREVALENCE OF RESEARCH FOCI

Figure 2 provides a visual summary of the number of articles per taxonomy category shown in Table 3. Unshaded sectors correspond to the taxonomy categories on subject matter knowledge (Categories 1-3), and shaded sectors show taxonomy categories dealing with pedagogical content knowledge (Categories 4-6). As noted earlier, these are approximate numbers from a collection of articles from a single journal, so Figure 2 should not be taken as a definitive overall summary of the current state of the field. Nonetheless, it does raise some preliminary questions about potential areas of over- or under-emphasis to look for in existing and future research.

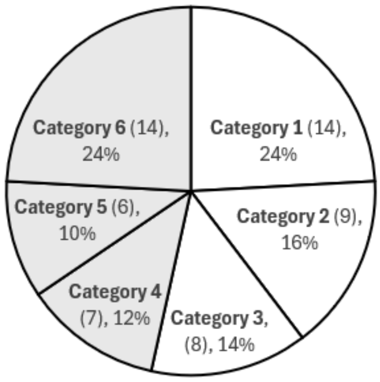

Figure 2. Approximate percentages of SERJ articles per taxonomy category; pedagogical content knowledge categories are shaded gray. The figure represents only the collection of SERJ articles that was selected for analysis (see Appendix A).

One salient feature of Figure 2 is that more than half the studies considered for this article focused on teachers' subject matter knowledge (Categories 1-3) rather than pedagogical content knowledge. Category 1 research was prevalent, in which researchers focused on a single component of subject matter knowledge (common content knowledge, specialized content knowledge, or horizon knowledge). So, it is worth considering what such studies potentially add to the field when making decisions about the foci for new research studies. With a plethora of new data science topics entering teachers' curricula, a case could be made that more emphasis on such studies is justified, as we are only beginning to learn about teachers' common content knowledge of contemporary topics they are asked to teach, such as decision trees, code-driven tools, and data wrangling (Fergusson & Pfannkuch, 2022; Lee et al., 2022; Zieffler et al., 2021). Well-designed, in-depth studies of teachers' understanding in these areas could provide valuable information for the practice of teacher education. However, caution is also warranted. If Category 1 research becomes too predominant, the field will lack information about how teachers' knowledge translates (or does not translate) to classroom practice. Unrealistic expectations about the transfer of teacher knowledge to practice can hasten the downfall of innovative curricula (Hill, 2009). So, it is important for the field overall to not lose sight of the dynamics of how teachers combine, or as Shulman (1987) put it, "amalgamate," their content knowledge and pedagogical knowledge (Categories 4-6).

A fair amount of research summarized in this article did attend to how teachers enact their knowledge in the context of teaching statistics; in fact, the Category 6 sector of Figure 2 is the same size as Category 1. Theoretically, having an even balance between Category 1 and Category 6 seems advisable, as it allows the field to know how teachers think about specific content as well as how they enact it in practice. An interesting trend within Category 6, however, was the predominance of articles that were small case studies. There is certainly a great deal of value in small case studies because they often bring out nuances of putting knowledge into practice that can easily be overlooked in larger-scale research. However, there is also value in scaling up this type of research to capture broader overall trends for statistics teacher educators to monitor. A classic example of large-scale aggregation of data from classroom case studies is the Third International Mathematics and Science Study (TIMSS). As research teams analyzed TIMSS video data, they found broad patterns that tended to characterize mathematics instruction in various countries, such as U.S. lessons being mainly concerned with "learning rules and practicing procedures" and Japanese lessons tending to exemplify "structured problem-solving" (Stigler & Hiebert, 1999). Scaling up Category 6 research on statistical knowledge for teaching may yield similar insights to inform teacher education efforts.

Along with considering the largest sectors in Figure 2 (Category 1 and Category 6), it is worth reflecting on the possible value of expanding the amount of research in the smaller sectors. For example, just as there can be value in focusing in detail on a single component of teachers' subject matter knowledge (Category 1), there may be value in focusing on a single component of their pedagogical content knowledge (Category 4). Knowledge of content and students, for example, has been found to help predict students' achievement in several studies (Callingham & Watson, 2016; Even & Tirosh, 2002; Fennema & Franke, 1992), but it rarely seems to be put under the microscope to the extent that teachers' common content knowledge of statistics often is. Researchers who are aware of this issue can be cognizant of opportunities to probe teachers' knowledge of content and students more thoroughly. For instance, it might be natural for researchers planning to study an aspect of teachers' common content knowledge (Category 1 research) to include some questions for participants on how they believe their students would think about that same content. Many other strategies for investigating aspects of knowledge of content and students could also be devised. In general, as researchers continue to frame and conduct studies of statistical knowledge for teaching, they can use the taxonomy categories to consider the value of engaging in research from sectors that may be under-emphasized in the field. The taxonomy proposed in this article, in combination with continuous systematic monitoring of the types of research being published (e.g., possibly via publisher or research team dashboards), provides a means to make such reflection possible.

5.2. USING THE TAXONOMY TO CLARIFY AND JUSTIFY RESEARCH FOCI

Although the taxonomy proposed in this article is not a way to rank the quality of studies relative to one another, considering where a new study may fall within the taxonomy can help researchers enhance their work by clarifying their foci. Explicitly naming the components of teacher knowledge under investigation not only makes it easier to put a given study in a taxonomic category; it also allows researchers to explain how their study adds to what the field already knows of the nature of statistical knowledge for teaching. Considering where a new study falls in terms of the taxonomy categories can also prompt researchers to include key details, such as whether the statistical content of the study is within the curriculum teachers are expected to implement or if it is in the realm of horizon knowledge. Such information was not included in all the studies reviewed in this article, but including it helps clarify what a given study contributes to the field.

As researchers clarify the foci of their studies in terms of the taxonomy, they should also be cognizant of justifying the foci. For example, if a research team decides to focus on horizon knowledge (or any other knowledge components), it is worth asking why the focus is justified. In the horizon knowledge example, for instance, researchers might consider whether there is theoretical support that their given aspect of horizon knowledge has a positive impact on students' learning. Such theoretical support can at times be difficult to find. However, providing the theoretical case for the foci can enhance their overall argument, along with helping situate the study in the broader field. In cases where theoretical support is lacking, they might add the study of other, complementary knowledge components described in the taxonomy to their research. In the process, researchers can enhance the value of their studies and make theoretical connections between different knowledge components and their impact on teachers' instruction.

5.3. DIRECTIONS FOR FUTURE DEVELOPMENT OF THE TAXONOMY

The acts of clarifying and justifying foci for studies of statistical knowledge for teaching with the taxonomy can lead naturally to proposing improvements to the taxonomy itself. At times, it can be difficult to precisely define the components of teacher knowledge at stake in a given situation. Researchers may disagree, for example, if teachers' knowledge of a representation used in teaching is indicative of their specialized content knowledge or knowledge of content and teaching. As research teams discuss such matters among themselves, working definitions and distinctions can be formulated to clarify the bounds among different knowledge components and recognize when one given component is in play rather than another. Disagreements will inevitably occur in this clarification process, but there is a great deal to gain by embracing rather than avoiding them. Even when full consensus is not obtained, a valuable byproduct of such discussions is systematic scholarly debate about which types of teacher knowledge are under consideration in a given study. Sharing the substance of such debates in scholarly venues can contribute to refining the taxonomy and giving insights to optimize its usefulness in practice.

Another potentially fruitful area for scholarship could be to fine-tune and modify the categories of the taxonomy. In some cases, for example, it may be beneficial to add new categories or split existing ones into multiple parts. For example, Category 5 currently encompasses studies that rely upon conventional research instruments such as surveys and tests, and those that use possibly more authentic approximations of practice, such as lesson plans. As this category of research further develops, it may become useful to distinguish more carefully between studies that use these different data gathering methods. As another example, Category 3 currently includes studies of teachers' statistical investigations related to school contexts as well as other contexts. If important differences become apparent between teachers' work across these different contexts, splitting Category 3 may be useful. Category 1 might also be considered for taxonomy refinements. Currently, Category 1 treats common content knowledge as a single component. It is conceivable that there could be advantages, however, to distinguishing between data science content that is new to the statistics curricula and content which has been included to a great extent in the past. Taxonomy changes that help draw out important distinctions or make key components of teachers' knowledge more visible could potentially improve the taxonomy's usefulness for taking inventory of developments in the field and helping researchers frame future studies. So, it is best to view the taxonomy that has been presented as being more akin to open-source software rather than text carved in stone.

At this point in time, with research on statistics teacher preparation still being a nascent field of endeavor (Weiland et al., 2024), it seems advantageous to have a taxonomy that is somewhat in flux and open to change based on the emerging needs of the field. Stress-testing the taxonomy categorization process (Figure 1) with collections of articles from journals other than SERJ is one step that could be taken by those who wish to further develop, refine, and extend the taxonomy. As subsequent stress tests are conducted and their results are compared across journals, points of consensus about taxonomy structure may emerge. If there is substantial overlap among the findings of such studies, it may be worthwhile to formulate a more general taxonomy in the future that is applicable across journals. This could be done using formal meta-analytic methods (e.g., Page et al., 2021) with features such as systematic database searches, interrater reliability measures, sensitivity analyses, and counts and percentages with confidence intervals. However, even if a more generalizable taxonomy is eventually created, there could still be value in retaining journal-specific taxonomies, as they may help define the scope and mission of any given journal regarding its role in fostering research on statistical knowledge for teaching.

6. CONCLUSION

Taxonomies can help professionals strategically focus, organize, prioritize, and refine their efforts. The taxonomy described in this article can be used as a tool within and across research teams to ensure that important aspects of statistical knowledge for teaching are not overlooked when research foci are chosen. It can also help researchers justify the focus for a study by connecting it to under-explored and high-priority aspects of teacher knowledge. As the taxonomy is applied across more collections of articles and those yet to be published and used to take inventory of studies via means like publisher dashboards and future literature reviews, it can more fully realize the purpose of presenting an aggregate portrait of the state of the field. It can also be used within research teams to reflect on their own work and decide if there are aspects of statistical knowledge for teaching they should address more extensively, just as teachers use taxonomies like Bloom's to reflect on and improve the questions they ask students. As the field continues to clarify and focus its research on statistical knowledge for teaching, we can progressively improve both theory and practice in statistics teacher education.

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APPENDIX A: TENTATIVE TAXONOMIC CLASSIFICATION OF SERJ RESEARCH ARTICLES

Taxonomy Category	SERJ Articles
Category 1: One Component of Subject Matter Knowledge	Common content knowledge focus: Canada (2006); Dollard (2011); Dolor & Noll (2015); Hobden (2014); Huber et al. (2024); Isoda et al. (2018); Kazak & Pratt (2017); Makar & Confrey (2005); Malaspina & Malaspina (2020) Specialized content knowledge focus: Biehler et al. (2017); Lee et al. (2016) Horizon knowledge focus: Bansilal (2014); Legacy et al. (2025); Magalhães & Magalhães (2021)
Category 2: Multiple Components of Subject Matter Knowledge	Casey & Wasserman (2015); Fergusson & Pfannkuch (2022); Frischemeier & Biehler (2018); Gómez-Torres et al. (2016); González (2021); Lesser & Winsor (2009); Noll (2011); Peters (2011); Ruz et al. (2021)
Category 3: Subject Matter Knowledge in the Context of Statistical Investigation	Bilgin et al. (2017); Gómez-Torres (2021); Gould et al. (2017); Green et al. (2018); Leavy (2006); Lee et al. (2014); Ubilla et al. (2021); Yilmaz et al. (2023)
Category 4: One Component of Pedagogical Content Knowledge	Knowledge of Content and Teaching: Carey & Dunn (2018); Ferguson et al. (2020); Groth & Bergner (2005); Leavy & Frischemeier (2022); Umugiraneza et al. (2018) Curriculum Knowledge: Groth (2008); Legacy et al. (2022)
Category 5: Multiple Components of Pedagogical Content Knowledge	Findley (2022); Gómez-Blancarte et al. (2021); Huey et al. (2018); Justice et al. (2017); Watson & Nathan (2010); Xu et al. (2020)
Category 6: Pedagogical Content Knowledge in the Context of Teaching Practice	Casey (2010); de Oliviera Souza et al. (2014); de Oliviera Souza et al. (2015); deSouza et al. (2014); deVetten et al. (2023); Groth (2017); Hammerman & Rubin (2004); Jaccobbe & Horton (2010); Leavy (2010); Makar & Rubin (2009); Pfannkuch (2006); Savard & Manuel (2016); Strayer et al. (2019); Verbisck et al. (2024)