

VARIABILITY IN STATISTICAL PROBLEM-SOLVING: THE CASE OF SWEDEN

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ABSTRACT

Understanding variability is key to statistical reasoning, yet research indicates that it receives insufficient attention in school mathematics textbooks. This study investigates how primary school textbooks for grades 4–6 (age 10–12 years) in Sweden present variability within the statistical problem-solving process. Using the Guidelines for Assessment and Instruction in Statistics Education and established variability practices, the analysis of two widely used textbooks reveals a strong emphasis on analyzing data, particularly tasks that involve reading data displays or calculating measures of central tendency. However, other practices, such as formulating statistical questions and interpreting results, are underrepresented. Tasks predominantly encourage implicit noticing and representation rather than deeper reasoning through explanation or investigation. Variability is predominantly introduced through descriptive statistics rather than as a fundamental concept for understanding data distribution and uncertainty. The findings underscore the need for textbooks to integrate all components of statistical problem-solving more evenly, thereby enhancing students' statistical reasoning and engagement with variability.

Keywords: *Statistics education research; Variability; School mathematics; Textbook analysis; Statistical problem-solving*

1. INTRODUCTION

Statistics is crucial for modern society. Understanding statistical concepts and methods is essential for making informed decisions in various industries, including medicine, finance, and social science. Given the importance of understanding statistics (Zieffler et al., 2008), as well as the consistent difficulties people have with statistical reasoning (Dolor & Noll, 2015), it is recognized widely recognized that the foundation of fundamental ideas of statistics should be laid in the early years of schooling (Meletiou-Mavrotheris & Paparistodemou, 2015; Nilsson, 2020).

Mathematics textbooks have long been recognized as essential tools for educators and students in teaching and learning mathematics (Remillard & Heck, 2014). Textbooks play a significant role in implementing standards and guiding instructional practice (Haggarty & Pepin, 2002) by presenting structured content, providing explanations, and offering tasks for exercises. However, as society and educational practice continue to evolve, it is crucial to continually examine the design of school mathematics textbooks to examine what learning opportunities these curricular materials afford students (Törnroos, 2005; Wijaya et al., 2015). By analyzing the statistical content and methods in two Swedish textbooks, the present study aims to provide insights into how mathematics textbooks for the compulsory school for students in grades 4–6, that is, students aged 10–12 years, offer students opportunities to learn statistics.

Variability is a key characteristic of statistics (Cobb & Moore, 1997) and central to the statistical problem-solving process (Bargagliotti et al., 2020; Franklin et al., 2007). Despite this, we observe that variability is largely absent in previous textbook research on statistics. One plausible reason for this could be that textbooks are limited in acknowledging variability in teaching statistics (Dierdorp et al.,

2017). However, part of the limitation may also be that research has not taken an explicit, critical analytical focus on how variability is addressed in primary school textbooks. In this study, we will take such a stance by applying the Guidelines for Assessment and Instruction in Statistics Education (GAISE) report¹ (Bargagliotti et al., 2020; Franklin et al., 2007) together with a framework about variability practices (Pfannkuch & Wild, 2004; Reading & Shaughnessy, 2004) to analyze how primary school textbooks in mathematics provide students with opportunities to learn statistics. In the present study, we focus on the dimension describing the statistical problem-solving process, henceforth referred to as the GAISE framework (Bargagliotti et al., 2020). This framework emphasizes context and the omnipresence of variability (Cobb & Moore, 1997) and structures the statistical problem-solving process into four components: “Formulate statistical investigative questions,” “Collect/consider data,” “Analyze the data,” and “Interpret the results” (Bargagliotti et al., 2020; Franklin et al., 2007). The framework of variability practices provides insight into how variability is addressed across these components. In this study, the GAISE framework is used to structure the components of the statistical problem-solving process, whereas variability practices are used as an analytical lens to examine the nature of students’ potential engagement with variability within each component. In this regard, the present study addresses the research question: How are statistical tasks in Swedish primary school mathematics textbooks distributed across the four components of the statistical problem-solving process and across the five variability practices?

2. PREVIOUS RESEARCH

2.1. REASONING ABOUT VARIABILITY

Traditionally, statistics education often overemphasizes measures of central tendency, such as the mean, at the expense of variability (Lehrer & English, 2018; Reading & Reid, 2004). Students frequently struggle to comprehend the role of variability due to limited exposure and engagement with this concept in the classroom. Many students initially perceive variability as error or noise rather than an essential feature of data analysis (Langrall et al., 2017). This limited understanding hampers their ability to interpret data and perform statistical reasoning effectively in comparing distributions or drawing statistical inferences (Nilsson, 2023).

Variability refers to the fluctuations within data and is central to statistical analysis. Variability is a core constituent in the teaching and learning of statistics, playing a fundamental role in helping students understand the nature of data, its central tendency, and distributions (Biehler et al., 2018). Recognizing and interpreting variability means moving from considering data as static points to understanding them as part of dynamic distributions. Variability must be understood not just as noise, but as an indicator of deeper patterns within data, enabling learners to differentiate between randomness (unexplained factors) and induced factors (systematic errors) (Wild & Pfannkuch, 1999). Peters (2011) showed that understanding variation involves recognizing its omnipresence, anticipating sources of variability, and interpreting how data fluctuate within and across contexts. She demonstrated that meaningful engagement with variability depends on reasoning that connects study design, data representation, and model-based interpretation throughout the statistical problem-solving process.

A variety of activities can help students learn about variability, including comparing distributions (Garfield & Ben-Zvi, 2005; Garfield et al., 2008; Zieffler et al., 2008), engaging in simulations (Nilsson & Eckert, 2023), and drawing statistical inferences in informal contexts (Makar & Rubin, 2014). Garfield et al. (2008) reflected on an activity involving students in analyzing histograms to determine which distribution had greater variability. Activities like these guide students to reason about variability by visualizing data spread and connecting these visualizations to measures such as the range and standard deviation. Thus, students can learn to interpret variability as both a distance from the centre and as an illustration of data spread. Working with random experimentation and simulation provides students with the opportunity to experience and get a sense of random variability (Fischbein, 1975).

¹ GAISE includes a two-dimensional framework with three developmental levels across four statistical problem-solving components.

Such experiences have been shown to encourage reflection on collecting and representing data and on how data is distributed due to randomness (Nilsson & Eckert, 2023).

Recognizing the omnipresence of variability and its importance in statistical reasoning (Langrall et al., 2017; Peters, 2011), alongside the documented challenges students face with this concept (Case & Jacobbe, 2018), it is widely acknowledged that the groundwork for understanding variability should be laid in the early years of schooling (Makar, 2016; Paparistodemou & Meletiou-Mavrotheris, 2008). Textbooks are heavily integrated into daily classroom instruction (Mullis et al., 2012) and are often assumed to have a considerable impact on student outcomes (van den Ham & Heinze, 2018). Therefore, a thorough examination of how primary school textbooks introduce and develop the concept of variability is essential to understand their potential influence on students' learning in statistics and to identify gaps or inconsistencies in these early educational resources.

2.2. TEXTBOOK RESEARCH IN STATISTICS EDUCATION

Despite variability being a fundamental characteristic of statistics, it often takes a backseat to measures of centre in the teaching of statistics (Gould, 2004; Shaughnessy, 2007). This can partly be attributed to a lack of emphasis on variability within curriculum materials such as textbooks (English & Watson, 2015; Reading & Shaughnessy, 2004). In the following section, we will report that it has similarly received little attention in research on textbook analyses.

Jones et al. (2015) analyzed the statistical content of five U.S. textbook series written for elementary students in grades 1–5. The researchers used the GAISE framework report to examine 17,688 pages and code 7,445 statistical tasks to determine the relative emphasis on the components of the statistical problem-solving process. The researchers found that most statistical tasks focused on the statistical problem-solving component of “Analyze the data.” In four of the five series, relatively little attention was given to the other three components. “Analyze the data” was classified into one of four analysis activities: “Read a display,” “Perform a mathematical calculation,” “Construct a display,” and “Use other statistical reasoning.” Moreover, most tasks in “Analyze the data” required students to “Read a display,” such as a table or a diagram, and “Perform a mathematical calculation.” The researchers provided a task example of “Read a display” as: Which sport(s) team has the fewest teams? An example task for “Perform a mathematical calculation” was: A black bear weighs 25 pounds more than a gorilla. Use the information in the table (which includes the gorilla’s weight) to find how much a black bear weighs (Jones et al., 2015, p. 7). From the analysis performed by Jones et al. (2015), it is not clear how these two tasks are specific to statistics according to the GAISE report. In other words, it is not clear how the tasks move beyond simple arithmetic to address variability, which is a key feature of statistics (Biehler et al., 2018).

Jones and Basyal (2019) reported results similar to Jones et al. (2015) when determining the relative emphasis on the components of the statistical problem-solving process in textbooks. They examined 1,755 tasks across 35 Nepalese textbooks for grades 4–10 and found that, although almost all tasks required students to analyze data, the other components were rarely addressed. Moreover, whereas Jones et al. (2015) observed that most tasks in “Analyze the data” required students to read a display and perform a mathematical calculation, the study does not establish clear conditions for what counts as a statistical task; nor does it specify how those conditions should relate to the concept of variability.

Büscher (2022) and Landtblom (2023) showed how textbooks tend to prioritize descriptive statistics in relation to reading a display and performing mathematical calculations of measures of centre. Büscher (2022), analyzing three German grade 6 textbooks, found that less space was allocated to more conceptually demanding tasks like interpreting models or analyzing and reflecting statistical arguments in favour of interacting with diagrams and calculations of measures. Landtblom (2023), analyzing seven Swedish textbook series for school years 4–6, found a varied focus on measures of centre; of all tasks analyzed, 62% were about the mean; 19% were about the median; and 18% were about the mode. These findings align with previous assertions that textbooks tended to overemphasize measures of centre (cf Shaughnessy 1997).

However, despite the importance of variability in statistical reasoning, we observed that variability is analytically underrepresented in previous textbook research on statistics. Although prior textbook studies have provided valuable insights into the distribution of statistical tasks across problem-solving components, they typically do not make explicit how variability is embedded within these tasks.

Consequently, it remains unclear whether, and in what ways, students are given opportunities to engage with variability as a core statistical idea. To this end, we focus on how aspects of variability practices (Reading & Shaughnessy, 2004; Wild & Pfannkuch, 1999) are expressed in the problem-solving process.

3. ANALYTICAL APPROACH

Variability is not simply one statistical concept among others; it is a defining characteristic of statistical reasoning and permeates the entire statistical problem-solving process. The GAISE report reflected this by positioning variability as a foundational idea addressed differently across the components of statistical inquiry (Franklin et al., 2007). However, identifying the statistical problem-solving component addressed by a task does not, in itself, reveal *how students are invited to engage with variability*. Two tasks may both belong to the “Analyze the data” component, for example, while providing fundamentally different opportunities for reasoning about variation: one may ask students merely to identify maximum and minimum values, whereas another may ask them to explain why the observations differ or to investigate possible sources of that variation. Thus, to address this limitation, we complement the GAISE framework with the notion of variability practices, drawing on Wild and Pfannkuch’s (1999) framework of statistical thinking and, specifically, their notion of consideration of variation. We use variability practices to refer to analytically distinguishable ways in which textbook tasks afford students opportunities to engage with variability in data.

Wild and Pfannkuch’s (1999) framework of statistical thinking has been taken up in GAISE II (Bargagliotti et al., 2020) as part of the core structure underpinning its recommendations for what should be taught as statistics in K–12 education. This provides a strong conceptual justification for using the two frameworks in combination: they are not competing perspectives but are closely connected accounts of statistical thinking and learning. To Wild and Pfannkuch (1999), variation was not treated as an isolated topic to be introduced at a particular point in an investigation. Rather, it was a pervasive feature of statistical inquiry that needed to be anticipated, recognized, described, explained, investigated, and used in reasoning. This makes their notion of variation particularly appropriate for textbook analysis, where the central question is not only whether variability is *present* in a task, but what students are actually afforded the opportunity to do with it. Indeed, a task can contain variable data without necessarily creating an opportunity for students to reason about that variability.

Reading and Reid (2004) found that students’ responses revealed different levels of consideration of variability: not considering variation, weak consideration of variation, emerging consideration of variation, and strong consideration of variation. Although our study does not examine the quality of students’ engagement with variability, we take their findings as an impetus to explore how textbooks allow students to engage with or address variability. By focusing on affordances, we do not analyze what students actually do, but rather what the tasks make *possible* for students to do in relation to variability.

3.1. STATISTICAL PROBLEM-SOLVING PROCESS

Before elaborating further on aspects of variability practices relevant to the present study, we describe the four components of the statistical problem-solving process (Bargagliotti et al., 2020). Significantly, a major premise of this study is that the proper treatment of the statistical problem-solving process is achieved when all components are meaningfully addressed. This view is based on the perception that the statistical problem-solving process is a complete, cyclical process (Pólya, 1957; Watson & English, 2018; Wild & Pfannkuch, 1999). However, we acknowledge that some of these problem-solving stages may receive less emphasis depending on the students’ educational level. Nevertheless, we argue that even at the primary level, curricular materials should provide structured exposure to all components (Bargagliotti et al., 2020; Makar, 2016).

Formulate statistical investigative questions Formulating statistical questions is often considered to be the initial component of the statistical problem-solving process. Students should be able to differentiate between a statistical question and a non-statistical question. A statistical question anticipates variability (Bargagliotti et al., 2020; Franklin et al., 2007). For example, given data on monthly rainfall over City X in the year 2024, a non-statistical question would simply ask for a specific

measure (value) such as “What was the total amount of rainfall in City X in October 2024?” This question has a single, definite answer. By contrast, a statistical question that anticipates variability might ask for a description of the expected monthly rainfall for the fall months, for example, “Describe the monthly rainfall that you would expect during the fall months in city X in the coming years.” Significantly, embedded in formulating a statistical question is an awareness of the possibility of different outcomes or answers, that is, anticipation of variability.

The process of formulating a statistical investigative question is not always straightforward. Arnold and Franklin (2021) distinguished between formally posed questions and spontaneously asked questions. Within the “Formulate statistical investigative questions” component of the statistical problem-solving process, the question posed is regarded as a statistical investigative question and often serves to guide the investigation and determine the data to be generated. In contrast, spontaneously asked questions typically emerge as one interacts with the data. In the “Formulate statistical investigative questions” component, particularly when working with secondary data (i.e., data that students have access to but which they did not collect themselves), one may pose “interrogative investigative questions” to clarify aspects such as the population or group of interest, and whether the data is sufficient to answer the question at hand (Arnold & Franklin, 2021; Bargagliotti et al., 2020).

Collect/consider the data The second component, collecting/considering the data, speaks to understanding both the data and the processes involved in data generation. As Cobb and McClain (2004) argue, the validity of statistical analyses is inherently tied to the quality of the data on which they are based; therefore, students must understand the relationship between the soundness of data collection methods and the legitimacy of any conclusions drawn from the data. Recognizing that data is rarely flawless and often contains errors or missing values is essential at this stage. In “Collect/consider the data,” students are expected to reflect on the data collection process, such as selecting representative samples and determining whether variability is induced by randomness or by systematic (measuring) errors (Bargagliotti et al., 2020). When measuring the height of students in classroom Y, the difference in the heights reflects the true variability; however, random variability can also arise from chance factors such as how the measuring tape is positioned or read. This type of error tends to be more prominent when analogue instruments are used. In contrast, a consistent bias in the measuring instrument itself would produce systematic variability.

Analyze the data Data-analysis tasks are designed to help students develop an understanding of variability within a data set. For instance, the question: “Looking at the diagram of monthly rainfall data for City X, which month has the highest rainfall amounts?” presents a meaningful opportunity for analyzing data. The initial analysis step involves examining tabulated data and graphical displays to describe and explore the pattern (distribution) of monthly rainfall to determine the month receiving the highest precipitation (Bargagliotti et al., 2020). This allows for the possibility of deeper reasoning and discussion about the city’s rainfall distribution, an aspect that can be used to answer questions such as “How do the rainfall patterns compare across seasons, and how can this information be valuable when planning a vacation?” Analyzing the data also allows students to revisit the original question that promoted data collection, reorganize the data, and potentially uncover new insights and discoveries. Researchers seem to attach several processes to this component. Friel et al. (2001), for instance, distinguished three types of tasks related to analyzing the data: “reading the data,” “reading between the data,” and “reading beyond the data.” Reading data concerns tasks that focus on a single outcome. Reading between data concerns comparing outcomes, that is, looking both across and within data sets, and reading beyond the data involves making predictions and inferences from the data. Jones et al. (2015) included tasks requiring the performance of a mathematical calculation and constructing a display, alongside tasks requiring reading a display in “Analyzing the data.” Actions requiring the data to be summarized (e.g., via measures of centre) or for part of the data to be delineated from the rest manifests sense-making with variability, which naturally maps into analyzing the data. A key aspect in this component of the statistics problem-solving process is the types of questions posed to the data. For example, it is not unexpected that single-answer questions will characterize tasks focusing on reading the data or reading a display, whereas tasks that seek to explore the data generally will focus on data involving several data points or a range of values.

Interpreting results The interpretation of results entails using the statistical evidence from analyses to answer the statistical investigative questions and communicate results through structured responses (Franklin et al., 2007). This may include making statements about the group or population from which the data were collected, while recognizing that conclusions are limited to these groups and cannot be generalized to other groups, and comparing results for different conditions in an experiment (Bargagliotti et al., 2020). For example, analyzing rainfall data in City X may reveal three major rainfall periods, thus revealing a multimodal pattern of rainfall. From a societal perspective, this could prompt discussions on how rainfall patterns impact day-to-day life, such as planning outdoor activities, agricultural work, or public transport schedules. It may be appropriate to use statistical concepts such as measures of centre and deviation in analyzing and interpreting data. Mooney et al. (2014) highly evaluated students who were able to give specific explanations of variability in relation to a specific measure of centre. In contrast, students who did not refer to exact measurements and focused on either centre or spread without connecting the two were perceived as manifesting a weak consideration of variability. Konold et al. (2014) posited that this was akin to perceiving data as aggregate, as contrasted to individual entities. Thus, the consideration of variability is realized in the reasoning and argument used in answering the statistical question

3.2. VARIABILITY PRACTICES

This study investigated how selected primary school textbooks characterized opportunities for students to consider variability within the statistical problem-solving process. We focused on the fundamental type of thinking, “consideration of variability” (Wild & Pfannkuch, 1999). Based on the works of Wild and Pfannkuch (1999) as well as Reading and Shaughnessy (2004), we developed a set of five analytical codes for noticing, measuring, explaining, investigating, and representing variability to operationalize how variability may be engaged within textbook tasks and thus to characterize the opportunities afforded to students to engage with variability. Although the labels used (i.e., noticing, measuring, explaining, investigating, and representing) are partly derived from the language of Wild and Pfannkuch (1999), we found them descriptive in the sense that they illustrated the process of engaging variability. In this study, we do not isolate controlling variability (e.g., Ko & Lee, 2011), considering it as a constituent of explaining variability (Wild & Pfannkuch, 1999). The analytical focus is therefore not only on the presence of statistical content, but also on the forms of engagement with variability that tasks make available. To this end, we draw on the GAISE framework in combination with aspects of variability practices identified by Wild and Pfannkuch (1999) as well as Reading and Shaughnessy (2004). We now turn to the variability practices framework.

Noticing Noticing variability refers to initial, informal engagement with data that supports awareness of variability within a distribution. Noticing tasks are designed to help learners see that the values of a statistical variable vary across cases (Garfield et al., 2008). Such tasks draw attention to how data points are distributed across a variable, for example, how the outcomes of 100 throws of two dice are distributed over the variable, ‘the sum of two dice.’ Noticing variability typically involves seeing differences among individual data points. Situations that afford simple comparisons of observed values have been considered appropriate for this purpose (English & Watson, 2015; Lehrer & English, 2018). For instance, calling students’ attention to the precipitation of a certain month in City X across all twelve months of the year provides an example of a noticing task because the aim is to allow the students to notice that the values vary.

Measuring Measuring variability involves applying formal, statistical methods to characterize how data vary, rather than simply noticing variation. This includes the use of statistical measures such as range and standard deviation (Garfield et al., 2008). For categorical data, variability may be described by how observations are distributed among different categories rather than from a fixed point such as the mean (Kader & Perry, 2007; cf. Olande, 2014). A central idea in measuring variability is the distinction between “signal” and “noise.” Signal refers to stable features of a distribution, such as modal clumps, mode, mean, and median, whereas noise refers to random variation (delMas & Liu, 2005;

Konold & Pollatsek, 2002; Konold & Kazak, 2008). In the case of statistical summaries, such as the mean or the regression line, signal is perceived as "...indicators of stable properties of a variable system..." (Konold & Pollatsek, 2002, p. 262). Acknowledging measures of centre as indicators of stable properties of a variable system underscores their intimate, inseparable connection to the notion of spread (Konold & Pollatsek, 2002). Lehrer et al. (2007) adopted a perspective of measurement that includes processes that reveal variation caused by measurement errors or instrument limitations. Similarly, regression analysis can make visible deviation from a model, thereby highlighting variability around a regression line (Dierdorp et al., 2017). Using City X's monthly rainfall, computing the difference between the highest and the lowest recorded rainfall amounts for a given year is an exercise in measuring variability.

Explaining Explaining variability is the act of seeking to understand why different phenomena behave differently and is considered a motivating factor for statistical inquiry. Students are encouraged to make sense of the causes of variation by considering contextual factors and distinguishing between random and induced elements (Ko & Lee, 2011; Wild & Pfannkuch, 1999). Importantly, Wild and Pfannkuch (1999) treated explaining variability as directly linked to controlling it: understanding why variability occurs is what motivates efforts to reduce or account for it. In this sense, the practice of explaining variability encompasses what some might separately label as "controlling variability" because the act of explaining naturally leads to identifying which sources of variation are induced, thus controllable, and which are random (Ko & Lee, 2011). Explanations may range from simple observations like noticing that "the data varied and they did not seem to follow a pattern" (Mooney et al., 2014, p. 3) to more sophisticated analyses that incorporate precise measurements of variability or connections between measures of centre and dispersion. However, students often begin with informal language and tend to focus on local and within-dataset variability rather than broader, global trends (Ben-Zvi, 2004). Explaining variability typically occurs on already collected data or data that has been analyzed, and where fundamental concerns are on reason-giving responses.

Investigating Investigating variability involves systematically finding out if variability is due to unexplained or induced factors (Wild & Pfannkuch, 1999). Such investigations seem more viable when working with realistic contexts where causal explanations are within reach (e.g., Dierdorp et al., 2017). In non-authentic contexts, investigative mindsets can be supported by allowing students to encounter such task items that require them to delve deeper into the data, allowing for identifying patterns or relations in stable data points. Konold and Pollatsek (2002) contended that even processes with stable data points, such as the averages, also have a variable factor. Recognizing this relationship can support a more robust evaluation of the distribution of observations. As an example with rainfall for City X, students might compare the specific cluster of months to the average precipitation to investigate whether deviations reflect meaningful patterns. Thus, investigating variability is characterized by procedural engagement with data, such as testing, manipulating, or reflecting on sampling and measurement decisions.

Representing Representing variability involves the use of tables, graphs, and symbolic representations to describe, summarize, and communicate the features of the data. These representational forms are central to statistical reasoning and should be used in answering the posed statistical investigative questions (Bargagliotti et al., 2020). Symbolic measures, for instance, the mean or the median, can help students reflect on how data vary around a central value. At the same time, models such as regression lines represent broader trends around which individual variation can be interpreted. By integrating tables, graphics, and symbolic representations, students can build flexible statistical reasoning. Tables provide precision; graphics highlight patterns and trends, and symbols allow for generalization (Cooper, 2018).

These five practices may not offer an exhaustive account of all dimensions of variation discussed in the literature (e.g., Ben-Zvi, 2004; Peters, 2011). However, they represent recurring ways in which variability becomes an object of attention within the statistical problem-solving cycle.

4. METHOD

To achieve the research goals, the present study employed a qualitative case-study design supported by content analysis, with the unit of analysis being the individual task item (Jones & Jacobbe, 2014). In line with Stake's (1995) emphasis on understanding particular cases in depth, the two mathematics textbooks are treated as separate cases situated within the Swedish curriculum context. This approach supports a close examination of how opportunities for engaging variability are organized through the tasks provided in each textbook. By analyzing how variability and components of the statistical problem-solving process are conveyed in curriculum material, our analysis offers insight into how textbook design may shape students' opportunities to develop statistical reasoning.

4.1. TEXTBOOK SAMPLE

The Swedish market for school textbooks is relatively unregulated: schools are responsible for selecting and providing students with necessary learning materials, including textbooks, in accordance with the Swedish Educational Act (SFS 2010:800). Thus, a set of criteria was established to guide the selection of textbooks for analysis. These criteria were informed, in part, by a policy document from the Swedish Association of Educational Publishers (Läromedelsföretagen, 2026). According to this document, school textbooks should enhance students' opportunities to achieve national curriculum goals, support self-directed learning and conceptual understanding, facilitate smooth progression to higher levels of learning, and stimulate, guide, challenge, and deepen students' learning. Textbooks produced by member publishers should also enable teachers to assess students' knowledge levels (Läromedelsföretagen, 2026).

Based on these considerations, two textbooks used in both private and public schools and aligned with the current curriculum for compulsory school, preschool class, and school-age education (typical age 10-12 years) were selected primarily through convenience sampling: *Mera Favorit Matematik 5B* (henceforth FM) and *Matte Direkt 5B* (henceforth MD) for analysis (Table 1). Although the sampling approach was convenient, an additional check was performed to ensure that the selected textbooks reflected and met curriculum requirements. Although reliable public information on textbook sales volumes is lacking, our experience of collaborating with teachers suggested that these textbooks were among the most used in Swedish schools. Moreover, they featured in previous research on Swedish mathematics textbooks (Bråting & Kilhamn, 2022; Bråting et al., 2019; Landtblom, 2023). Selecting textbooks from different publishers also provides variation in authorship, thereby offering a complementary perspective on how variability is addressed (e.g., Bråting et al., 2019).

Table 1. Selected textbooks, detailing authors and publisher

Textbook series	Authors	Publisher	Abbreviation
Mera favorit matematik 5B	Katariina Asikainen Kimmo Nyrhinen Pekka Rokka Päivi Vehmas	Studentlitteratur	FM
Matte direkt	Synnöve Carlsson Pernilla Falck	Sanoma Utbildning	MD

The present study focused on task items requiring an active response from students; introductory explanations and worked examples were excluded. Focus was particularly placed on the statistics problem-solving process and variability addressed in this process in a Swedish school context. Given this observation, it is worth noting that neither the statistics problem-solving process (i.e., GAISE) nor measures of variability (e.g., range and standard deviation) are explicitly highlighted in the formal curriculum for grades 4-6 (age 10-12 years) in Sweden (Skolverket, 2024). In particular, at the end of

year 6, the students are expected to have covered the following aspects in statistics and probability, which are treated as a single strand: tables and diagrams for describing results from statistical investigations both with and without digital tools; interpreting data in tables and diagrams; and measures of centre—mean, median, and mode, including how these are utilized in statistical analyses.

By making explicit how two common primary school mathematics textbooks provide students with opportunities to encounter and make sense of a statistics problem-solving process and variability, this study can inform curriculum developers and textbook constructors. It can also help teachers in higher grades align their teaching in statistics to their students' statistical dispositions and prior knowledge. Additionally, by comparing textbook analyses from different countries, educators and researchers can gain a broader understanding of the topics covered and instructional strategies used, in relation to the educational system, culture, and values of a country (Son & Diletti, 2017).

Favorit Matematik 5B FM includes five chapters, and statistics is part of chapter 2, “Percent, statistics, and probability.” This chapter is organized into four sections, labelled: (a) fractions, decimal numbers, and percentages; (b) tables and diagrams with and without digital aids; (c) mean, median, and mode; and (d) probability and simulation of probability events. Each section starts with a general explanation of the key concept and/or procedures of the section, including worked-out examples. Next follows a set of tasks. The chapter ends with summary questions at different levels entitled “What have I learnt?”

Matte Direkt 5B MD includes four chapters (5–8), with chapter 6 titled “Statistics.” This chapter is organized into sections that primarily cover statistical content. Each section begins by explaining the key concept, illustrated by a worked-out example. The sections are (a) tables; (b) bar-charts; (c) pie-charts; (d) histograms; (e) negative numbers; (f) line charts; and (g) mean, median, and mode. There is also a section (h) that includes mixed exercises with task items with similar content from sections (a)–(e). Section (h) presents a “palette” containing tasks that promote problem-solving, conceptual understanding, reasoning, collaborative activities, and true or false task items. At the end of this sequence of tasks, there is a section titled “What can you do now?” that functions as a diagnostic text. Based on the results from this test, students can choose to continue with similar tasks or move on to other tasks that require less scaffolding.

4.2.METHOD OF ANALYSIS

The analysis followed the principles of content analysis (Holsti, 1969; Hsieh & Shannon, 2005), guided by the GAISE framework and the framework of variability practices developed for this study. Textbooks were first examined for their statistical content in order to identify chapters with a high concentration of statistical tasks. Within these chapters, each sub-item was treated as an individual unit of analysis (Jones & Jacobbe, 2014): a task comprising items (a), (b), (c), and (d), for instance, was coded as four separate units rather than as a single task. Nevertheless, the reported frequencies should be interpreted as the distribution of coded tasks rather than independent observations. Although such items share a common task context analogous to a testlet structure in educational measurement (Wainer & Kiely, 1987), coding them individually made it possible to aggregate items across tasks and sections irrespective of their originating context. This, in turn, allowed systematic patterns in statistical demand to be traced across the textbooks as a whole, rather than being confined to the level of individual tasks (cf. Organization for Economic Co-Operation and Development [OECD], 2023).

We analyzed the selected chapters in four steps. Firstly, we distinguished statistical task items from non-statistical task items. Statistical task items centred around a statistical topic (Jones et al., 2015) requiring students to organize, represent, interpret, summarize, and reason about a set of values. For instance, tasks involving survey questions and tables that presented the resulting data were coded as statistical task items. Task items asking exclusively for determining percentages or fractions from charts were excluded, as were number strings with no reference to statistical concepts such as mean or median. Task items related to probability and combinatorics were also excluded unless applied in a statistical context, such as data collection for an experiment. FM included a total of 298 task items meeting these criteria; MD included 296 task items that met the criteria. Secondly, we examined how task items aligned with the components of the statistical problem-solving process according to the GAISE

framework (Bargagliotti et al., 2020). We used Friel et al.'s (2001) distinction between reading the data, reading between the data, and reading beyond the data to differentiate task items in the component of "Analyze the data." Thirdly, we looked into how variability was addressed within each statistical problem-solving component using the five variability practices outlined above (Ben-Zvi, 2004; Reading & Shaughnessy, 2004; Wild & Pfannkuch, 1999). Task items that were coded as statistical tasks but did not fit into any dimension of the variability practices were coded as "others." An example of "other" was a task item asking students to consider the structure of a diagram without calling the students' attention to account for or describe variability. Fourthly, the two researchers discussed and reached consensus on task items whose categorization was uncertain or problematic.

A case illustrating the four-step process is given for item A in Table 2, for which students were provided with statements describing characteristics of different graphical instruments. First, the task was identified as a statistical task because it concerned the representation of data points and required students to distinguish characteristics of statistical representations. Second, although students were not asked to think about what data were needed, how they should be collected, or which measurements would be appropriate, the task offered students the opportunity to reflect and show understanding that variability was to be represented using appropriate displays (Bargagliotti et al., 2020). Although no actual graphical display was at hand, the ability to identify and interpret characteristics of graphical data representations was perceived as a precursor to graph comprehension (Friel et al., 2001), which is a central part of "Analyze the data." In step three, the task was coded as "other" within the variability practices because it did not engage students in considering variability in any data. Step four involved a discussion between the two researchers. As the analysis proceeded, it became evident that some task items did not squarely map onto a single component of the statistical problem-solving processes. For such tasks, we categorized them according to the part of the statistical problem-solving process in which they were primarily intended to engage students.

Table 2. A task illustrating the coding process

Task A (MD: 20 p. 77) Read the statements provided in the table on your right. Which of them describes: a) Bar chart b) Histogram	A: It has two axes with numbers B: It has wider bars C: There are arrowheads on both axes D: On one of the axes is shown the categories
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Two researchers in statistics education—the authors of the present paper—did the coding. The process was two-pronged, combining individual and collaborative coding. It was carried out in two stages: to code for the GAISE framework, then for the variability practices. For coding according to the GAISE framework, segments of tasks were first coded individually. The researchers then met to review and discuss the coding to ensure consistency and to achieve consensus on task items where there was disagreement. This procedure was repeated until all tasks were coded. For coding variability, the first author initially coded a segment of the tasks. After coding each segment, the two researchers met to review and discuss the coding to reach consensus. The process was repeated for variability practices until all task items were coded regarding variability.

5. RESULTS AND ANALYSIS

The results are organized in two parts. We first present the distribution of tasks across the four components of the statistical problem-solving process according to the GAISE framework, followed by how variability practices are expressed across the components. The tasks are then presented in chronological order of their appearance within each component. To support traceability, source references are provided in brackets. For example, the source reference for Task 1 (MD: 9 p. 51) indicates that the task labelled Task 1 was selected from the textbook MD and can be traced back to the source material as task number 9 on page 51.

Although the two textbooks showed small differences in how the tasks were distributed, they displayed a broadly similar overall distribution of task items. See Figure 1. A majority of tasks were concentrated in the component of “Analyze the data” (MD 89%; FM 82%). The remaining three components received substantially less emphasis. Tasks involving “Collect/consider data” accounted for 10% in MD and 16 in FM. Considerably less emphasis was placed on the components of “Formulating statistical investigative questions” and “Interpreting the results.” The textbooks diverged somewhat in how they allocated tasks across the statistical problem-solving process. Compared with FM, MD contained slightly more tasks classified as “Formulate statistical investigative questions” (MD 1.4%; FM 0.7%). For “Interpret results,” the pattern was reversed, with no items coded in this category for MD (0%) compared with FM (2%).

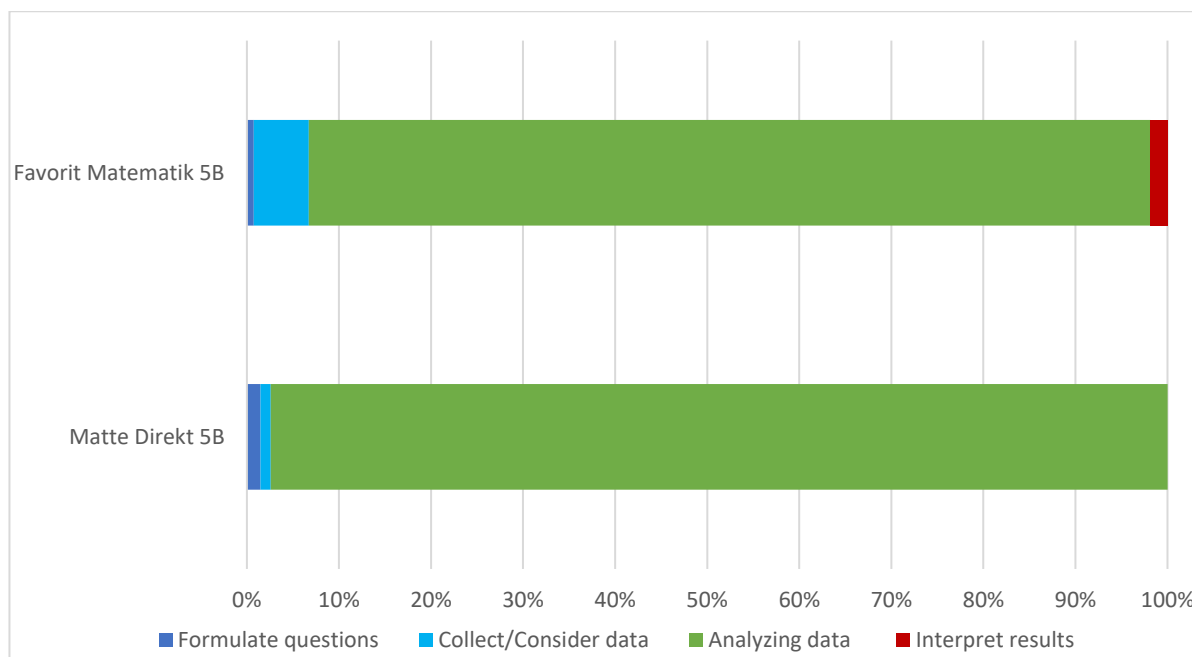


Figure 1. Distribution of task items according to the GAISE framework as a proportion total number of items for respective textbook

5.2 EXPLORING VARIABILITY PRACTICES ACROSS COMPONENTS

A closer analysis of the textbooks revealed that task items categorized as “Analyze the data” offered a wide range of opportunities for students to engage with the five variability practices. Table 3 shows that both textbooks provided students with opportunities to engage with all five variability practices, although their frequencies differed substantially. “Noticing” (NOT) was the most frequently occurring variability practice, followed by “Representing” (REP) and “Investigating” (INV) variability. Under “REP” we included tasks in which students calculated measures of central tendency (e.g., the mean or median) because these tasks inherently invited consideration of variability around a central value, even when variability was not explicitly referenced.

Formulate statistical investigative questions A total of six task items were coded as formulating statistical investigative questions: four task items from MD and two task items from FM. Common to these tasks was posing a question *to* the data, rather than *for* the data. When posing a question to data, the data is already given (e.g., Tasks 1 & 2 in Table 4). Notably, these “questioning” tasks (Arnold & Franklin, 2021) often involve elements of analyzing the data.

Table 3. Variability practices distributed within the statistical problem-solving process

GAISE components	Textbook	NOT	MEA	EXP	INV	REP	Others	Total
Formulate statistical investigative questions	MD	0	0	4	0	0	0	4
	FM	0	0	0	2	0	0	2
Consider/collect the data	MD	1	0	0	0	12	17	30
	FM	5	0	0	13	26	3	47
Analyze the data	MD	157	3	2	16	56	28	262
	FM	83	4	1	12	93	51	244
Interpret the results	MD	0	0	0	0	0	0	0
	FM	0	0	4	1	0	0	5
Total		246	7	11	44	187	99	594
%		42	1	2	7	31	17	100

Except possibly for Task 3 (see Table 4), no tasks required students to formulate a statistical investigative question for the data; students were not explicitly required to pose a question and then consider what data would be necessary to answer it. However, Task 3 can be regarded as presenting students with examples of survey questions to choose from rather than requiring them to formulate a statistical investigative question themselves. Nevertheless, we regard the task as a relevant step in educating students to formulate a statistical investigative “question-for-data.” Although no data is provided, the task presents a situation in which conducting an investigation on the chosen subject requires at least an implicit statistical investigative question.

We therefore perceive these tasks, despite their open-ended nature, as having great potential in relation to Arnold and Franklin’s (2021) criteria for statistical investigative questions. Specifically, for Task 1 and Task 2, both the population or group and the variable of interest are clearly defined. The intent of the query is also explicit, whether it requires a summary description, a comparison, or an analysis of associations between variables. Moreover, the question is interesting and worth investigating, and its answer can be derived from the data. Finally, the question allows for analysis of the entire group. In the case of Task 3, its open-ended design implicitly asks students to define the population and the variable of interest. The task forces students to reflect on the intent of a statistical question, evaluate whether it is worthwhile to investigate, and recognize that answers need to be settled by data collected from the whole group or a representative sample. Because formulating a well-posed question without guidance imposes high cognitive demand, this task highlights why most students in years 4–6 may still require teacher assistance to formulate statistical investigative questions. Although Task 2 is open-ended, it includes a deterministic question as an example, thereby providing an opportunity to discuss the distinction between deterministic and statistical investigative questions. Task 2 is perceived as having the potential to prompt such a discussion. However, because Task 1 and Task 2 include data for analysis, it is also possible to perceive these tasks as transitional between “Formulating statistical questions” and “Analyze the data.” Given the focus on formulating a statistical question, we perceived the classification of the tasks in this category as more natural, given the cognitive demand imposed by the tasks.

Table 4. Tasks promoting variability practices for “Formulate statistical investigative questions”

Task 1 (MD: 9 p. 51)								Travel type		Number of students		
Look at the chart showing how students travel to school: a) write two questions with answers for the chart.								Bike		6		
								Walk		10		
								Bus		5		
								Car		3		
Task 2 (FM: 3 p. 94)												
Construct a bar chart from the information below												
Months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Expenses	8000	7600	7000	7300	8500	9200	9100	7800	8100	8300	9200	9800
a) Study the bar chart. Come up with three questions about the chart. <i>For example</i> , “Which months did the budget exceed SEK 9000?”												
Task 3 (FM: 3 p. 95)												
Conduct an investigation and collect statistical data												
a) Choose a subject for your survey. - What is your favorite color? - What is your favorite dish? - What is your favorite animal? - What is your shoe size?												

Concerning variability, Task 1 was characterized by the variability practice of “explaining” variability. Coding for explaining variability within the “Formulate statistical investigative questions” component was guided by the assumption that teaching and learning aim to promote statistical thinking. In Task 1, students are provided with opportunities to pose questions that allow them to delve deeper into understanding the fluctuations in the data and to seek reasons for the differences in the selection of travelling modes. Task 2 was categorized as “investigating” variability. Significantly, the task involved investigating the months meeting a given condition that can be considered stable, namely, SEK 9000, onto which there are other fluctuating values. Consequently, the stable reference point makes the variability around it visible and investigable.

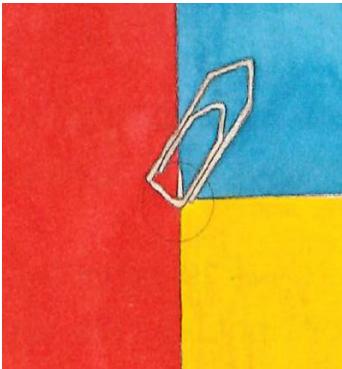
Task 3 is an example of a task that has the potential to engage students in all components of the problem-solving process. The nature of the survey topics signals to students that the responses will vary. As a result, the task has prospective noticing qualities, given that the students are prepared to anticipate variability before the data is collected. However, because no data is provided, the anticipated variability is more appropriately characterized as investigating rather than noticing (Wild & Pfannkuch, 1999).

Collect/Consider the data About 13% of all task items (77 of 594) coded from the textbooks fell under the component of “Collect/consider the data.” In MD, 30 items out of 296 (10%) and in FM, 47 items out of 298 (16%) belonged to this component. Although comparatively less frequent, these tasks represented the second most common component in both textbooks, indicating some attention to data generation and representation. For secondary data, there seems to be a focus on teachers helping students to pose data collection questions that are necessary before answering the statistical investigative question. These data collection questions can help students to deal with issues of data type, such as numerical and categorical data, and how the data can be displayed in visual representations (Bargagliotti et al., 2020). Arnold and Franklin (2021) considered the practices of using appropriate

tools strategically and attending to precision as crucial for the process of collecting or considering data (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Table 5, containing Tasks 4–6, illustrates the various occurrences of variability practices when collecting and considering data. Task 4 requires students to conduct a statistical investigation to potentially answer an investigative question. In particular, the task requires the students to reflect on the sample and how the data is to be recorded. By inviting the students to reflect on the sampling process and subsequent data presentation, the task exemplifies an intention to enable students to focus on investigative aspects of data collection, thus reducing bias errors such as induced errors (Wild & Pfannkuch, 1999). Task 5 provides a clear case of “Collecting data.” By recording repeated trials, students produce or generate and record data. In this process, the students are provided with the opportunity to notice variation in the outcomes.

Table 5. Tasks promoting variability practices in “Collect/consider data”

Task 4 (FM: 3b p. 95). 3. Conduct an investigation and collect statistical data (see, Task 3) b. Think about who you’re going to interview and how you’ll record their answers
Task 5 (FM: 2a p. 95). Investigate using a thumbtack and paperclip. Push the thumbtack up from underneath so that the point sticks up and place the paperclip on it as shown in the picture.  a. Spin the paperclip 10 times and write down in the table which color it stops on each time.
Task 6 (FM: 3a p. 103) Children in a class listen to music and award score points (as illustrated below) 8 7 9 6 8 7 9 5 8 9 a) Write all the score points in ascending order.

Task 6 (Table 5) is also representative of considering data. By asking students to order data points, the task highlights variability as the simple differences between values (English & Watson, 2015; Lehrer & English, 2018). Bargagliotti et al. (2020) posit that interrogating a set of data to understand the context of the variable and how they may relate to the statistical investigative question belongs to

“Collect/consider the data.” We coded this task, and similar ones, under “Collect/consider the data” because it concerns making variability visible through ordering—a precursor to guiding analysis and interpretation.

Of the 77 items (30 from MD and 47 from FM) belonging to “Collect/consider the data,” 36 (47%) focused on diverse aspects of visual displays using data already collected. Often, students are asked to represent data provided by the textbook using a particular type of representation (e.g., Task 5a, Task 14 part 3). Only 5 tasks (2 from MD and 3 from FM) involved students selecting the form to represent data. It was also observed that all 16 items considered as “Other” were from MD, and all 13 items considered as investigating variability were from FM. A closer look at the tasks from MD indicated that although these tasks do not deal directly with variability, they tend to deal with conceptual aspects of visualization, such as characteristics of the different types of graphical representations and aspects to consider in their construction/creation. The tasks from FM on investigating were characterized by aspects that challenged the students to focus on aspects of data collection and how these might impact the validity of the statistical results. For example, Task 4 asked students to reflect on the sample of students to be interviewed, and Task 14 provided a set of questions that could be used for statistical investigation.

Analyze the data Overall, 85% of all statistical task items were classified within the “Analyze the data” component. Within this component, approximately 29% (149) of the 506 items involved reading the data, approximately 69% (347) involved reading between or beyond the data (Friel et al., 2001), and the remaining 2% (10) were coded as other. Most of the items dealt with pointing out values in a visual display, comparing data points, and calculating measures of centre (e.g., Jones et al., 2015).

Table 6 illustrates the various occurrences of variability in “Analyze the data.” Among the items within this component, 240 of 506 items were focused on noticing variability, with 157 from MD and 83 from FM. Upon closer examination of these 240 items, we differentiated between implicit and explicit variability. Task 7 challenges students to experiment with data to fulfil different predetermined conditions (Wild & Pfannkuch, 1999). By explicitly invoking measures of center, the task is clearly within the realms of “Analyze the data.” Significantly, the students must read between the data (Friel et al., 2001). In analyzing the data, the task also provides students with an opportunity to reflect and investigate on what we consider to be the effects of “induced variability” on data. In other words, Task 7 encourages students to recognize and reason about how changes in variables of interest can influence the outcome of data collection. The latter illustrates how task classification is not straightforward and that there is a possibility to identify transitional tasks between components. In the context of the present study, Task 7 was primarily viewed as providing an opportunity for students to experiment with data to achieve a targeted statistical outcome and was thus classified as “Analyze the data.”

Table 6. Tasks promoting variability practices in “Analyze the data”

Task 7 (FM: 9 p. 117)									
Look at the representation of colored shapes below:									
Shape									
Color	blue	yellow	blue	red	red	yellow	red	yellow	blue
Draw and color three figures so that the statements are true.									
• The mode for the color is blue • The mode for the shape is a circle • There are yellow, blue, and red circles • There are one less of triangles than of squares									

Task 8 (MD: 1a p. 48)	River	Length (Km)
Look at the table showing the name and lengths of four well-known rivers. Identify a) The longest river	A	750
	B	1100
	C	1600
	D	350

Task 9 (MD: 1d p. 48)

Look at the table in **Task 8** showing the name and lengths of four well-known rivers:

- Compare the lengths of two rivers B and D.

Task 10 (FM: 1 p. 86)

Given a table of information about the average temperature and amount of rainfall for his city, to enhance clarity, a student, Isa constructs a graphical climate chart.

Based on the table and the graph

- How much more did it rain in August than in July?

Task 11 (FM: 2 p. 95)

In a random experiment students are asked to spin a circular spinner and record the outcome over 10 trials. The results should be recorded in a frequency table and then represented in a pie chart. The spinner is divided into three colored sectors.

 $\frac{1}{2}$ of the circle is red
 $\frac{1}{4}$ of the circle is blue
 $\frac{1}{4}$ of the circle is yellow

- In which color did the spinner stop the most?

Task 12 (MD: 37 p. 81)

what is the mode

- a) 6 3 16 8 3 18.
- b) white red white red red white red

Task 13 (FM: 5 p. 100)

what number is x?

- a) The mean of the number x and 5 is 8.
- b) The mean of the number x and 7 is 7.
- c) The mean of the number x, 1, 3 and 6 is 5.
- d) The mean of the number x, 7, 11 is 15.

Task 8 (Table 6) serves as an example of implicit variability noticing. In this task, the noticing is implicit because the question does not explicitly ask for a comparison of outcomes. Instead, the task prompts students to identify the longest river, requiring them to infer what constitutes 'longest.' Hence, this inference naturally involves comparing lengths even though the comparison is not directly stated but implied by the term, longest. Task 9, on the other hand, exemplifies explicit noticing of variability. This task directly prompts students to compare different outcomes of the variable 'length,' guiding students to notice how this variable can vary.

Notably, 149 tasks in “Analyze the data” were categorized as representing variability because they represented aspects that can be related to determining spread in data. The majority of these 149 tasks focused on mathematical calculations; although these tasks involved comparing data sets, they mainly focused on calculating summary values such as measures of centre. Calculation tasks categorized as representing exemplify the transformation of data from one representation form (graphs or tables) to another (symbolic) (Pfannkuch & Rubick, 2002; Pfannkuch & Wild, 2004). Task 12 and Task 13 (Table 6) are typical examples of such tasks.

Interpret the results In this section, we provide task items that meet the criterion of “Interpret the results,” the fourth component of the statistical problem-solving process. We observed that this component has no items from one of the textbooks, MD. It was also observed that most of the task items for this component in FM were connected to aspects of statistical activities.

Table 7. Tasks promoting variability practices in “Interpret the results”

Task 14 (FM: 3d p. 95). 3. Conduct an investigation and collect statistical data based on the following questions (see Task 3) ● What is your favorite colour? ● What is your favorite dish? ● What is your favourite animal? ● What is your shoe size? Choosing the set of supplied statistical investigation questions, students are encouraged to conduct a survey and collect statistical material. Guided through the statistics investigation process the students are asked to: ... d. Lastly write with your own words findings from your survey.
Task 15 (FM: 3 p. 111) Carry out your own survey! Interview students at your school and ask how many hours they spend on homework during a typical school week. 1. Plan how you will record the responses. 2. Collect the data through interviews. 3. Organize the data in a table. 4. Write a report of your investigation: ... d. Reflect on whether there is anything you could do differently to improve the investigation next time.

Five tasks in total from FM were categorized as “Interpret the results.” Two tasks focused on explaining variability, whereas the other three focused on investigating variability. None of the two tasks on explaining variability explicitly required students to reflect on how the variation in outcomes can be explained. However, these tasks are considered cases of explaining variability because they may (implicitly) encourage students to reflect on how variation in outcomes can be explained by prompting

them to express (explain) their survey findings. It is reasonable to assume that Task 14d challenges students to articulate their findings from the survey “with their own words.” This question was perceived as seeking to elicit an answer that is preceded by a process that challenges students to synthesize and communicate findings derived from already collected and analyzed data. Thus, the task may allow for explanations that reference variability. For example, a possible student response could be, “*There were more students owning dogs, followed by cats, not so many had birds, and a handful owned reptiles.*” Task 15d was perceived as implicitly asking the students to consider causes that contribute to variability in their analysis. In this way, the task follows the notion of explaining and controlling variability as stipulated by Wild and Pfannkuch (1999), where explaining variability leads to exploring ways of controlling it. Although the task requires reflection on the data collection process, the primary cognitive demand is evaluating and communicating conclusions. As a result, it was classified as “interpret the results.” Notably, sub-items b and c of Task 14 and sub-item a from Task 15 were mapped and coded as Collect/consider data.

6. DISCUSSION

The present study aimed to examine how primary school textbooks in Sweden characterized the potential for considering variability across the statistical problem-solving process. Although the analysis reveals that these textbooks emphasize different components of the statistical problem-solving process to varying degrees, we observed more areas of convergence than divergence, suggesting that these textbooks exemplify a consistent approach to statistical education. Consistent with previous textbook reviews (Jones et al., 2015; Jones & Basyal, 2019), significantly more space is devoted to “Analyze the data,” followed by “Collect/consider the data.” This pattern observed in two textbooks widely used in Swedish schools suggests that many students encounter statistics in textbooks in ways similar to those identified in our analysis. Although the present study does not claim statistical representativeness, the findings provide insight into common approaches to statistical education in Swedish primary school textbooks.

Our analysis aligns with previous observations about the strong emphasis on data analysis in textbooks and underscores an imbalance in how different components of the statistical problem-solving process are addressed (e.g., Jones et al., 2015). Highlighting this pattern informs curriculum development and teacher training on the necessity of ensuring that all components of the statistical problem-solving process receive adequate attention. For instance, consistent with textbook analyses from other educational contexts (Jones et al., 2015), our analysis showed that few tasks encourage students to formulate or reflect on statistical problems. Even tasks identified as potentially promoting the formulation of statistical questions underscored the need for teacher guidance. This finding suggests that textbooks alone are not sufficient and that guided instruction is also necessary to help students develop these critical skills.

Formulating a statistical investigative question involves anticipating variability, which is a fundamental aspect of the statistical problem-solving process. If problem posing does not account for variability, data collection and analysis might not lead to meaningful investigation (Bargagliotti et al., 2020). Furthermore, without a well-defined statistical question, there is no meaningful context for interpreting results. Research highlights the need for a balanced approach to statistics education that includes all components of the statistical problem-solving process to support students’ development of a more comprehensive statistical literacy (Watson & English, 2018). Accordingly, our analysis underscores the need for a more balanced distribution of the problem-solving components in textbooks, not only to ensure students encounter and practice all aspects of the process, but also to help them reason about data holistically by understanding how variability is integrated across all components in the statistical problem-solving process (Wild & Pfannkuch, 1999).

This study also deepens the understanding of how variability is addressed in mathematics textbooks by shedding new light on how variability is expressed across the different components of the statistical problem-solving process in textbooks. Our strong focus on variability aligns with research emphasizing its central role in developing statistical reasoning in primary education in that early exposure to variability helps students build a deeper understanding of data, uncertainty, and inference (e.g., Garfield & Ben-Zvi, 2007; Makar & Rubin, 2009). Our analysis highlights how textbooks express different variability practices and reveal an imbalance in their representation, with some practices receiving

greater emphasis than others. This imbalance may influence how students develop their understanding of variability, shaping their ability to recognize and interpret variation in data. Identifying these differences provides valuable insights for improving textbook design and instructional approaches to ensure a more comprehensive treatment of variability in the statistical problem-solving process.

Taking this close look at variability, we find that across the 594 statistical items, noticing (42%) and representing (31%) accounted for 73% (433 of 594 task items) of all coded variability practices. Focus on these practices was particularly pronounced within the “Analyze the data” component. However, the textbooks differed in their relative emphasis; noticing accounted for 60% of MD’s “Analyze the data” compared with FM’s 34%, and “Representing the data” accounted for 21% and 38%, respectively. These tasks often prompt students to identify extreme values such as the “highest” or “the lowest,” compare pairs of data points, or calculate measures of centre. Although these are important for developing initial awareness of variability, their dominance risks conveying a procedural rather than a conceptual view of variability. Aspects such as explaining variability were less represented in the tasks included in the textbooks, indicating that students were less often invited to engage with tasks that explicitly invite them to reason about the causes of variability or reflect on the sources of error. This finding aligns with Ben-Zvi (2004) as well as Ko and Lee (2011), who emphasize the challenges students face in progressing from local observations to broader explanations and control of variability.

Our findings echo Peters’ (2011) concern that “not all statistics textbooks make the primary role of variation in statistics explicit” (p. 52). Peters argued that learners often remain at procedural or surface-level treatments of variation, struggling to move beyond intuition and calculation. Similar concerns have been raised by Konold and Pollatsek (2002), who showed that students may be able to compute statistical measures without developing an understanding of how such measures represent and help interpret variability in data. Moreover, Garfield and Ben-Zvi (2005) conceptualize a deep understanding of variability as involving reasoning about how variation is represented, compared, interpreted, and connected to other statistical concepts rather than simply describing or measuring it. Taken together, these perspectives support our interpretation that textbook tasks focusing primarily on noticing or measuring variability may provide limited opportunities for students to develop a deeper understanding of variability. To further illustrate the variability practices identified in the analysis, selected textbook tasks were redesigned (see Appendix). The redesigned tasks are linked to some of the tasks in the analysis (e.g., 1, 2, 12 and 13). The tasks aim to draw attention to ways in which existing tasks may be adapted to promote deliberate engagement with variability.

Concerning noticing variability, we observe a distinction between implicit and explicit noticing of variability. When the noticing of variability is explicit, a task directly prompts students to compare different outcomes. When it is implicit, it comes by comparison language such as asking students to infer on the meaning of “longest/shortest,” “greatest/smallest,” “higher/lower,” etc. The distinction between implicit and explicit noticing of variability contributes to discussions on scaffolding statistical learning. An implicit language is also observed in tasks asking students to measure range. The textbooks do not include items that explicitly ask about range. Instead, tasks are formulated in contextual terms, such as asking students to measure the difference between highest and lowest values (e.g., Dierdorp et al., 2017). Research on primary education emphasizes that young students benefit from structured guidance before they can independently discern patterns in data (Pfannkuch, 2011). Hence, if textbooks rely heavily on implicit language cues on variability, teachers may need to make the variability an explicit topic of discussion and provide additional support to ensure students develop a robust understanding of variability beyond surface-level comparisons.

7. LIMITATIONS OF THE STUDY

Although the present study analyzed two widely used textbooks in Sweden, it did not aim for representativeness in the selection of textbooks. Rather, the selection of these textbooks enabled an in-depth exploration of how variability was made visible in curricular materials. The intention was thus to provide analytical, rather than statistical, generalization by identifying patterns in how opportunities to engage with variability were structured. To extend and validate these patterns, future research could examine a broader range of curriculum materials across different contexts.

Moreover, focusing exclusively on textbooks entails inherent limitations, particularly when considering curriculum in a broader sense. Our analysis highlighted the potential opportunities afforded by the textbooks; however, this perspective remains partial because it does not account for how these materials are used in classroom practice. Teachers' implementation of lessons, their instructional strategies, and the discussions they facilitate all substantially shape students' engagement with statistical concepts. Consequently, future research should also investigate how textbook tasks are enacted in classroom settings, and how instructional practices may complement or transform the opportunities identified in the present study.

From a methodological perspective, subdividing items and clustering them without regard to their broader context presented certain challenges because statistical reasoning is inherently context dependent. Although this is a valid concern, our approach was consistent with methodologies used in large-scale assessment such as OECD's Programme for International Student Assessment survey. Analyzing subitems separately enabled a closer look at how opportunities to engage in variability practices were distributed across different task components. However, this approach also has limitations. Students' responses to a given subitem may be shaped by preceding subitems. Thus, there is the possibility of our coding not fully capturing how students would engage with a task as an integrated whole. Moreover, the five variability practices are not always readily distinguishable in practice. For example, a task coded for noticing variability may simultaneously invite measuring or explaining variability. Future research examining actual student engagement with complete tasks would help clarify the interactions between contexts and the application of different variability practices.

8. CONCLUSION

This study highlights the imbalance in the representation of the statistical problem-solving process and variability practices in Swedish mathematics textbooks, which predominantly focus on analyzing data while marginalizing question formulation and result interpretation. From a variability perspective, a cornerstone of statistical reasoning, the textbooks primarily focused on implicit noticing and representation, and implicit measures of range. The limited emphasis on explaining and investigating variability suggests a need for a more holistic approach in textbook design. By integrating variability practices more evenly across the statistical problem-solving process, textbooks can better support students in developing a comprehensive understanding of variability as a central statistical idea. These findings underscore the importance of instruction and curriculum that address variability explicitly, moving beyond procedural tasks towards activities that foster deeper statistical reasoning and aligning textbook content with broader educational goals in statistics education. Future research should explore how variability practices are unfolded in classroom enactments and examine how instruction can supplement textbook content to promote a more robust conceptualization of variability across the statistical problem-solving process among primary school students.

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APPENDIX

This section presents examples of redesigned tasks (RD) informed by the analytical framework used in the present study. The aim is to illustrate how the tasks can be redesigned to engage students more explicitly with variability.

FORMULATE STATISTICAL INVESTIGATIVE QUESTIONS

Task A – RD: Task 1 (*Explaining variability*)

Look at the chart.

- a) Write two questions about the data that cannot be answered with a single number or word.
- b) For each question, explain what you think causes the differences you can see in the chart.

Travel type	Number of students
Bike	6
Walk	10
Bus	5
Car	3

Rationale: The task directs students toward formulating questions that anticipate and *explain variability*, moving from a descriptive to an analytical orientation.

Task B – RD: Task 2 (*Investigating variability*)

Construct a bar chart from the information below

Months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Expenses	8000	7600	7000	7300	8500	9200	9100	7800	8100	8300	9200	9800

Look at the chart. Come up with three questions about the bar chart. Include at least:

- a) one question that asks about why the values might differ across months, and
- b) one question that asks what you would expect the values to look like in a future month.

Compare your questions with a classmate. With a classmate, decide:

- c) Which of your questions could be answered using only the data in the chart;
And
- d) which would require collecting new data?

Rationale: The task redirects students to formulate questions that *investigate the sources and implications of variability*. It also challenges students' reasoning by asking them to evaluate their own questions.

Task C (*Noticing variability*)

Look at the list of your classmates' name on the class list. Write one question you could investigate about your class. For example: Do all students' name have the same number of letters ?

Rationale: The task grounds students in an observable context where variability is visible (noticeable) before any question is formulated. It is this aspect that characterizes the task as *noticing variability*. The awareness of differences *in the data* can influence the nature of statistical questions posed.

COLLECT/CONSIDER THE DATA

Task D – RD New task (*Representing and noticing variability*)

Rebecka asks some friends who they are voting for as new leader of the cheerleader team. On the sheet below you can see the voting results. The result is to be shown in a frequency table.

Sarah, Tina, Sarah, Leon, James, James, Tina, Leon, Newton,
Leon, Sarah, Leon, Tina, James, James, Leon, Sarah, James,
Leon, Sarah, Newton, Leon, Tina, Leon, Sarah, James, Newton,
Leon, Tina, James, Newton, Sarah, Newton, Leon, Sarah.

Create a frequency table from the voting data. Once you have made the table, answer the following:

- Describe how the data are spread out in your own words
- Did one candidate receive many more votes than the others, or were the votes fairly evenly distributed?
- How spread out are the votes?

Rationale: The task not only ask the students to create a *new representation – a frequency table* but explicitly focuses their attention on *the distribution of the data*, thus giving statistical meaning to the data.

Task E – RD: Task 6 (*Noticing and measuring variability*)

Children in a class listen to music and award score points (as illustrated below)



Write the score points in ascending order.

- What is the lowest score?
- What is the highest score?
- Calculate the difference between them.
- Do you think the scores are spread out or mostly similar? Explain your answer.

Rationale: By asking students to order the data, the task encourages them to *notice* variability. The scaffolding questions then support students in identifying and *measuring* the range of the data.

Task F – RD: Task 7 (*Investigating variability*)

Look at the representation of colored shapes below:

Shape									
Color	blue	yellow	blue	red	red	yellow	red	yellow	blue

Draw and color three figures so that all the conditions are met.

Now change your drawing so that the mode for color becomes red while keeping all the other conditions the same

- which changes were necessary?
- were you able to do this in more than one way?
- what can you say how changes to single values can affect the distribution of the shapes?

- The mode for the color is blue
- The mode for the shape is a circle
- There are yellow, blue and red circles
- There are one less of triangles than of squares

Rationale: The task challenges students to reason explicitly about variability by asking them to *investigate how targeted changes in one variable can affect the distribution* of the data.

ANALYZE THE DATA

Task G – RD: Task 8 (*Noticing variability*)

Look at the table of river lengths.

- Which river is the longest?
- Which is the shortest?
- How much longer is the longest river than the shortest?
- Do you think the differences in length are large or small compared to the length of Sweden?

River	Length (Km)
A	750
B	1100
C	1600
D	350

Rationale: The task *explicitly asks the students to notice* and reflect on the variability.

Task H – RD: Task 10 (*Explaining variability*)

Look at the table in **Task G** showing the name and lengths of four well-known rivers:

Compare the lengths of the two rivers (B and D).

- How much longer is one than the other?
- Can you think of any of reason from nature (e.g., mountains or rainfall) why these two rivers might have different lengths?

Rationale: The task explicitly allows the students to reason about sources of variability thus *enhancing their ability to explain variability within and between the river lengths*.

Task I – RD: Task 12 (*Representing variability*)

1. Consider two data sets: (6 3 16 8 3 18), and (3 3 4 5 6 7) both with mode 3.
2. Another data set consists of the following colors: white red white red red white red:
 - a) Look at the list of values in the data set (1). Are the datasets alike?
 - b) Create a data set using colors. What proportion of the observations falls into the most common color?

Rationale: The task sought to use the mode as a summary measure – *another way of representing the variation in data*. The extra questions allow students to reflect on what the mode reveals and conceals about variability, connecting the measure of centre to the concept of spread.

Task J – RD: Task 13 (*Measuring variability*)

What number is x?

- a) The mean of x, 1, 3 and 6 is 5.
- b) Write down how far each of the four numbers is from 5. Which number is the farthest from the mean?
- c) The mean of the number x and 7 is 7. What is the number x? Explain how the spread for these values compares with that of the values in a) above,
- d) Write four numbers with mean 5 but that are more spread out than those in a) above.

Rationale: The task encourages students to reason about how a single missing value influences a measure of centre. By determining the missing value, students must consider the relationship between individual observations and the overall average. This supports engagement with measuring variability.

INTERPRET THE RESULTS

Task K – RD: Task 14 (*Explaining variability*)

Conduct a survey and collect statistical data based on the following set of questions

- What is your favorite color?
- What is your favorite dish?
- What is your favorite animal?
- What is your shoe size?

Write up your findings from the survey in your own words.

- a) Describe how much the answers varied.
- b) Think about factors such as age and where students live. Could any of these explain why some students gave different answers from others?

Rationale: The task scaffolds students in *describing and explaining central tendency and reflecting on the spread of the data*.

Task L – RD: Task 15 (*Investigating variability*)

Interview students at your school and ask how many hours they spend on homework during a typical school week.

1. Plan how you will record the responses.
2. Collect the data through interviews.
3. Organize the data in a table.

Write a report of your investigation:

Reflect on your survey.

- a) Did the way you selected the students to interview affect the results?
- b) How do you think the results would turn out if you had interviewed different students?
- c) Is there anything in how you collected the data that might have introduced inconsistencies?
- d) Write a short sentence outlining one thing you would change and predict how it might affect the results.

Rationale: The task allows students to think systematically about sampling, measurement error, and the relationship between data collection decisions and variability in outcomes.