# WHICH MEASURE OF CENTRAL TENDENCY IS MOST USEFUL? GRADE 6 STUDENTS' EXPRESSED STATISTICAL LITERACY

KARIN LANDTBLOM
*Stockholm University*
*karin.landtblom@su.se*

LOVISA SUMPTER
*Stockholm University*
*lovisa.sumpter@su.se*

**ABSTRACT**

*Recently, the importance of statistical literacy has been stressed, and three central concepts in statistical literacy are the measures of central tendency: mean, median, and mode. This study explores aspects of statistical literacy expressed by 12–13-year-old students, focusing on mean, median, and mode. Their responses were analysed using a framework of statistical literacy that includes knowledge and dispositional elements. The results showed that students' descriptions of the measures were mainly based on mathematical and vocabulary knowledge. When discussing what measure was easiest or hardest to explain, a variety of conceptions were expressed. Some explanations about the usefulness of the measures were related to context knowledge. Here, the median was an exception as students gave neither examples of contexts nor found the median useful outside the classroom.*

***Keywords****: Conceptions; Mean, median, and mode; Measures of central tendency; Primary students; Statistical literacy*

## 1. INTRODUCTION

Statistics as a subject has grown over the past decades (Chick & Pierce, 2012), and several researchers have concluded that statistical literacy is necessary for functioning in today's society (Carmichael et al., 2010; Çatman Aksoy & Işıksal Bostan, 2021). Statistical literacy can be framed in several ways (Callingham & Watson, 2017; Gal, 2002). What different statistical literacy frameworks have in common is that they include several knowledge elements (Gal, 2002), such as mathematical knowledge (Bond et al., 2012; Büscher, 2019) and context knowledge (Büscher, 2019; Leavy & Hourigan, 2016), and several dispositional elements (Gal, 2002), such as beliefs or attitudes (Bond et al., 2012; Ramirez et al., 2012). Studies suggest that knowing the procedures for calculating measures does not automatically entail understanding (Bond et al., 2012). Gal (2002) concluded that facts and rules do not lead to skills such as interpreting and communicating statistical information, meaning that mathematical knowledge alone is insufficient for becoming statistically literate.

Mean, median, and mode are key concepts essential for understanding statistics. These statistical concepts are often the first ones that students meet, where they have to consider both data and context, and it is often in middle school when the averages are introduced (Landtblom, 2018). As concepts, they have clear definitions that differ from the procedures to calculate the measures and how data should be interpreted. These measures do not make sense without correctly interpreting data and context. For example, one can have an average of 2.1 children, but no one can have exactly 2.1 children. Hence, the concepts are part of statistical literacy. Previous studies indicate that many in-service teachers, prospective teachers, and students of different ages struggle to distinguish among the different measures (Jacobbe, 2012; Leavy & O'Loughlin, 2006), to define the various measures (Groth & Bergner, 2006; Watson, 2007), to decide which measure is best to use with which context (Büscher, 2019), and to calculate the measures (Jacobbe, 2012). In addition, it has been reported that students state that a lot of lesson time is focused on procedures when learning about measures of central tendency (Mokros & Russell, 1995). Consequently, many students use procedural descriptions for mean, median, and mode

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(Clark et al., 2007; Groth & Bergner, 2006), meaning that students may not learn about averages in the sense of statistical literacy—knowing the names of the measures of central tendency and knowing the procedures does not necessarily entail knowledge of how to use the measures (Clark et al., 2007; Leavy & Hourigan, 2016) or how to interpret the measures (Leavy & Hourigan, 2016; Mokros & Russell, 1995). Many studies have in common that they study students' reasoning about or understanding of one of the measures or about only one aspect of statistical literacy. Few studies simultaneously look at students' reasoning or understanding of all three measures concerning several aspects of statistical literacy. In conclusion, we do not know much about students' collective statistical literacy regarding mean, median, and mode. In addition, most research focuses on cognitive aspects of students' statistical literacy, and few have "explicitly [attended to] the associated dispositional component" (Sharma, 2017, p. 130). The dispositional aspects tend to be considered in research about statistics in general but not in research specifically about the mean, median, and mode (Groth & Meletiou-Mavrotheris, 2018). Therefore, the present study aims to address these gaps and explore grade 6 (ages 12–13) students' expressed statistical literacy about three measures of central tendency: mean, median, and mode.

## 2. BACKGROUND

Before formulating specific research questions, we frame this study within a statistical literacy framework. We also present previous empirical research related to the measures of central tendency.

### 2.1. STATISTICAL LITERACY

Statistical literacy is described differently in different frameworks (Sharma, 2017). For example, whereas Gal (2002) built a framework with cognitive and dispositional elements, the framework of Watson and Callingham (2003) differentiated between hierarchical levels of statistical literacy. In this study, we explored the statistical literacy of a group of sixth-grade students. Because we were not looking for different levels of knowledge but rather what elements appeared in students' answers, we used Gal's (2002) framework as a starting point (see Figure 1).

| **Knowledge elements** | **Dispositional elements** |
|---|---|
| Mathematical knowledge | Beliefs and Attitude |
| Statistical knowledge | Critical stance |
| Context knowledge | |
| Literacy skills | |
| Critical Questions | |
| **Statistical literacy** | |

*Figure 1. A framework of statistical literacy (adapted from Gal, 2002, p. 4)*

Concerning dispositional elements that fall under the umbrella term of affect (Gal, 2002), researchers have reported on studies investigating students' attitudes (Bond et al., 2012), self-efficacy (Carmichael et al., 2010), self-concept, and interest in statistics (Sproesser et al., 2016). There are studies on student engagement in statistics (Watson & Callingham, 2003), how attitudes and beliefs affect students' statistical behaviour, and several aspects of different dispositional elements (Ramirez et al., 2012). One issue among the studies is that the affective construct under consideration is seldom defined, meaning that data can, in some studies, be interpreted as an attitude and, in others, as a belief. This inconsistency has been identified as a central problem in mathematics education research (Diego-Mantecón et al., 2019). In addition, it is difficult to distinguish between affective constructs, such as how true a belief is (Philipp, 2007) or how stable it is (Goldin, 2002). Moreover, ontologically, disposition should be interpreted as "an umbrella term that refers to a cluster of related but distinct concepts, including motivations, beliefs, attitudes, and emotions" (Gal et al., 2023, p. 48). We use this definition of disposition as our theoretical starting point for how we view dispositional elements of this kind. However, we would also like to acknowledge that affective constructs, such as beliefs and attitudes, have some commonalities and overlap, as pointed out by Diego-Mantecón and colleagues (2019). Instead of using the terms for these affective constructs, we use the theoretical term, *conception*, with its more overarching construction, to consider dispositions. Conception, broadly framed, is defined as “a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images and preferences” (Philipp, 2007, p.259). Hence, conceptions involve beliefs, attitudes, critical stance, and mental images built on available experiences, aligning with Gal (2002) and Gal et al. (2023).

Beyond dispositional elements, statistical literacy includes the following knowledge elements: mathematical knowledge, statistical knowledge, context knowledge, literacy skills, and critical questions (Gal, 2002). Starting with mathematical and statistical knowledge, the decision in the present study was to join them into one, mathematical knowledge. In Sweden, which is the context of the study, statistics education is part of mathematics education (Landtblom & Sumpter, 2021). This means that for the students who participated in the study, statistics was part of their mathematics education. Mathematical knowledge includes knowledge about concepts and procedures, such as how definitions contain different mathematical properties (Strauss & Bichler, 1988). It also includes knowledge of variables as a prerequisite for developing knowledge about measures of central tendency (Watson, 2006).

Context knowledge is crucial for learning statistics (Chick & Pierce, 2012; Fry et al., 2024; Leavy & Hourigan, 2016; Watson & Callingham, 2003) and for engaging with mathematical processes such as modelling (Sumpter et al., 2024). Context knowledge requires insight into situated knowledge related to recognised situations. Consequently, one should understand how to apply statistical concepts in a social context (Fry et al., 2024; Leavy & Hourigan, 2016), providing a kind of awareness beyond numbers (Callingham & Watson, 2017). Contextual knowledge contributes to an understanding of how to use the mean for comparing distributions (Leavy & Hourigan, 2016), for instance~~,~~ to compare trends in data between players in different sports. Developing contextual knowledge can be problematic for students because statistics are seldom linked to real-world contexts in the classroom. Contexts, therefore, should be introduced consciously (Fry et al., 2024). Outside of school, the mean is used more widely than the median, which can influence students’ knowledge of other measures of central tendency (Batanero et al., 2020). Thus, students should encounter context in their school study of statistics because doing so can afford opportunities to develop students’ notions of statistics as meaningful and useful (Büscher, 2019; Chick & Pierce, 2012).

Gals’ definition of statistical literacy builds on two interrelated components: people’s abilities to interpret and critically evaluate statistical information and their abilities to discuss or communicate their reactions to it (Gal, 2002). Knowledge in the form of literacy skills underlies the abilities needed to achieve proficiency with both components. These literacy skills involve conveying and understanding statistical messages orally or through written text. The focus of literacy skills in this study involves grasping the meaning of statistical language and the statistical systems essential for communicating a sense of statistics (Arnold & Pfannkuch, 2022; Callingham & Watson, 2017; Gal, 2002), specifically in relation to measures of central tendency. Generally, familiarity with language, such as statistical vocabulary, is a precursor to understanding (Usiskin, 2012). However, “[s]everal difficulties of learning statistics that are documented in research are consistent with a perspective of statistics as isolated facts” (Makar & Confrey, 2005, p. 49). For example, in their study about teachers’ articulated notions of variation, preservice teachers expressed rich views of variation using nonstandard language—with or without standard statistical language—that connected ideas of variation with distribution (Makar & Confrey, 2005). In contrast, teachers’ use of standard statistical language tended to consist of isolated facts related to mean, skewness, or dispersion without connecting the ideas to consider relationships in the data or to capture the desired concept of variation. Even though standard deviation could be an inclusive term for describing variation, two preservice teachers used the term without articulating the meaning of variation, indicating that statistical vocabulary does not always entail understanding. More recent studies also provide evidence that using nonstandard language can promote students’ understanding of statistics concepts (e.g., Arnold & Pfannkuch, 2022). With respect to measures of central tendency, students encounter averages often expressed in nonstandard, colloquial terms outside the school setting (Mokros & Russell, 1995), such as middle rather than median. Such nonstandard language at times can provide a bridge for students to develop statistical language (Watson, 2007) and develop meaning, such as helping students intuit the meaning of median. However, the use of nonstandard language does not always lead to meaning, such as using a word like typical to describe a measure of central tendency—a single description for multiple measures that does not have statistical meaning (Watson, 2007). Nor does using colloquial words in a tautological way provide statistical meaning (Clark et al., 2007), and colloquial words can have connotations other than statistical meanings that may interfere with students' learning (Watson, 2006). Additionally, when students confuse words such as the mean and median, the confusion may be related to terminology, which can be described as a semiotic notational conflict (Madrid-García et al., 2023; Mayén et al., 2009), rather than conceptual understanding. As described, terminology and normative understanding of concepts associated with the terms are critical for developing literacy skills. This study focuses on the communicative parts of literacy skills, which involve standard and nonstandard words—both colloquial and descriptive—that the students use when expressing knowledge about the measures of central tendency. We have named this vocabulary knowledge a subset of literacy skills.

Finally, critical questions are closely related to context knowledge. This knowledge element emphasises the importance of presenting different contexts of mathematical skills in teaching to develop students' higher-order statistical literacy skills (Watson & Callingham, 2003; Weiland, 2019). Including critical questions in teaching is beneficial for developing engagement and providing opportunities to interpret data in familiar and unfamiliar contexts (Callingham & Watson, 2017; Gal et al., 2023; Leavy & Hourigan, 2016). However, in the current study, critical questions were not addressed because the format of the items seemed prohibitive for students to provide any substantive attention to critical questions.

Against this backdrop, this study uses a framework containing the following knowledge elements: mathematical knowledge (including statistical knowledge), context knowledge, and vocabulary knowledge. Dispositional elements are limited to conceptions. Figure 2 depicts these elements.

| Knowledge elements | Dispositional elements |
|---|---|
| Mathematical knowledge<br>Context knowledge<br>Vocabulary knowledge | Conceptions |

*Figure 2. Relevant elements of statistical literacy in this study*

## 2.2. PREVIOUS RESEARCH ON MEASURES OF CENTRAL TENDENCIES

Previous research has indicated that students at different school levels often lack mathematical knowledge about averages (Mathews & Clark, 2003). One reason for students' difficulties with measures of central tendency may be related to their views of distribution. When trying to make sense of measures, one needs to view a dataset as a whole because measures are representations of a dataset, including any outliers (Konold et al., 2015; Mokros & Russell, 1995). Students tend to look separately at each observation in a dataset instead of making statements about the distribution as an entity (Konold et al., 2015; Leavy & Hourigan, 2016). Reasoning distributionally requires understanding the context and the formal statistics involved, including measures of central tendency, to analyse data distributions and interpret the meaning of statistics (Büscher, 2019; Gal et al., 2023; Leavy & Hourigan, 2016). Reasoning about data in a context requires knowledge in the intersection of mathematical and context knowledge (Chick & Pierce, 2012). When students learn to make sense of measures and, at the same time, learn about specific contexts where the measures can be applied, a more advanced knowledge of the measure can be developed (Büscher, 2019; Watson & Callingham, 2003). An aggregated view of distribution is necessary when, for instance, comparing different groups of data (Konold et al., 2015). However, studies indicate that using statistical measures such as mean, median, and mode to make comparisons is problematic for students despite many years of schooling.

Multiple studies provide evidence of the different meanings that students attribute to the arithmetic mean. For example, Garfield and Ben-Zvi observed, "it appears that many students, who complete introductory statistics courses, are unable to understand the idea of the mean" (p. 383). Additionally, eight first-year university students expressed discrepancies between their concept images and their definitions of the mean (Mathews & Clark, 2003). This means that just asking for definitions will likely not give a valid picture of students' statistical literacy; other aspects of their concept images, such as conceptions, are also essential.

Fewer studies provide insights into students' understanding of the median and mode (Jacobbe, 2012; Landtblom, 2023). One suggested reason for fewer studies on the median is that its calculation is simpler than the calculation for the mean (Lesser et al., 2014); similar reasoning might also explain fewer studies on the mode. Concerning the median, one study identified preservice teachers who struggled to identify the median in unordered data and when the median was not a value in the data set (Groth & Bergner, 2006). Schnell and Frischemeier (2019) posited that primary students' struggles when reasoning to compare distributions might be related to knowledge about the median, involving understanding the distribution and its meaning. In general, research suggests that procedural knowledge for finding a median is not sufficient for conceptually understanding median. Concerning mode, there are indications that students, teachers, and prospective teachers struggle to distinguish between mode and frequency (Groth & Bergner, 2013; Landtblom & Sumpter, 2021). Understanding the mode includes the understanding that the mode is the only measure of central tendency for nominal data; the mean and median are not applicable.

### 2.3. RESEARCH QUESTIONS

This study aimed to explore grade 6 (ages 12–13) students' expressed statistical literacy about measures of central tendency: mean, median, and mode. The research questions were as follows: (1) What aspects of different knowledge elements do the students use in their descriptions of the measures of central tendency?; (2) What characterises the explanations given by grade 6 students about which measure is easiest or hardest to explain?; and (3) What characterises the explanations given by grade 6 students about which measure is most or least useful?

## 3. METHODS

To study students' expressed statistical literacy, we used a web-based questionnaire to generate data (Mason, 2018). Using a questionnaire can save time, facilitate geographical spread, and benefit students with lower self-esteem by allowing them to answer anonymously (Braun et al., 2020). Because the concepts of mean, median, and mode are introduced in school years 4–6 (ages 10–13) in Sweden, the students in these grades should have had several possibilities to learn about the measures (Landtblom & Sumpter, 2021). However, the curriculum in Sweden does not state how measures of central tendency should be introduced and taught, and textbooks vary significantly in how statistics are presented (Landtblom, 2018; Landtblom, 2023). This means that when students are introduced to different procedures, the affordances of conceptual tasks vary between textbooks. Depending on which textbooks the teachers choose to use or if they decide to use their own material, some children might work with the concepts throughout the three school years, whereas others might only be introduced to them, for instance, in grade 5. The curriculum states that the students should have learned the measures not just as procedures by grade 6 but also as part of different processes, such as problem-solving, mathematical reasoning, and communication, and the national tests in grade 6 test for both mathematical products and processes.

The sample of students was decided based on convenience and purpose, which is appropriate for exploratory research (Denscombe, 2017). One hundred thirty students participated, which can be seen as a suitable range for providing a rich amount of study data (Braun et al., 2020). The student participants were from schools in different parts of a large city. They attended three city schools—two independent and one public—and two public suburban schools, and came from different socioeconomic backgrounds. All students were in grade 6 (12–13 years old). This diversity matters for the kind of knowledge the study informs and from an inclusion perspective (Braun et al., 2020). The study followed the ethical guidelines provided by the Swedish Research Council (2017), which means that students participated voluntarily and anonymously; written informed consent was obtained from each student's guardian; and data were treated and stored according to Stockholm University's data management plan.

The questionnaire instrument was piloted and then developed in two studies with prospective and in-service teachers (Landtblom, 2018; Landtblom & Sumpter, 2021). Similar questions were used in this study, with some minor adjustments in language made to make the instrument more inclusive to younger students. The open-ended questions generated qualitative data to capture the respondents' answers as fully as possible and quantitative data to reflect the distribution of data (Braun et al., 2020; Denscombe, 2017). The first three items (1–3) aimed to generate data on what knowledge elements—mathematical, context or vocabulary—students used when explaining the different measures. The introduction to the items was: Imagine that another student in your class has been ill and missed a mathematics class where you have been working with the measures mean, median, and mode. The introduction was followed by the following question: How would you explain these measures to your friend? This introduction served as a stimulus (Mason, 2018) and was followed by three questions. (1) How would you explain the concept of mean? (2) How would you explain the concept of median? (3) How would you explain the concept of mode? Items four and five intended to explore conceptions, knowledge elements, and potential links between the two. These items were: 4 a) Which of the averages was easiest to explain? b) Explain your answer; c) Which of the averages was hardest to explain? and d) Explain your answer; 5 a) Which average is most useful? b) Explain your answer; c) Which average is least useful? and d) Explain your answer. The response options were mean, median, mode, or other. In this way, the instrument aimed to cover aspects of students' statistical literacy, namely knowledge elements and conceptions.

Regarding the validity of interpretations from the questionnaire, we considered both theoretical and empirical evidence. Items 1, 2, and 3 were originally developed based on existing theoretical and empirical expositions in the literature. The responses to items 1, 2, and 3 were analysed using an inductive analysis in a pilot study (Landtblom, 2018). The results indicated difficulties with mathematical and vocabulary knowledge. The items were then revised and tested again in a second study (Landtblom & Sumpter, 2021). The results from the pilot study also started a discussion with other researchers about how to generate data for the dispositional element of conceptions. At that point, few studies were published in this area, so we formulated items 4 and 5 as a result of this discussion. The theoretical basis for items 4 and 5 was grounded in research suggesting that awareness of pedagogical affordances of data requires awareness of the data in a particular context (Chick & Pierce, 2012). Engaging with data in a context requires drawing on all aspects of statistical literacy, including conceptions (Callingham & Watson, 2017). Items 4 and 5 were tested in a second study using an inductive thematic analysis (Landtblom & Sumpter, 2021), and except for results categorised as mathematical and vocabulary knowledge, both items revealed conceptions. Item 5 generated responses coded as contextual knowledge, which was demonstrated through connections between usability and different situations. See Landtblom and Sumpter (2021) for a more extended discussion.

Thematic deductive analysis (Braun et al., 2020) using the framework depicted in Figure 2 was conducted to analyse the data. The results were interpreted as themes indicating different aspects of students' knowledge and dispositional elements. We cannot say that the themes represent an element fully, and indicate this shift in epistemology by writing the themes using capital letters. Table 1 displays the themes, categories, characteristics of responses that would be associated with the categories, and examples of coded responses from the data. The theme of Mathematical Knowledge (MK) includes categories connected to procedures and definitions. Context Knowledge (CxK) includes categories such as notions of situations expressed implicitly (investigations without any variable explicitly stated) or explicitly (a specific investigation in a specific context, such as an election or exemplifying using a variable). Vocabulary Knowledge (VK) includes standard vocabulary aligned with definitions and nonstandard vocabulary. Categories of nonstandard vocabulary are colloquial and descriptive words to explain measures. When coding VK, the word "typ" occurred, which is the first syllable of the word "typvärde" (mode in Swedish). Because this word was used with its homonym meaning, homonyms were abductively added as a category in Table 1. The Conceptions (Co) theme includes categories that could indicate some beliefs, attitudes, or other mental images. For any instance in the data where the explanations did not align with knowledge elements, the data was compared to the definition of conceptions suggested by Philipp (2007). The categories in this theme are Knowledge and skills, Usefulness and relevance, Statistics as a subject, and Others. Finally, the theme of "Do Not Know" related to responses in which students expressed that they did not know in some way. Not Relevant (NR) refers to explanations without evidence of knowledge or dispositional elements.

*Table 1. Themes, categories, characteristics of categories for the analysis, and examples from the data*

| Theme | Category | Characteristics of category | Example from the data |
|---|---|---|---|
| Mathematical Knowledge (MK) | A correct procedure, Complete | Gives correct definition, including mathematical properties of the measure of central tendency | • For example, to calculate the mean of 8 people's ages, you add all their ages and then divide by the number of people they are.<br>• For data sets with uneven numbers, the median is the middle number if arranged in order of magnitude. For sets with an even number of numbers, the median is the mean of the two middle numbers.<br>• The value that occurs most. If we have the numbers 1, 1, 3, 4, 5 and 7, we take the number repeated most often. In this case, 1, but if two numbers are repeated twice, for example, 1 1 2 4 4 6, then both 1 and 4 are the mode. |
| | A correct procedure, incomplete | Does not include all mathematical properties, for example: | • If you have 4 and 8, then the mean is 6. |
| | | Considers median only for data sets with an odd number of values | • If you write down some numbers, the median is the one that is in the middle. |
| | | Determines median from middle value without rank-ordering the data | • 1 2 8 9 11, 8 is the median value, the one that is in the middle. |
| | | Considers only datasets with a single mode | • The value that occurs most often |
| | A correct procedure, incorrect answer | Makes mistakes in calculating the measure of central tendency | • For example, if you have the numbers 1–10, then the median is 5. (not paying attention that there are ten numbers: 1, 2, … 8, 9, 10)<br>• The median is the number between two other numbers. If we take 11 and 12 as an example. Between 11 and 12 is 10. Then, the median is 10. |
| | Wrong procedure | Mixes up the measures of central tendency, for example:<br>Calculates the mean and refers to it as the median | • It is what everyone has and the average of it. [about median]<br>• The median is the most common number of a set of numbers.<br>• Mean is a number in the middle. |
| | No procedure | Does not make connection to any mathematical property | • An approximate value. [about mode]<br>• Average is what everyone has approximately. [about mean] |
| Context Knowledge (CxK) | Implicit situations | Does not state which variable is being investigated | • Mean is the most relevant measure of central tendency in most investigations. |
| | Explicit situations | Refers to explicit variables, such as age, length, etc., or explicit situations connected to a measure, such as an election | • I find the mean most useful because I can use it to determine how many goals I make in a year.<br>• The mode is useful as comparing election results and similar events is vital. |
| | No CxK | Does not make connection to any context | • Most people know what it is. [about mode] |

| Theme | Category | Characteristics of category | Example from the data |
|---|---|---|---|
| Vocabulary Knowledge (VK) | Colloquial | Uses colloquial terms | • Cut off several numbers. [about mean]<br>• It is an approximate value. [about mode] |
| | Descriptive | Uses words describing the measure: most, typical, usual, middle, and other | • The median is the most common number out of a set of numbers.<br>• Middle. [about median] |
| | Homonyms | Uses homonyms | • As it is just about correct, you don’t have to be entirely right. [about mode, typ meaning approximative.] |
| | Standard vocabulary | Uses words or numerals close to the definition | • 13+4+15+4+4=40, 40/5=8, the mean is 8.<br>• If you have several numbers in order, the median is the number in the middle. |
| Conceptions | Knowledge and skills | Describes what is difficult or easy | • Mode is easy to explain because you only take what there is most of.<br>• The median is easy because no math is involved in calculating the median.<br>• The mean is easy to calculate but difficult to explain.<br>• I knew what it was [median], but it was hard to explain with my own words.<br>• [Mode is difficult] because the others I can at least guess. |
| | Usefulness and relevance | Describes the usefulness of mean, median, or mode | • It is pretty easy to use. [about mean]<br>• Most people know what it is [about mode]<br>• I don´t even know what it is suitable for or what one uses it for. [about mean]<br>• I don´t think it is so essential. [about median] |
| | Statistics as a subject | Relates to learning statistics in school | • [Mean is easiest to explain] because we have worked a lot with it.<br>• [Median is hardest to explain] because we only worked with the median in the 5th grade. |
| | Other | Provides other conceptions related to personal feelings or experiences different from those above | • I don’t think the mode is so important.<br>• I don’t know because I have not used them.<br>• Median, no one has explained it to me. |
| Do not know | | Expresses that they cannot answer | • Honestly, I have forgotten.<br>• I don’t know right now, but I knew it before.<br>• I don’t know how to explain it. |
| Not relevant | | Provides response unrelated to the item, using different signs with no meaning | • -<br>• Dw<br>• … |

Both authors compiled the categories and the characteristics of categories in Table 1. Based on this compilation, the first author coded all data. In order to increase validity, interpretations of data were discussed with the second author during peer debriefing (Nowell et al., 2017).

The descriptive statistical analysis aimed to describe and compare present levels of statistical literacy regarding measures of the central tendency among the students. Descriptive statistics combined with examples of how students express themselves can give comprehensive results compared to using only qualitative analysis (Mason, 2018).

The results are presented with excerpts from the data, where student responses are given with square brackets and a number associated with a unique student (e.g., [91]). The excerpts thus provide a fuller narrative and interpretation of the categories. Frequency analysis was used in the quantitative analysis, which provides both frequencies and percentages for comparing the proportions in each category.

## 4. RESULTS

The results are presented in three sections based on the research questions. First, we discuss knowledge elements MK, CxK, and VK used in students' descriptions for the measures of central tendency. Then, we present the knowledge elements and conceptions used in students' explanations for what is considered easiest/ hardest to explain and perceived usefulness. In some tables (Tables 6, 7, 9, and 10), the total exceeds the total number of students, indicating that some responses were coded for multiple instances. Not all sums of listed percentages equal 100% due to rounding errors.

### 4.1. STUDENTS' DESCRIPTIONS OF MEAN, MEDIAN, AND MODE

First, we have the results on the theme of Mathematical Knowledge (MK). The students' descriptions were analysed regarding mathematical knowledge concerning the algorithm that was used. See Table 2 below.

*Table 2. Frequency (and percentage of total) of mathematical knowledge categories for mean, median, and mode*

| Mathematical Knowledge | Mean | Median | Mode |
|---|---|---|---|
| A correct procedure, complete | 59 (45.4) | 16 (12.3) | 6 (4.6) |
| A correct procedure, incomplete | 7 (5.4) | 73 (56.2) | 60 (46.2) |
| A correct procedure, incorrect answer | 0 (0) | 4 (3.1) | 0 (0) |
| Wrong procedure | 23 (17.7) | 3 (2.3) | 6 (4.6) |
| No procedure | 41 (31.5) | 34 (26.2) | 58 (44.6) |
| Total | 130 (100) | 130 (100.1) | 130 (100) |

Many student responses included the correct procedure for each measure. About half of the students used the correct procedure for mean and mode; for the median, 93 students indicated knowledge of the correct procedure (approximately 72%). However, more than half of the responses were coded as incomplete procedures for the median. Students did not consider the possibility of an even number of values in their description for finding the median or rank-ordering the data values to find the median. For instance: "You rank order the numbers and then circle the middle number" [27; about median]. This excerpt was coded as incomplete because the student only exemplified finding the median using an odd number of values. A similar ratio was observed for the mode concerning properties for data sets with no or more than one mode. Many students gave correct but incomplete responses. One example to illustrate such a response is the following: "It is what you have the most of; for instance, if you have 1 2 3 3 3 4, then the mode is 3" [37; about mode]. A response coded as complete was the explanation given by student 116: "The mode is the numbers repeated most times of several numbers in a number sequence" [116; about mode]. The conclusion was that although students appeared reasonably sure about the procedure in their definitions, their explanations covered only some of the relevant mathematical properties. Notably, many students used the wrong procedure when explaining the mean. A high proportion also described the measures without using a procedure.

When investigating Context Knowledge (CxK), the analysis revealed different aspects of placing values in contexts. The determination of context knowledge is then based on whether contexts were described explicitly in relation to measures of central tendency, such as mean referring to age or mode referring to an election. Context can also be implicit, such as referring to an investigation without naming a particular variable. The results are presented in Table 3.

*Table 3. Frequency (and percentage of total) of context knowledge categories for mean, median, and mode*

| Context Knowledge | Mean | Median | Mode |
|---|---|---|---|
| Implicit situations | 11 (8.5) | 5 (3.8) | 25 (19.2) |
| Explicit situations | 18 (13.8) | 4 (3.1) | 8 (6.2) |
| No CxK | 101 (77.7) | 121 (93.1) | 97 (74.6) |
| Total | 130 (100) | 130 (100) | 130 (100) |

As shown in Table 3, most of the students did not include any context in their descriptions for the mean (77.7%), median (93.1%), and mode (74.6%). For the mode, there were more implicit allusions to context in the explanations. In implicit situations, the responses indicated some situation or competition without exemplifying what was being measured. One example coded as implicit is the following: “This is what you use more often in different investigations” [58; about mean]. Another example coded as implicit indicated that the mode has a connection to some choice: “The most chosen option” [49; about mode]. This can be compared to examples that did not refer to any context, such as “The number that there is the most of.” [16; about mode]. When describing mean, explicit mention of contexts was more common. This student connected the mean to football: ”I think the mean is most useful because then I can calculate the mean of how many goals I strike in a year” [111; about mode]. For all three measures, students included explicit examples concerning age. One example was: “If four people are 3, 4, 6, and 7 years old, the median is five years because it is in the middle” [107; about median]. Some responses referenced data generated when throwing a die for both the mean and median. Explicit situations specific to the mode were voting situations or investigations of preferences, such as ice cream flavours, or most occurring, such as the colour of dogs. Two answers about the median mentioned the presence of extreme outlying data values. One talked about money: “It will generate a fairer answer because if someone gets 700 Swedish crowns as pocket money one month, and the rest get 200 Swedish crowns, it might look as if everyone gets more, although only one person receives more money” [59; about median]. Implicit in the response was a comparison of the median to the mean. Because context was not explicitly asked for in the question, the conclusion was that few students would consider the context when defining without any further stimuli.

For Vocabulary Knowledge (VK), every word used as a synonym for the measure or description of the measure of central tendency was considered. The categories generated in the VK theme were colloquial words, descriptive words, homonyms, and no explanation (see Table 4).

*Table 4. The frequency (and percentage of total) of vocabulary knowledge categories for mean, median, and mode*

| Vocabulary Knowledge | Mean | Median | Mode |
|---|---|---|---|
| Colloquial | 15 (11.5) | 3 (2.3) | 0 (0) |
| Descriptive | 32 (24.6) | 97 (74.6) | 91 (70.0) |
| Homonyms | 0 (0) | 0 (0) | 23 (17.7) |
| Standard vocabulary | 83 (63.8) | 30 (23.1) | 16 (12.3) |
| Total | 130 (99.9) | 130 (100) | 130 (100) |

The results presented in Table 4 show that the students added synonymous words when explaining the median and the mode more often than when explaining the mean. Colloquial words were used to describe the mean, such as "cut" as short for average (in Swedish, snitt). One example was: "The cut of several numbers" [40; about mean]. Descriptive words like middle, most, and highest were also used. These words are similar to words in the definitions of the terms, especially regarding the median and the mode. However, the middle was frequently used for the mean as well. There are several similar descriptive examples where the word middle was used: "The number in the middle in an investigation" [70; about median] and "The mean is a number that is in the middle" [94, about mean]. Many students exemplified this by giving two numbers: "If you have 7 and 9, the mean is 8" [100; about mean]. Students used similar explanations in these answers to describe mean and median, namely the word 'middle.' In that sense, the explanations can be considered incomplete given that the central mathematical properties that distinguish mean from median are not explicit.

Sometimes, the descriptive words were used tautologically. The most frequent descriptive word was "middle" to describe the median. The results also showed that approximately 18% of the students described the mode using a homonym. The students recognised and used the first syllable of the word typvärde, the Swedish word for the mode. This word, typ, is frequently used to mean "kind of." Using it to explain the mode leads to answers as follows: "It is an approximate value" [111; about mode].

## 4.2. EASIEST AND HARDEST TO EXPLAIN

The following results are about whether students see a measure as easy or hard to explain (see Table 5).

*Table 5. Frequency (and percentage of total) of easiest and hardest measure to explain*

| | Mean | Median | Mode | Other | Total |
|---|---|---|---|---|---|
| Easiest to explain | 34 (26.2) | 12 (9.2) | 80 (61.5) | 4 (3.1) | 130(100) |
| Hardest to explain | 49 (37.7) | 57 (43.8) | 15 (11.5) | 9 (6.9) | 130 (100) |

Almost two-thirds of the students (62%) expressed that mode was the easiest measure of central tendency to explain, and also the measure that the fewest students identified as hardest to explain. For the median, it was the opposite relationship: few found it easy to explain, and it was the measure that most found hardest to explain. The responses to the mean were more diverse. As a follow-up analysis, the students' written explanations were analysed using the themes Conceptions (Co), Mathematical Knowledge (MK), Contextual Knowledge (CxK), Vocabulary Knowledge (VK), Do not know, and Not relevant (NR). The distribution between the categories on what was easiest to explain is shown in Table 6:

*Table 6. Frequency (and percentage of total) of themes in students' explanations for the easiest measure to explain*

| Theme | Mean | Median | Mode | Other | Total |
|---|---|---|---|---|---|
| MK | 7 (4.3) | 2 (1.2) | 23 (14.0) | 0 (0) | 32 (19.5) |
| CxK | 1 (0.6) | 0 (0) | 6 (3.7) | 0 (0) | 7 (4.3) |
| VK | 3 (1.8) | 2 (1.2) | 24 (14.6) | 0 (0) | 29 (17.6) |
| Co | 24 (14.6) | 6 (3.7) | 54 (32.9) | 2 (1.2) | 86 (52.4) |
| Do not know | 0 (0) | 0 (0) | 2 (1.2) | 1 (0.6) | 3 (1.8) |
| NR | 3 (1.8) | 2 (1.2) | 1 (0.6) | 1 (0.6) | 7 (4.2) |
| Total | 38 (23.1) | 12 (7.3) | 110 (67) | 4 (2.4) | 164 (100) |

*Note*. MK = Mathematical Knowledge, CxK = Context Knowledge, VK = Vocabulary Knowledge, Co = Conceptions, NR = Not Relevant

In Table 6, the theme with the highest proportion of responses is Conceptions (52.4%). Here, we find explanations based on students' self-evaluation of their knowledge and perceived confidence.

Some students related the conceptions to their own experiences, as illustrated with the following response: “Because it was easier to find an example” [11; about mode]. In this reply, the key objective is to exemplify. A few respondents found all measures easy to explain, as will be illustrated with two comments about the mean. The first student referred to how much schooling they had about the concept: “Because we have worked with it a lot” [99; about mean]. The other reply highlighted student confidence: “Because that is what I know best” [91; about mean]. These answers were based on familiarity but provided no further information about why the measure was easiest to explain.

There are also explanations connected to various mathematical properties of the concepts. One example is from student 112, who wrote about mean: “There will be no decimal sign” [112; about mean]. This explanation signals confusion between the measures because the mean can have decimals. Nevertheless, this particular student found it easier to understand a measure that did not have decimals.

One category of Vocabulary Knowledge in Table 6 is homonyms, which were used to explain the concept of mode: “Because the answer is in the word” [125; about mode]. This reply illustrates how the student tried to find a meaning for the measure using a homonymous meaning for the word.

Looking at results for the hardest to explain, the distribution among themes is presented in Table 7.

*Table 7. Frequency (and percentage of total) of themes in students’ explanations for the hardest measure to explain*

| Theme | Mean | Median | Mode | Other | Total |
|---|---|---|---|---|---|
| MK | 22 (15.5) | 6 (4.2) | 2 (1.4) | 0 (0) | 30 (21.1) |
| CxK | 1 (0.7) | 0 (0) | 0 (0) | 0 (0) | 1 (0.7) |
| VK | 0 (0) | 6 (4.2) | 0 (0) | 1 (0.7) | 7 (4.9) |
| Co | 30 (21.1) | 36 (25.4) | 7 (4.9) | 5 (3.5) | 78 (54.9) |
| Do not know | 1 (0.7) | 13 (9.2) | 4 (2.8) | 3 (2.1) | 21 (14.8) |
| NR | 1 (0.7) | 2 (1.4) | 2 (1.4) | 0 (0) | 5 (3.5) |
| Total | 55 (38.7) | 63 (44.4) | 15 (10.5) | 9 (6.3) | 142 (99.9) |

*Note*. MK = Mathematical Knowledge, CxK = Context Knowledge, VK = Vocabulary Knowledge, Co = Conceptions, NR = Not Relevant

As in Table 6, the Conception theme has the most significant percentage of responses in Table 7 (54.9%), especially for the mean and the median. Some explanations were about how students thought about the procedures. For example: “It is hard to explain precisely what you get and express it” [53; about mean]. Student 53 focused on the procedure and how to describe the answer. The procedure was also part of the Mathematical Knowledge theme. When looking at the theme, most responses were about the mean. One example is: “You need to explain in a way that the other person understands and not believe that you should divide by five all the time; it depends on how many numbers there are” [86; about mean]. The response signalled mathematical knowledge about the specific transformations. Looking at the next theme, Vocabulary Knowledge, we found a variety of responses. To illustrate this variation, we present two responses about the median. The first response illustrates that the word stands for something complicated: “Because the median does not sound like something you can guess if you have forgotten it” [76; about median]. From the student’s response, we see that procedures connected to the word (here, median) are not revealed in the word itself. The second response was that the word in itself was considered odd: “The word is strange” [117; about median]. Responses similar to these last two signalled that the students might struggle to connect the concept’s name and meaning.

### 4.3. MOST AND LEAST USEFUL

The results of the measures the students considered most and least useful are presented in Table 8.

*Table 8. Frequency (and percentage of total) for most and least useful measure of central tendency*

| | Mean | Median | Mode | Other | Total |
|---|---|---|---|---|---|
| Most useful | 64 (49.2) | 12 (9.2) | 34 (26.2) | 20 (15.4) | 130 (100) |
| Least useful | 5 (3.8) | 71 (54.6) | 26 (20.0) | 28 (21.5) | 130 (99.9) |

As shown in Table 8, almost half of the students picked mean as the most useful measure and median as the least useful. Only 3.8% of the students in this study found the mean to be least useful. The responses for mode did not show a majority for either most or least useful, but there were approximately equal numbers of responses in both categories. The results produced two preliminary hypotheses. First, students appear to perceive measures as different, and second, usage does not primarily depend on context. To see if these two preliminary results were correct, we further analysed the responses using some aspects of statistical literacy. The results are presented in Table 9.

*Table 9. Frequency (and percentage) of themes in students' explanations for most useful measure*

| Theme | Mean | Median | Mode | Other | Total |
|---|---|---|---|---|---|
| MK | 24 (14.3) | 2 (1.2) | 1 (0.6) | 3 (1.8) | 30 (17.9) |
| CxK | 40 (23.8) | 0 (0) | 20 (11.9) | 1 (0.6) | 61 (36.3) |
| VK | 11 (6.5) | 9 (5.4) | 14 (8.3) | 0 (0) | 34 (20.2) |
| Co | 8 (4.8) | 4 (2.4) | 6 (3.6) | 2 (1.2) | 20 (11.9) |
| Do not know | 3 (1.8) | 0 (0) | 2 (1.2) | 12 (7.1) | 17 (10.1) |
| NR | 2 (1.2) | 1 (0.6) | 0 (0) | 3 (1.8) | 6 (3.6) |
| Total | 88 (52.4) | 16 (9.6) | 43 (25.6) | 21 (12.5) | 168 (100) |

*Note*. MK = Mathematical Knowledge, CxK = Context Knowledge, VK = Vocabulary Knowledge, Co = Conceptions and NR = Not Relevant

Looking at Table 9, approximately one-third of the responses for mean and mode were coded for the Context Knowledge theme. For the mean, context knowledge codings resulted when the mean was considered useful in a context from the real world, in real life, in society, in work situations, or in everyday situations. Student 67 wrote, "It can be useful for everyday life, for instance, the mean for the weather during one week. Alternatively, if you want to know the mean for approximately how old people are in a football team" [67; about mean]. Another word that students used to refer to everyday situations was "general," illustrated by the following response: "One can easily see what people generally think, such as their favourite ice cream, what kind of dog people have, etc." [69; about mean]. It should be noted that the example from student [69] indicated confusion between mean and mode. Similar responses were found in students' explanations about mode: "It is important to compare results in an election and so on" [29; about mode]. What these responses have in common is that they refer to usage in society and real-life situations.

Some explanations stressed various mathematical properties to argue why the measures were most useful. Such responses were categorised as Mathematical Knowledge. The following two responses about mean illustrate this theme: "You should use the mean in investigations as it gives you an average of all results. You can also vary which method you use depending on whether you want to affect someone in a particular way" [65; about mean]; "To achieve an average, but if there is a high number that pushes up the mean, it could be good to use the median or the mode" [66; about mean]. In the latter response, student 66 used mathematical knowledge to show an understanding of data, in this case that extreme values affect the mean. In the former response, student 65 combined MK with CxK.

Again, as signalled in previous tables, in the theme for Vocabulary Knowledge, several students referred to the word "typ" ("kind of") in a homonymous way when speaking about the mode. This is illustrated by the following explanation: "Because it is the correct answer, kind of " [120, about mode]. Here, speaking about usefulness, the student expressed a conception about the measure that is neither mathematically correct nor signals an understanding of data.

When looking at the responses about the least useful measure in Table 10, the results indicated differences between how the measures were perceived. Most of the responses had explanations categorised as under the Context Knowledge theme (28.5%). Most responses in this theme are about the median, and several students questioned whether the median existed or saw it as a concept only existing in school. We illustrate this with two replies: "Since the median is probably used only in school assignments and not outside school, who wants to know what number is in the middle?" [30; about median] and "This word seems useless for anyone except if you will become a teacher" [101; about median]. The last quote is interesting, we think, because it can be interpreted as that the median is only useful for teachers and, hence, not useful for everyone else. There were a few students whose responses were exceptions by indicating that the central tendencies were all useful independent of whether you were aware of it or not. One was student 128: "None of the measures, because you use all of them in school and in your free time even if you do not think about it" [128; about Other]. These replies could be interpreted as understanding the role of statistics (and mathematics) in society.

*Table 10. Frequency (and percentage) of themes in students' explanations for least useful measure*

| Theme | Mean | Median | Mode | Other | Total |
|---|---|---|---|---|---|
| MK | 1 (0.6) | 10 (6.1) | 5 (3.0) | 3 (1.8) | 19 (11.5) |
| CxK | 1 (0.6) | 35 (21.2) | 8 (4.8) | 3 (1.8) | 47 (28.4) |
| VK | 1 (0.6) | 20 (12.1) | 5 (3.0) | 0 (0) | 26 (15.7) |
| Co | 2 (1.2) | 17 (10.3) | 11 (6.7) | 4 (2.4) | 34 (20.6) |
| Do not know | 1 (0.6) | 11 (6.7) | 4 (2.4) | 16 (9.7) | 32 (19.4) |
| NR | 0 (0) | 2 (1.2) | 2 (1.2) | 3 (1.8) | 7 (4.2) |
| Total | 6 (3.6) | 95 (57.6) | 35 (21.1) | 29 (17.6) | 165 (100) |

*Note.* MK = Mathematical Knowledge, CxK = Context Knowledge, VK = Vocabulary Knowledge, Co = Conceptions, NR = Not Relevant

## 5. DISCUSSION

The present study explored students' statistical literacy in grade 6 (ages 12–13) regarding mean, median, and mode. Results for the first research question, 'What aspects of the different knowledge elements do students use in their descriptions of the measures of central tendency?' showed that the aspects differed among the measures of central tendency. A relatively high percentage of students (55.4–73.9%) used different mathematical knowledge, including wrong procedures, to describe each of the three measures. However, for median and mode, vocabulary knowledge was at the core of the responses in ways that align with previous research that concludes that measures are often presented using colloquial terms (Mokros & Russell, 1995). Overall, many students gave incomplete definitions or connected measures to incorrect definitions, similar to both students of similar age and students at the university level in previous studies (e.g., Clark et al., 2007; Madrid-García et al., 2023; Mayén et al., 2009; Mokros & Russell, 1995). In addition, there are examples of students using colloquial words tautologically without explanations—results that have been reported earlier (e.g., Callingham & Watson, 2017; Clark et al., 2007). Interestingly, although these studies have been conducted in countries where context and language differ, similar struggles have been found. Looking closer at students' attempts to explain the median and mode, we saw how unfamiliar many students seemed to be with these concepts. This unfamiliarity seemed to hinder them from communicating their understanding (e.g., Madrid-García et al., 2023; Usiskin, 2012; Weiland, 2017). Such a conclusion aligns with Makar and Confrey's (2005) observation that the use of standard terminology for statistical concepts may not advance making connections or uncovering relationships in data when concepts are treated as isolated facts. One implication is that students should be encouraged to use their language to make sense of concepts. Relatedly, we want to acknowledge one limitation of this study: respondents might have found it difficult to formulate a definition/description of a concept (Watson, 2007). Hence, the responses captured only what participants could articulate at one particular point in time.

The second research question was: 'What characterises the explanations given by grade 6 students about which measure is easiest or hardest to explain?' The results revealed that most students found the mode to be the easiest measure of central tendency to explain, similar to the responses of prospective and in-service elementary teachers who were asked the same questions (Landtblom & Sumpter, 2021), whereas the median was considered the hardest to explain. In addition, almost one-fifth of the students appeared to struggle to separate the mean from the median. Similar results for preservice teachers, in-service teachers, and students at the secondary and university levels have been reported earlier (e.g., Groth & Bergner, 2006; Jacobbe, 2012; Madrid-García et al., 2023; Mayén et al., 2009). Given this, when teaching measures, students might benefit from instruction that pushes for understanding beyond memorising the procedures for calculating the mean and the median for data sets. Here, several aspects of statistical literacy can be helpful (Gal, 2002). Students should know why the procedures work the way that they do, that is, to connect mathematical knowledge with contextual knowledge (Sumpter et al., 2024).

The third research question was, ‘What characterises the explanations given by grade 6 students about which measure is most or least useful?’ Students’ responses were often about the context, which—at first glance—can be interpreted as productive because several studies have stressed the importance of understanding measures of central tendency within the context of the data (e.g., Büscher, 2019). Our results support such a conclusion because some students evidenced an understanding of the mean and the mode connected to the context. However, our study contributes additional results about students’ expressed conceptions. More than one-fourth of the students thought the median was less useful because it did not appear in real-life situations. They described the median as only existing in (school) mathematics. This result is similar to what Batanero and colleagues (2020) observed. The results imply that teachers and researchers in mathematics education have a challenge in convincing students that the median is relevant in everyday society (e.g., Callingham & Watson, 2017; Watson & Callingham, 2003). This challenge is crucial given that context knowledge supports students’ awareness of what is represented in a dataset (e.g., Chick & Pierce, 2012). One example of support could be addressing students’ understandings about when a measure of central tendency is useful and when it is not.

Few studies have investigated students’ conceptions, including attitudes towards statistics and their relation to knowledge elements (Bond et al., 2012; Carmichael et al., 2010; Konold et al., 2015). Therefore, findings related to the third research question contribute new knowledge. Here, the results present several examples where students’ conceptions and context knowledge appeared to be intertwined. One example is from student 30, who knew the mathematical procedure for the median but also expressed a conception that the median was only valid in school. Philipp’s (2007) definition of conception encompasses preferences and meanings that can contain objective and subjective knowledge, which is different from knowing when a measure is valid. Although we did not focus on a critical stance in this study, we see that some students provided explanations that signalled such awareness and could have been coded as critical stance. One example was student 65, who wrote: “It can also vary which method you use if you want to affect someone in a specific way.” Here, the student signalled a connection between different aspects of Gal’s (2002) framework: dispositional elements (limited to conceptions), context knowledge, and critical stance. Given that statistical literacy is often stressed as an essential tool for members of future society (e.g., Carmichael et al., 2010; Çatman Aksoy & Işıksal Bostan, 2021), we concluded that we need further studies on how context knowledge, including a critical stance, can be developed together with mathematical knowledge and dispositional elements. Because research studies already indicate that students tend to learn measures at a higher grade level, where mathematics and contexts are brought into a parallel learning process (Büscher, 2019; Watson & Callingham, 2003), like Weiland (2019), we would like to suggest that critical stance and critical questions are two elements relevant for further research and design of teaching and junior students.

## REFERENCES

Arnold, P., & Pfannkuch, M. (2022). Engaging novice statisticians in statistical communications. *Mathematics Education Research Journal*, *36*(Suppl 1), 1–27. https://doi.org/10.1007/s13394-022-00442-w

Batanero, C., Valenzuela-Ruiz, S. M., & Gea, M. M. (2020). Significados institucionales y personales de los estadísticos de orden en la Educación Secundaria. [Institutional and personal meaning of order statistics in secondary education] *Matemáticas, Educación y Sociedad, 3*(2), 21–39. https://journals.uco.es/mes/article/view/12912/11744

Bond, M. E., Perkins, S. N., & Ramirez, C. (2012). Students' perceptions of statistics: An exploration of attitudes, conceptualizations, and content knowledge of statistics. *Statistics Education Research Journal*, *11*(2), 6–25. https://doi.org/10.52041/serj.v11i2.325

Braun, V., Clarke, V., Boulton, E., Davey, L., & McEvoy, C. (2020). The online survey as a qualitative research tool. *International Journal of Social Research Methodology*, *24*(6), 641–654. https://doi.org/10.1080/13645579.2020.1805550

Büscher, C. (2019). Students' development of measures. In G. Burrill, & D. Ben-Zvi (Eds.), *Topics and trends in current statistics education research* (pp. 27–50). Springer. https://doi.org/10.1007/978-3-030-03472-6_2

Callingham, R., & Watson, J. M. (2017). The development of statistical literacy at school. *Statistics Education Research Journal*, *16*(1), 181–201. https://doi.org/10.52041/serj.v16i1.223

Carmichael, C., Callingham, R., Hay, I., & Watson, J. (2010). Statistical literacy in the middle school: The relationship between interest, self-efficacy and prior mathematics achievement. *Australian Journal of Educational & Developmental Psychology, 10*, 83–93.

Çatman Aksoy, E., & Işıksal Bostan, M. (2021). Seventh graders' statistical literacy: An investigation on bar and line graphs. *International Journal of Science and Mathematics Education*, *19*(2), 397–418. https://doi.org/10.1007/s10763-020-10052-2

Chick, H. L., & Pierce, R. (2012). Teaching for statistical literacy: Utilising affordances in real-world data. *International Journal of Science and Mathematics Education*, *10*(2), 339–362. https://doi.org/10.1007/s10763-011-9303-2

Clark, J., Kraut, G., Mathews, D., & Wimbish, J. (2007). *The fundamental theorem of statistics: Classifying student understanding of basic statistical concepts.* [Unpublished manuscript].

Denscombe, M. (2017). *The good research guide: For small-scale social research projects* (6th ed.). Open University Press.

Diego-Mantecón, J. M., Blanco, T. F., Chamoso, J. M., & Cáceres, M. J. (2019). An attempt to identify the issues underlying the lack of consistent conceptualisations in the field of student mathematics-related beliefs. *Plos One*, *14*(11), Article e0224696. https://doi.org/10.1371/journal.pone.0224696

Fry, K., English, L., & Makar, K. (2024). Cognitive tuning in the STEM classroom: Communication processes supporting children's changing conceptions about data. *Mathematics Education Research Journal*, *36*(Suppl 1), 67–89. https://doi.org/10.1007/s13394-023-00465-x

Gal, I. (2002). Adults' statistical literacy: Meaning, components, responsibilities. *International Statistical Review, 70*(1), 1–52. https://doi.org/10.2307/1403713

Gal, I., Nicholson, J., & Ridgway, J. (2023). A conceptual framework for civic statistics and its educational applications. In J. Ridgway (Ed.), *Statistics for empowerment and social engagement: Teaching civic statistics to develop informed citizens* (pp. 37– 66). Springer. https://doi.org/10.1007/978-3-031-20748-8_3

Garfield, J., & Ben-Zvi, D. (2007). How students learn statistics revisited: A current review of research on teaching and learning statistics. *International Statistical Review, 75*(3), 372–396. https://doi.org/10.1111/j.1751-5823.2007.00029.x

Goldin, G. A. (2002). Affect, meta-affect, and mathematical belief structures. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 59–72). Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47958-3_4

Groth, R. E., & Bergner, J. A. (2006). Preservice elementary teachers' conceptual and procedural knowledge of mean, median, and mode. *Mathematical Thinking and Learning, 8*(1), 37–63. https://doi.org/10.1207/s15327833mtl0801_3

Groth, R. E., & Bergner, J. A. (2013). Mapping the structure of knowledge for teaching nominal categorical data analysis. *Educational Studies in Mathematics, 83*(2), 247–265. https://doi.org/10.1007/s10649-012-9452-4

Groth, R., & Meletiou-Mavrotheris, M. (2018). Research on statistics teachers' cognitive and affective characteristics. In D. Ben-Zvi, K. Makar, & J. Garfield (Eds.), *International Handbook of Research in Statistics Education* (pp. 327–355). Springer. https://doi.org/10.1007/978-3-319-66195-7_10

Jacobbe, T. (2012). Elementary school teachers' understanding of the mean and median. *International Journal of Science and Mathematics Education*, *10*(5), 1143–1161. https://doi.org/10.1007/s10763-011-9321-0

Konold, C., Higgins, T., Russell, S. J., & Khalil, K. (2015). Data seen through different lenses. *Educational Studies in Mathematics, 88*(3), 305–325. https://doi.org/10.1007/s10649-013-9529-8

Landtblom, K. K. (2018). Prospective teachers' conceptions of the concepts mean, median and mode. In H. Palmér & J. Skott (Eds.), *Students' and teachers' values, attitudes, feelings and beliefs in mathematics classrooms* (pp. 43–52). Springer. https://doi.org/10.1007/978-3-319-70244-5_5

Landtblom, K. (2023). Opportunities to learn mean, median, and mode afforded by textbook tasks. *Statistics Education Research Journal*, *22*(3), Article 6. https://doi.org/10.52041/serj.v22i3.655

Landtblom, K., & Sumpter, L. (2021). Teachers and prospective teachers' conceptions about averages. *Journal of Adult Learning, Knowledge and Innovation*, *4*(1), 1–8. https://doi.org/10.1556/2059.03.2019.02

Leavy, A., & O'Loughlin, N. (2006). Preservice teachers' understanding of the mean: Moving beyond the arithmetic average. *Journal of Mathematics Teacher Education*, *9*(1), 53–90. https://doi.org/10.1007/s10857-006-9003-y

Leavy, A., & Hourigan, M. (2016). Crime scenes and mystery players! Using driving questions to support the development of statistical literacy. *Teaching Statistics*, *38*(1), 29–35. https://doi.org/10.1111/test.12088

Lesser, L. M., Wagler, A. E., & Abormegah, P. (2014). Finding a happy median: Another balance representation for measures of center. *Journal of Statistics Education, 22*(3), 1–27. https://doi.org/10.1080/10691898.2014.11889714

Madrid-García, A. E., Valenzuela-Ruiz, S. M., & Batanero, C. (2023). Is the calculation of the median for university students simple? In L. Gómez Chova, C. González Martínez, & J. Lees (Eds.), *INTED2023 Proceedings: 17th International technology, education and development conference* (pp. 1253–1257). IATED Academy. https://doi.org/10.21125/inted.2023.0362

Mason, J. (2018). *Qualitative researching* (3rd ed.). Sage.

Makar, K., & Confrey, J. (2005). "Variation-talk": Articulating meaning in statistics. *Statistics Education Research Journal*, *4*(1), 27–54. https://doi.org/10.52041/serj.v4i1.524

Mathews, D., & Clark, J. (2003). *Successful students' conceptions of mean, standard deviation, and the central limit theorem.* [Unpublished manuscript].

Mayén, S., Díaz, C., & Batanero, C. (2009). Students' semiotic conflicts in the concept of median. *Statistics Education Research Journal*, *8*(2), 74–93. https://doi.org/10.52041/serj.v8i2.396

Mokros, J., & Russell, S. J. (1995). Children's concepts of average and representativeness. *Journal for Research in Mathematics Education*, *26*(1), 20–39. https://doi.org/10.2307/749226

Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic analysis: Striving to meet the trustworthiness criteria. *International Journal of Qualitative Methods*, *16*(1). https://doi.org/10.1177/1609406917733847

Philipp, R. A. (2007). Mathematics teacher's beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–315). Information Age Publishing.

Ramirez, C., Schau, C., & Emmioglu, E. (2012). The importance of attitudes in statistics education. *Statistics Education Research Journal*, *11*(2), 57–71. https://doi.org/10.52041/serj.v11i2.329

Schnell, S., & Frischemeier, D. (2019). Primary school students' reasoning about and with the median when comparing distributions. In U.T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education* (pp. 1075–2082). Freudenthal Group & Freudenthal Institute, Utrecht University, and ERME.

Sharma, S. (2017). Definitions and models of statistical literacy: A literature review, *Open Review of Educational Research 4*(1), 118–133. http://doi.org/10.1080/23265507.2017.1354313

Sproesser, U., Engel, J., & Kuntze, S. (2016). Fostering self-concept and interest for statistics through specific learning environments. *Statistics Education Research Journal*, *15*(1), 28–54. https://doi.org/10.52041/serj.v15i1.256

Strauss, S., & Bichler, E. (1988). The development of children's concepts of the arithmetic average. *Journal for Research in Mathematics Education*, *19*(1), 64–80. https://doi.org/10.2307/749111

Sumpter, L., Eriksson, H., Hedefalk, M., & Markkanen, P. (2024). Ethical reasoning as part of mathematical modelling: young children's work with sharing and division. In P. Ernest (Ed.). *Ethics and Mathematics Education: the Good, the Bad and the Ugly*. (pp. 443–462). Springer. https://doi.org/10.1007/978-3-031-58683-5_21

Swedish Research Council (2017). *Good research practice*. https://www.vr.se/english/analysis/reports/our-reports/2017-08-31-good-research-practice.html

Usiskin, Z. (2012). What does it mean to understand some mathematics? In S. J. Cho (Ed.), *Selected regular lectures from the 12th International Congress on Mathematical Education* (pp. 821–841). Springer. https://doi.org/10.1007/978-3-319-17187-6_46

Watson, J. M. (2006). *Statistical literacy at school: Growth and goals*. Routledge.

Watson, J. M. (2007). The role of cognitive conflict in developing students' understanding of average. *Educational Studies in Mathematics*, *65*(1), 21–47. http://doi.org/10.1007/s10649-006-9043-3

Watson, J., & Callingham, R. (2003). Statistical literacy: A complex hierarchical construct. *Statistics Education Research Journal*, *2*(2), 3–46. https://doi.org/10.52041/serj.v2i2.553

Weiland, T. (2017). Problematizing statistical literacy: An intersection of critical and statistical literacies. *Educational Studies in Mathematics*, *96*(1), 33–47. https://doi.org/10.1007/s10649-017-9764-5

Weiland, T. (2019). Critical mathematics education and statistics education: Possibilities for transforming the school mathematics curriculum. In G. Burrill & D. Ben-Zvi (Eds.), *Topics and Trends in Current Statistics Education Research* (pp. 391–411). Springer. https://doi.org/10.1007/978-3-030-03472-6_18

KARIN LANDTBLOM
IÄD, Stockholm University
106 91 Stockholm, SWEDEN