TEACHING STATISTICS WITH POSITIVE ORIENTATIONS BUT LIMITED KNOWLEDGE? TEACHERS' PROFESSIONAL COMPETENCE IN STATISTICS

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ABSTRACT

Research suggests teachers have positive motivational and emotional orientations regarding statistics but little statistical knowledge. How does this fit together? Since teachers' professional competence in statistics has not been well explored, we asked 88 in-service mathematics teachers about their orientations regarding teaching statistics and tested their statistical content knowledge. First, we investigated how "positive" their orientations were by comparing them to their orientations regarding teaching fractions. Then, we analyzed relationships between teachers' orientations and content knowledge in statistics using mixed-effects logistic regression models. The results showed that teachers' orientations regarding teaching statistics were: (1) poorer than those regarding teaching fractions and (2) related to their statistical knowledge. Teachers with high self-efficacy showed higher knowledge than teachers with low self-efficacy, and anxious female teachers had higher knowledge than less anxious female teachers. We also found that knowledge decreased with increasing age of the teachers. The findings underscore the need to strengthen statistics in teacher education, including both content knowledge and the development of positive orientations.

Keywords: *Statistics education research; Professional competence; Motivational and emotional orientations; Content knowledge; In-service teachers; Teacher education*

1. INTRODUCTION

In the wake of the *COVID19*-pandemic, the importance of statistics and data science was growing in the everyday life of citizens worldwide; statistical measures enabled comparisons of infection incidence, and data analyses served as a basis for decision-making about pandemic restrictions. This relevance also reinforced the importance and urgency of promoting statistical literacy in schools. Teachers are crucial in making this happen, as they can affect student achievement (Callingham et al., 2016). In order to fulfill their role as multipliers of statistical literacy properly, teachers need *professional competence*, which is considered a personal requirement for successfully coping with professional demands (Kunter et al., 2009). These requirements are not assumed unchangeable personal characteristics but rather learnable skills and orientations that teachers acquire and deepen during their vocational training and professional career (Kunter, 2014). Following the model of professional competence proposed by the *COACTIV* study, which examined *COgnitive ACTIVation* in mathematics

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classrooms and mathematics teachers' professional competence (Baumert et al., 2013), professional competence includes both cognitive and affective aspects (Baumert & Kunter, 2013).

Cognitive aspects of professional competence refer to teachers' *knowledge* (Baumert & Kunter, 2013), which comprises content knowledge, pedagogical content knowledge, and pedagogical knowledge (Shulman, 1986). There is wide agreement that teachers' knowledge is a key component of professional competence (Baumert & Kunter, 2013). It influences teachers' decisions about instructional activities and is fundamental to effective teaching (Walshaw, 2012). Moreover, teachers' knowledge is related to students' achievement (Callingham et al., 2016; Campbell et al., 2014; Hill et al., 2005).

Affective aspects of professional competence refer to "a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition" (McLeod, 1992, p. 576). They can be divided into more cognitive, motivational, and emotional *orientations* (Hannula, 2011). While cognitive orientations comprise mental ideas to which a truth value can be attributed (e.g., beliefs), motivational orientations reflect personal preferences and determine behavior (e.g., self-efficacy), and emotional orientations include feelings and moods (e.g., joy, anxiety; Hannula, 2011). Teachers' orientations are considered important for instruction (McLeod, 1992): They influence teaching practices (Groth & Meletiou-Mavrotheris, 2018; Kunter et al., 2008; Reinhold et al., 2021) and impact student learning (Groth & Meletiou-Mavrotheris, 2018; Schumacher, 2017) and student achievement (Schwarzer & Warner, 2014; Ulug et al., 2011).

Cognitive and affective aspects as part of teachers' professional competence have also received attention in the field of statistics. Within the framework of the *Joint International Commission on Mathematical Instruction*/*International Association for Statistical Education Study*, a working group investigated teachers' orientations regarding statistics, their statistical knowledge, and the effects of these factors on the teaching of statistics (Batanero et al., 2011; Part III). The working group revealed a need to increase and improve the quality of research regarding teachers' cognitive and affective aspects in statistics, as research in this area is scarce (Batanero, 2011). Previous findings suggested that teachers have positive orientations regarding statistics (Eichler & Zapata-Cardona, 2016) but, at the same time, lack statistical knowledge and encounter numerous challenges when teaching statistics (Garfield & Ben-Zvi, 2008). How does this fit together?

To answer this question, it seems relevant, on the one hand, to clarify how specific positive orientations are to statistics, for example, by comparing orientations regarding statistics with those regarding other mathematical content areas. The resulting findings may help teacher educators address certain topics during mathematics teacher training to a greater or lesser extent. For instance, if teachers have less self-efficacy regarding teaching statistics than regarding teaching other topics, it might be beneficial to spend more time during teacher education to address different teaching strategies for statistics instruction. Indeed, such courses about different teaching strategies increased prospective teachers' self-efficacy in the fields of natural sciences and technology (Schwarzer & Warner, 2014).

On the other hand, to answer the question above, it seems relevant to analyze teachers' orientations and knowledge jointly within the same sample, for example, identifying the extent to which teachers' self-efficacy matches their content knowledge or how self-efficacy and knowledge influence each other (Fives, 2003; Kunter, 2014; Sharp et al., 2016). A survey exploring factors contributing to teachers' self-efficacy regarding teaching statistics indicated relationships between teachers' orientations and knowledge; prospective teachers had to select one statistical item they felt least self-effective to teach and explain their reasons for doing so. Eighty-nine percent attributed their lack of content knowledge and 19% their lack of pedagogical content knowledge, while other reasons were rarely mentioned (Lovett & Lee, 2017). If such a relationship between teachers' self-efficacy and their knowledge can be identified empirically, it could imply that more in-depth teaching of knowledge in teacher education or professional development programs increases teachers' self-efficacy. Thus, research regarding teachers' orientations and knowledge in statistics holds the potential to understand relationships among teachers' professional competence and enhance teacher education and professional development programs.

Previous studies on teachers' orientations and knowledge in statistics have often investigated prospective teachers rather than in-service teachers. Studies with in-service teachers, who have full responsibility for a class, provide better insights into teachers' professional competence concerning their teaching practice (Eichler & Zapata-Cardona, 2016). For these reasons, we examined *in-service* teachers' professional competence in the present study. First, we investigated_how positive teacher motivational and emotional orientations regarding teaching statistics are by comparing them to their orientations regarding teaching fractions. Then, we analyzed the relationships between teachers' orientations regarding teaching statistics and their statistical content knowledge.

1.1. TEACHERS' ORIENTATIONS REGARDING STATISTICS

Motivational and emotional orientations as part of teachers' affect may explain differences in teachers' professional competence. Teachers' *motivation* may be related to their engagement in the profession, which may be reflected in their intention to participate in professional development activities (Kunter, 2014; Pelletier & Rocchi, 2016) or in their willingness to invest cognitive resources (e.g., systematic course improvements, efforts at class preparation; Kunter, 2014; Neves de Jesus & Lens, 2005). It may also have effects on their instructional quality (Han & Yin, 2016; Kunter & Holzberger, 2014), for example, through better subject mastery (Pelletier & Rocchi, 2016), a wider range of teaching strategies (Neves de Jesus & Lens, 2005), or improved rationales and examples provided to students (Pelletier & Rocchi, 2016). Teachers' *emotions*—such as joy over a successful lesson, anger over a student's behavior, or fear of not being able to adequately answer students' questions—may be expressed in the relationships and interactions with their students and in their instructional behavior (Frenzel, 2014; Hascher & Krapp, 2014), such as lesson design (Hascher & Krapp, 2014), subject knowledge (Frenzel, 2014), creativity and use of vivid examples (Frenzel, 2014; Hascher & Krapp, 2014), handling of mistakes (Frenzel, 2014; Hascher & Krapp, 2014), or intensity of student support (Hascher & Krapp, 2014).

Concerning the content area statistics, teachers' orientations are under-researched (Estrada et al., 2011; Shaughnessy, 2007) and less considered than teachers' cognitive aspects. Moreover, studies predominantly focused on the orientations of prospective teachers who did not yet have full teaching experience (Eichler & Zapata-Cardona, 2016). Many studies with prospective teachers (e.g., Estrada & Batanero, 2008; Hannigan et al., 2013; Nasser, 2004; Zientek et al., 2011) measured their orientations regarding statistics with (modified versions of) the *Survey of Attitudes Toward Statistics* (SATS), which comprises cognitive, motivational, and emotional orientations (i.e., feelings, cognitive competence, value, difficulty, interest, and effort; Schau, 2019; Schau et al., 1995). In a recent study with prospective teachers, Ruz et al. (2021) used the *Attitudes' Scale Towards Probability and Its Teaching* (Estrada et al., 2018) and extended its assessment beyond orientations in probability to encompass orientations in statistics (i.e., feelings, cognitive competence, value, and behavior). The *Attitudes' Scale Towards Probability and Its Teaching* is particularly noteworthy because it assesses orientations regarding statistics and *teaching* statistics. Such assessments focusing on teaching statistics are rare but relevant to better understanding teachers' mediating role between statistical content and students (Groth & Meletiou-Mavrotheris, 2018). Another well-known instrument that focuses on orientations regarding the teaching of statistics is the *Self-Efficacy to Teach Statistics* (SETS) scale (Harrell-Williams et al., 2014) along with its version for high school (SETS-HS; Harrell-Williams et al., 2019). These two scales have been used in studies, again with prospective teachers, by Harrell-Williams et al. (2015) and Lovett and Lee (2017), respectively. A major result of all the previous studies was that prospective teachers tend to hold positive orientations regarding (teaching) statistics. Recent research suggests that in-service teachers also tend to hold positive orientations regarding statistics. Regarding their teaching of descriptive statistics, in-service teachers reported positive motivational and emotional orientations (i.e., self-efficacy, joy, anxiety, anger, and boredom; Schumacher, 2017). It is important, however, to mention that the purpose of Schumacher's (2017) study was to pilot the corresponding test instrument, and the sample of in-service teachers was positively selected, as pointed out by the author. This fact could potentially limit the generalizability of the results.

Previous research suggested teachers generally hold positive orientations regarding statistics, but what constitutes positive remains open. In the literature, teachers' orientations are thought to be context-specific, meaning they can vary depending on specific settings or circumstances, such as teaching certain subjects or students (Furinghetti & Pehkonen, 2002; Tschannen-Moran et al., 1998). Moreover, research has shown that teachers' orientations are domain-specific, meaning they may differ across different content areas within a subject (Eichler & Erens, 2015; Törner, 2002). Hence, comparisons to other mathematical content areas seem appropriate for better classifying teachers' orientations regarding statistics. Such comparisons are particularly relevant because it is common practice to teach statistics in school as part of mathematics and to train teachers in statistics as part of their mathematics teacher training. Previous surveys have indicated that teachers' orientations regarding statistics might be poorer than their orientations regarding other mathematical content areas. The majority of prospective teachers (63%) surveyed by Lovett and Lee (2017) rated themselves as least confident in teaching statistics. Although generally liking teaching stochastics (which comprises statistics), in-service teachers surveyed by Schumacher (2017), ranked stochastics lower in popularity than other mathematical content areas. For a more precise understanding of teachers' positive orientations regarding statistics, further systematic investigations and comparisons to mathematical content areas are required.

1.2. RELATIONSHIP BETWEEN TEACHERS' ORIENTATIONS AND KNOWLEDGE IN STATISTICS

In addition to studying teachers' motivational and emotional orientations, examining relationships between these orientations and cognition, that is, teachers' knowledge, is also relevant. For instance, research examining self-reported *motivation* as a predictor of teachers' professional behavior is important in clarifying how to achieve professionalism in the teaching profession (Kunter, 2014). General findings regarding *emotions* state that emotional orientations are closely related to cognitive processes, such as positive emotions supporting cognitive performance. Research regarding these relationships between emotions and cognition, specifically regarding the teaching profession, is largely lacking (Hascher & Krapp, 2014).

Concerning the content area statistics, little is known about the relationship between teachers' orientations and knowledge (Eichler & Zapata-Cardona, 2016; Groth & Meletiou-Mavrotheris, 2018). Our study addresses this relationship, focusing on the relationship between teachers' orientations and their *content knowledge* in statistics, as prior studies have documented low performance of statistics teachers even on tasks that do not go beyond the school level, suggesting that teachers' statistical content knowledge does not always meet the requirements of their profession (Lovett & Lee, 2017; Ruz et al., 2021; Schumacher, 2017). Given teachers' generally positive orientations regarding statistics reported in the previous section, this may seem contradictory. A joint consideration of teachers' orientations and content knowledge in statistics may help to clarify this contradiction.

Previous findings on the relationship between teachers' orientations and content knowledge in statistics primarily stem from studies with prospective teachers and, therefore, do not relate to teaching practice (Eichler & Zapata-Cardona, 2016). These studies with prospective teachers yielded contradictory results. Hannigan et al. (2013) observed no associations between teachers' orientations and knowledge in statistics, whereas Nasser (2004) as well as Estrada and Batanero (2008) found weak correlations. Zientek et al. (2011) and Ruz et al. (2021) reported moderate associations.

This discrepancy could be due to differences in the test instruments. While the studies (except from Ruz et al., 2021) used similar test instruments to measure teachers' orientations (i.e., modifications of the SATS; see Section 1.1), they all used different instruments to measure teachers' content knowledge. Two studies used standardized tests. Hannigan et al. (2013) used the multiple-choice format *Comprehensive Assessment of Outcomes in a First Statistics course* (CAOS; delMas et al., 2007), and Estrada and Batanero (2008) used the multiple-choice format *Statistical Reasoning Assessment* (SRA; Garfield, 2003). The other three studies used proprietary measures. Nasser (2004) used scores of open-ended format course exams, Zientek et al. (2011) used final course grades, and Ruz et al. (2021) used self-developed test items in multiple-choice format. Hence, approaches for measuring teachers' content knowledge differed in content and methodology. Moreover, the instruments used to measure orientations and content knowledge were not aligned, which is unfortunate because instruments that assess cognitive and affective aspects on the same items have more potential to reveal relationships between different aspects of teachers' professional competence. Such a holistic framework that assesses both orientations and knowledge in statistics is the *BeSt Teacher* framework, an instrument with evidence of validity developed to measure in-service teachers' professional competence in descriptive statistics. In a pilot study to evaluate this framework, in-service teachers showed moderate correlations between orientations and content knowledge in statistics (Schumacher, 2017). Due to the positively selected sample, the results, however, are of limited significance.

In addition to examining correlative relationships between teachers' orientations and content knowledge in statistics, some of the aforementioned studies with prospective teachers used more sophisticated methods. Using structural equation models, Nasser (2004) found that more positive orientations regarding statistics were aligned with increased teachers' statistical knowledge. Using a multiple regression model, Zientek et al. (2011) concluded that different facets of teachers' orientations regarding statistics (i.e., feelings, cognitive competence, value, difficulty, interest, and effort) explained their statistical knowledge.

In summary, existing findings on the relationship between prospective statistics teachers' orientations and content knowledge are inconclusive. Additionally, this relationship still requires analysis among in-service teachers. Since orientations and knowledge are assumed to be formed during training and on the job (Kunter, 2014), further insights into their relationship can help to optimize teacher education and professional teacher development in a more targeted way. It could be, for example, that more intensive teaching of statistical content knowledge during teacher training reduces teachers' anxiety when teaching statistics because they feel better prepared, or that conducting a data evaluation with digital tools increases teachers' motivation and thus encourages them to engage more intensively with statistical content.

1.3. THE PRESENT STUDY

We aimed to complement existing research about teachers' professional competence. Therefore, our study analyzed in-service teachers' motivational and emotional orientations regarding teaching statistics compared to teaching other mathematical content areas. Moreover, our study clarified the relationship between teachers' motivational and emotional orientations regarding teaching statistics and their statistical content knowledge.

Previous studies of professional competence in statistics with predominantly prospective teachers did not consider teachers' experiences in the classroom (Eichler & Zapata-Cardona, 2016). Our study addresses this issue in several ways: (1) We studied *in-service* teachers. Unlike prospective teachers, they have full control of a class and are in the midst of their professional practice, which could impact their orientations and knowledge (Eichler & Zapata-Cardona, 2016). (2) Instead of focusing on teachers' orientations regarding statistics, we considered teachers' orientations regarding *teaching* statistical content in school. For mathematics, such a distinction between teachers' orientations regarding the subject and orientations regarding *teaching* the subject was important when studying effects on instructional behavior (Kunter et al., 2008). In regard to statistics, research has not yet made a substantial distinction between these two approaches, making it difficult to study the effects of teachers' orientations (Groth & Meletiou-Mavrotheris, 2018). (3) We examined teachers' statistical content knowledge at a level that does not go beyond *school subject matter*. This level reflects the content requirements for teachers when teaching statistics. It is, therefore, particularly interesting to analyze relationships with teachers' orientations regarding *teaching* statistics, especially because teachers appear to have similar difficulties with statistical concepts as their students (Garfield & Ben-Zvi, 2008; Shaughnessy, 2007). (4) We included demographic *teacher characteristics* (i.e., teachers' gender, age, and teaching experience) as control variables when analyzing the relationships between teachers' orientations and their content knowledge. Teachers' gender and teaching experience are relevant predictors for teachers' mathematical content knowledge (Haroun et al., 2016) and thus might also be responsible for differences in teachers' statistical content knowledge. Furthermore, it is conceivable that differences in statistical knowledge are due to teachers' age as there have been curricular changes in the school subject statistics (Ben-Zvi & Makar, 2016) and adjustments in teacher training (Eichler & Erens, 2015) since the beginning of this century.

We used the *BeSt Teacher* framework (Schumacher, 2017) as an assessment method because it was designed specifically for in-service teachers and allowed for simultaneous investigation of both teachers' orientations and knowledge in the field of statistics. We assessed teachers' *self-efficacy* regarding statistics as an aspect of their *motivation* to understand how well they can support and promote their students' learning (Harrell-Williams et al., 2014; Kunter, 2014; Schwarzer & Warner, 2014). As aspects of teachers' *emotions* when teaching statistics, we assessed teachers' *joy* as their response to good teaching (Emmons, 2020) and their *anxiety* as their worries and tension (Zeidner, 1991).

For comparing teachers' motivational and emotional orientations regarding teaching statistics with another mathematical content area, we selected the content area fractions, which we considered appropriate for several reasons. Our study was conducted in Colombia (further explanations provided in the next section), where the Colombian *Basic Standards of Competences in Mathematics* (Ministerio de Educación Nacional, Colombia, 2006) define the fundamental competences for the subject of mathematics that students should achieve by the end of certain grade levels. The competences comprise five types of mathematical thinking: *numerical thinking and numerical systems*, *spatial thinking and geometric systems*, *metric thinking and measurement systems*, *probabilistic thinking and data systems*, *variational thinking and algebraic and analytic systems*. Statistics falls under *probabilistic thinking and data systems*, while fractions belong to *numerical thinking and numerical systems*. Thus, fractions are a sufficiently distinct content. On the other hand, fractions and statistics are similar because both are typically taught in early secondary education, often at the same or a similar grade level. Also, statistics and fractions both often require analyzing and interpreting different representations.

Our study addresses the following research questions:

Research Question 1. Do in-service teachers' motivational and emotional orientations regarding teaching statistics differ from those regarding teaching fractions?

Since previous surveys have indicated that teachers' orientations regarding statistics might be poorer than their orientations regarding other mathematical content areas, the hypothesis was that there are differences in teachers' motivational and emotional orientations to the disadvantage of the content area statistics. In particular, when teaching statistics, we expected teachers to be less self-effective, less joyful, and more anxious compared to teaching fractions.

Research Question 2. Are in-service teachers' motivational and emotional orientations regarding teaching statistics related to their statistical content knowledge?

As previous findings with in-service teachers are limited and previous studies with prospective teachers have yielded inconsistent results regarding the relationship between orientations and knowledge, we analyzed this question on an exploratory basis.

2. METHOD

2.1. PARTICIPANTS AND PROCEDURE

We conducted a cross-sectional study with 88 in-service mathematics teachers (34 female, 53 male, 1 did not specify). These teachers participated in a 4-hour professional teacher development program on statistics in Medellín, Colombia, organized by a division of the local Ministry of Education. Within one week, this paper's first author delivered eight identical workshops on statistics. Before each workshop, we asked the teachers for their demographic data (i.e., their gender, age, and teaching experience) and assessed their motivational and emotional orientations regarding teaching statistics and fractions, as well as their statistical content knowledge. As data collection took place before each workshop, the study results were not affected by any learning induced by the workshops. Teachers participated in the study voluntarily, without reimbursement, and based on informed consent. They were 24–59 years old (M = 38.1, SD = 8.5) and had 1–37 years of teaching experience (M = 10.5, SD = 6.9). Female and male teachers did not differ in age or teaching experience (see Table 8 in the Appendix).

2.2. INSTRUMENTS

The instruments to measure teachers' motivational and emotional orientations and their statistical content knowledge were based on the *BeSt Teacher* framework (Schumacher, 2017). According to Schumacher, the framework was developed based on theoretical and empirical studies, and a high level of construct validity was established. Experts assessed the appropriateness of the instruments and, thus, their content validity. The reliability of the instruments was also high (Schumacher, 2017). We will provide the original values of Cronbach's alpha, to assess the scales' internal consistency, obtained from Schumacher's pilot study when introducing the different scales.

While the original items were in German, this study's questionnaire was presented in paper-based format in Spanish, the native language of Colombia. The translation process involved collaboration between this paper's first author and native Spanish-speaking mathematicians and mathematics educators to ensure linguistic and content-related correctness. Data were coded using Schumacher's (2017) coding manual, which provides unambiguous solutions to the content knowledge test. According to Schumacher, the solutions were supported by experts and the literature and thus leave no room for interpretation.

Motivational and emotional orientations. We adapted the original scales that assessed teachers' orientations regarding teaching *descriptive* statistics to assess orientations regarding teaching statistics more generally and to assess orientations regarding teaching fractions (see Section 1.3). We decided not to restrict the orientation scales solely to descriptive statistics as we were interested in obtaining insights into teachers' domain-specific orientations regarding statistics overall, as there has been limited research on this topic so far, particularly among in-service teachers (see Section 1.1). Additionally, we aimed to ensure consistency with the professional teacher development program within which we collected our data covering topics beyond the scope of descriptive statistics.

Teachers' motivational orientations were assessed in terms of their *self-efficacy* regarding school content in statistics and fractions at the level of early secondary education, respectively. For this purpose, we adapted items from the *BeSt Teacher* scale *self-efficacy*, with Cronbach's alpha of .82 (internal consistency) in the original version (focusing on descriptive statistics; Schumacher, 2017). The scales, both for statistics and fractions, were introduced by an identical stimulus: "How confident do you feel in …? Please estimate how confident you are in being able to solve tasks on the following topics.", where the placeholder was replaced by the words *statistics* and *fractions*, respectively. Each scale consisted of seven items (e.g., *statistics*: "tasks on absolute and relative frequencies"; *fractions*: "tasks on expanding and reducing fractions"). The items are listed in full in Table 4 in the Appendix. Each item had to be rated on a 4-point Likert scale (1 = "unconfident", 2 = "rather unconfident", 3 = "rather confident", 4 = "confident"). Thus, higher scores indicate higher self-efficacy, which reflect more favorable motivational orientations. The scales also showed good internal consistency in the adapted version (see Table 1) and can therefore be considered reliable. Unlike the other scales for measuring teachers' orientations, which are presented below, the self-efficacy scale addressed the content itself instead of teaching the content. To recall, in a study with prospective teachers, 89% cited their lack of statistical content knowledge as a reason for their low self-efficacy when teaching certain statistical items (Lovett & Lee, 2017). It stands to reason, therefore, that self-efficacy regarding statistical content could be a confounding variable because someone who feels little self-effective regarding statistical content will also respond negatively to questions about their self-efficacy regarding teaching statistics. To avoid confounding, we measured teachers' self-efficacy directly regarding the content.

Table 1. Reliabilities of orientation scales measured with Cronbach's Alpha

			Content area	
	Scale	Number of items	Statistics	Fractions
Motivational orientation	Self-efficacy	7	.91	.93
Emotional orientation	Joy	4	.94	.95
	Anxiety	4	.78	.80

Teachers' emotional orientations were assessed in terms of the *joy* and *anxiety* they feel regarding teaching statistics and fractions, respectively. For this purpose, we adapted items from the two *BeSt Teacher* scales *joy* and *anxiety*, with Cronbach's alpha of .88 and .92 (internal consistency), respectively, in the original version (focusing on descriptive statistics; Schumacher, 2017). The scales, both for statistics and fractions, were introduced by an identical stimulus: "How do you feel about teaching …? Please indicate how much you agree with the following statements.", where, again, the placeholder was replaced by the words *statistics* and *fractions*, respectively. Each scale consisted of four items (e.g., *joy*: "In general, I enjoy teaching …"; *anxiety*: "When teaching …, I am tense and nervous in general."). The items are listed in full in Table 5 and Table 6 in the Appendix. Items had to be rated on 4-point Likert scales (1 = "I totally disagree", 2 = "I rather disagree", 3 = "I rather agree", 4 = "I totally agree"). Thus, higher scores for joy indicate higher joy, which reflect more favorable emotional orientations, and higher scores for anxiety indicate higher anxiety, which reflect less favorable emotional orientations. The scales also showed good internal consistency (see Table 1) in the adapted version of the questionnaire and can therefore be considered reliable. The scales used for assessing teachers' orientations (*self-efficacy*, *joy*, and *anxiety)* were correlated (see Table 7 in the Appendix), as is typical for affective characteristics (Hannula, 2019).

Content knowledge. To assess teachers' *statistical content knowledge*, we used an adapted version of the *BeSt Teacher* content knowledge test, which in the original version had a Cronbach's alpha of .82 (internal consistency; Schumacher, 2017). All test items are published in Schumacher (2017) but are only available in German. Due to time constraints during data collection, we used only 16 out of the initial 33 test items. We omitted entirely specific tasks and eliminated individual items of some tasks. Moreover, we slightly modified some tasks (e.g., change of currencies, update of years, change of numerical values, change of wording) to adapt them to the Colombian sample. As a result, the adapted test version for our study included items on various concepts of descriptive statistics taught in early secondary education (i.e., absolute and relative frequencies, arithmetic mean, median, and boxplot), which were also related to the items asked for in the self-efficacy scale (see Table 4 in the Appendix). In Schumacher's (2017) pilot study, the solution rates of the selected test items (in their original version) ranged from 66% to 98%. The test created for this study is available in the supplementary material, translated into English solely for clarity within this article, while the participants received the test in Spanish. The items had different answer formats: multiple-choice, multiple-response, and numerical open-ended. Teachers' answers to the items were coded dichotomously (i.e., correct vs. incorrect) and included as separate observations in the data analysis (see Section 2.3). Cronbach's alpha (internal consistency) in our adapted content knowledge test was .64. Considering that the test covered different areas of statistics, this is acceptable (for discussion, see Stadler et al., 2021).

2.3. DATA AND STATISTICAL ANALYSIS

All analyses were conducted in *R* (Version 4.0.3; *R* Core Team, 2020), and graphs were constructed using the *ggplot2* package (Version 3.3.2; Wickham, 2016).

To answer Research Question 1, we compared teachers' motivational and emotional orientations regarding teaching *statistics* and *fractions*. It is recommended to measure complex constructs (such as teachers' orientations) using multiple Likert-type items and to calculate a mean score for the scale items despite the ordinal nature of Likert scales (Sullivan & Artino, 2013). Comparing ordinal Likert scale data is possible with parametric tests such as *t*-tests, even if they assume normality of the data because *t*-tests do not require any assumption of normal distribution in sufficiently large samples (Fagerland, 2012; Lumley et al., 2002; Sullivan & Artino, 2013). A rough guideline is a sample size of at least 30 observation pairs for a paired sample *t*-test (Bortz & Schuster, 2010). Our study comprises 88 teachers, which exceeds this guideline. In this case, parametric tests should be preferred over nonparametric tests as they offer advantages in terms of statistical power and interval estimation (Fagerland, 2012; Lumley et al., 2002; Sullivan & Artino, 2013). Therefore, we performed paired sample *t*-tests with participants' individual mean scores for the Likert scales of the scales under study (*self-efficacy*, *joy*, and *anxiety*) and calculated corresponding effect sizes.

To answer Research Question 2, we studied relationships between teachers' motivational and emotional orientations regarding teaching statistics and their statistical content knowledge while considering demographic control variables. We analyzed these relationships with *linear mixed-effects models* (LMEMs), which, unlike conventional linear regression models, include *random effects* in addition to *fixed effects*. Random effects are possible sources of error for nonindependent data, as present in this study, in which the same subjects (here: teachers) rated the same set of items (Brauer & Curtin, 2018). Failing to include appropriate random effects, as is the case with traditional ANOVA- and regression-based approaches, can lead to high type-I error rates (i.e., a high probability of mistakenly rejecting a null hypothesis; Brauer & Curtin, 2018; Judd et al., 2012) and consequently to a high proportion of effects that do not exist in reality (Brauer & Curtin, 2018). LMEMs, however, are the only models that provide unbiased parameter estimates with acceptable type-I and type-II error rates (Brauer & Curtin, 2018). Furthermore, linear mixed-effects models offer the notable benefit of allowing the estimation of variance components, which can provide insights into the factors that contribute to unexplained variability in the data (Judd et al., 2012). LMEMs were calculated using the *lme4* package (*R* package Version 1.1.23; D. Bates et al., 2015).

We analyzed Research Question 2 on an exploratory basis. The aim was to identify those facets of motivational and emotional orientations as well as those demographic control variables that best approximated teachers' statistical content knowledge. To that end, we conducted an automated model selection with the function *dredge* from the *MuMIn* package (*R* package Version 1.43.17; Bartón, 2020). The function generates candidate models with subsets of the fixed effect terms in a given global model (presented in detail below) and ranks them according to a specified criterion. We chose the *Akaike information criterion* (AIC), which is appropriate for LMEMs in general (Buscemi & Plaia, 2020; Müller et al., 2013; Nakagawa & Schielzeth, 2013) but especially for LMEMs with focus on the overall effect of the predictors on the outcome variable, as is the case in our study, rather than on subsamples (Vaida & Blanchard, 2005; for further discussion about model selection approaches for LMEMs see also Buscemi & Plaia, 2020, and Müller et al., 2013). The model with the minimum AIC value formed the resulting model in the automated model selection.

The global linear mixed-effects model for the model selection process was set up to model teachers' *statistical content knowledge* (outcome variable) by estimating their probability of correctly solving a task of average difficulty in the statistical content knowledge test. It included random intercepts (i.e., random effects) for *teacher* and *item* to account for general differences in the solution rates of individual *teachers* (e.g., due to teacher characteristics like previous knowledge or education) and individual *items* (e.g., due to item characteristics like difficulty, task type, or topic). In addition, the model included fixed effects for teachers' *self-efficacy*, *joy*, and *anxiety* regarding teaching statistics (metric predictors standardized at the sample mean), their *gender* (dichotomous factor with *male* as the baseline), *age*, and *teaching experience* (metric predictors standardized at the sample mean), as well as the interactions of *gender* and *self-efficacy*, *joy*, and *anxiety*. Low *p*-values of the predictors *self-efficacy*, *joy*, and *anxiety* suggested unique effects of teachers' motivational or emotional orientations on their statistical *content knowledge* after controlling for teachers' *gender*, *age*, and *teaching experience*. Based on this global model, the automatic model selection generated various candidate models that consisted of all possible combinations of subsets of the fixed effects while always including the random effects. The resulting model from the model selection process contained the subset of fixed effects (i.e., main and interaction effects) that best estimated teachers' statistical content knowledge.

For the outcome variable *content knowledge*, we did not calculate a mean score of the test items. Instead, we considered teachers' answers to each item as separate observations, as the test asked about various content areas of descriptive statistics. As items were coded dichotomously (i.e., correct vs. incorrect answer), we resorted to *mixed-effects logistic regression models*, which are generalized linear mixed-effects models appropriate for such binary outcome variables.

As is customary for logistic regression models, the parameter estimates of coefficients are presented as their corresponding exponential values. They represent the *odds* (of the fixed intercept) or *odds ratios* (of the fixed slopes) of the predictors in the model. *Odds* express ratios between probabilities, specifically, the ratio between the probability of success (here: correctly solving an item of average difficulty in the statistical content knowledge test) and the probability of failure (here: not correctly solving an item of average difficulty in the statistical content knowledge test; Sperandei, 2014). For example, an "odds of 4" indicates a four times higher probability of solving a task correctly than not solving it correctly. *Odds ratios* express ratios between odds. Specifically, the odds ratio corresponding to a predictor indicates the extent by which the odds of success are higher for a group with the specific predictor characteristic compared to the reference group without this predictor characteristic. For instance, an odds ratio of 3 corresponding to the predictor *gender* (with males as the reference group) would indicate that female teachers have three times higher odds of correctly solving a task in the statistical content knowledge test than male teachers. Hence, odds ratios greater/less than 1 imply greater/lower odds and, correspondingly, a higher/lower success probability due to the predictor characteristic. Therefore, odds ratios as predictor estimates reflect unique associations between the changes in the outcome variable and the shifts when a factorial predictor changes from the baseline to another level or when a metric predictor is 1 *SD* above the sample mean. Odds ratios are also used to calculate predicted probabilities of the outcome at various, levels of the predictor variables (Persoskie & Ferrer, 2017). In general, the probability of success can be calculated as:

$$\mathrm{P(success)} = \frac{\mathrm{P(success)}}{1} = \frac{\mathrm{P(success)}}{\mathrm{P(success)} + \mathrm{P(failure)}} = \frac{\frac{\mathrm{P(success)}}{\mathrm{P(failure)}}}{\frac{\mathrm{P(success)}}{\mathrm{P(failure)}} + \frac{\mathrm{P(failure)}}{\mathrm{P(failure)}}} = \frac{\mathrm{odds}}{\mathrm{odds} + 1}.$$

We further report two relevant summary statistics of generalized linear mixed-effects models. The *marginal* R^2-value, which gives the variance explained by the fixed effects, and the *conditional* R^2-value, which gives the variance explained by the fixed and random effects. Hence, the *conditional* R^2-value is the variance explained by the entire model and thus provides a value for the model's goodness of fit (Nakagawa & Schielzeth, 2013).

Furthermore, the aim was to observe specific changes in the variance component of the random effect *teacher*, which reflects variance attributable to teachers' individual characteristics. In the null model (intercept-only model), this variance component accounts for all such individual characteristics since no predictors are included. However, when adding fixed effect predictors to the model that specifically represent teacher characteristics (i.e., any of the fixed effects in our global model, e.g., teachers' *self-efficacy*), they might explicitly explain part of this variance. Thus, the variance component *teacher* only accounts for those individual teacher characteristics that the added predictors do not cover. We quantify this change in the variance component of the random intercept *teacher* via the *Proportion Change in Variance* (PCV, Nakagawa & Schielzeth, 2013), that is,

$$1 - \frac{\text{variance of } \textit{teacher} \text{ in the resulting model}}{\text{variance of } \textit{teacher} \text{ in the null model}}$$

The odds ratios and their confidence intervals, the marginal and conditional R^2-values, and the variances of the random intercept *teacher* were calculated using the *sjPlot* package (*R* package version 2.8.5; Lüdecke, 2021).

3. RESULTS

3.1. TEACHERS' MOTIVATIONAL AND EMOTIONAL ORIENTATIONS REGARDING TEACHING STATISTICS

We compared teachers' motivational and emotional orientations regarding teaching statistics and fractions (see Research Question 1). We had expected differences in teachers' orientations to the disadvantage of the content area statistics. The results confirmed this expectation; teachers reported being less self-effective, less joyful, and more anxious when teaching statistics compared to teaching fractions (see top row of Figure 1, where orientations are depicted on 4-point Likert scales, with higher values corresponding to higher agreement; thick crossbars represent respective scale means; error boxes represent ± 1 standard deviation). We found these differences to the disadvantage of the content area statistics also when considering teachers' individual *mean differences* in their orientations regarding teaching the two content areas (see bottom row of Figure 1, where mean differences are depicted as differences of the Likert scale values in the top row, with positive values indicating that teachers reported higher means of self-efficacy, joy, or anxiety regarding teaching statistics than regarding teaching fractions). For the scales *self-efficacy* and *joy*, mean differences were predominantly negative, indicating that teachers responded less affirmatively to these scales regarding teaching statistics than fractions. For the scale *anxiety*, mean differences were predominantly positive, indicating that teachers responded more affirmatively to this scale regarding teaching statistics than fractions. Thus, teachers' orientations regarding teaching statistics were poorer across all three examined scales than regarding teaching fractions.

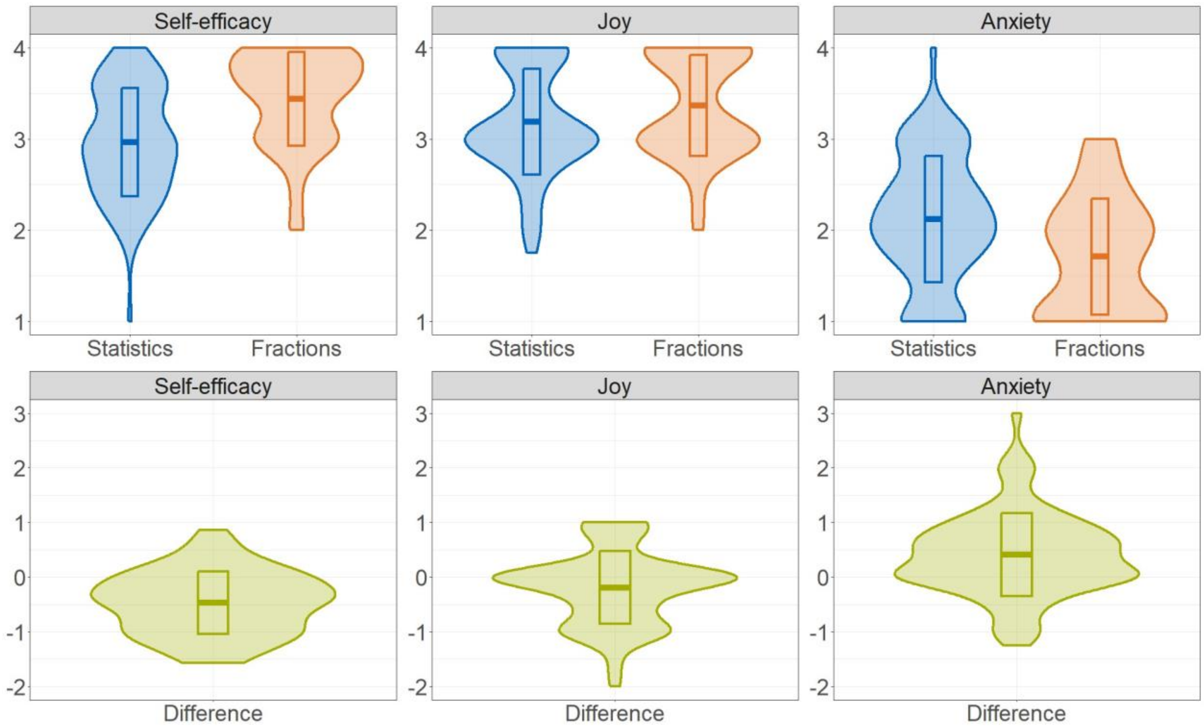

Figure 1. Violin plots showing teachers' motivational and emotional orientations regarding teaching statistics and fractions (top row) and corresponding mean differences (bottom row).

Paired sample *t*-tests confirmed the mean differences in teachers' orientations between the content areas statistics and fractions, with effect sizes of small (*joy*), medium (*anxiety*), and large (*self-efficacy*) magnitude (see Table 2; Cohen, 1988). Table 2 shows the corresponding descriptive data and the *t*-test statistics.

Table 2. Differences in teachers' motivational and emotional orientations regarding teaching statistics and fractions

	Content area				Difference						
	Statistics		Fractions								
Scales	*M*	*SD*	*M*	*SD*	△*M*	*SD*	95% CI	*t*	*p*	*df*	Cohen's *d*
Self-efficacy	2.97	0.59	3.44	0.51	–0.47	0.57	[–0.59, –0.35]	–7.57	< .001	85	–0.82
Joy	3.19	0.58	3.37	0.56	–0.19	0.67	[–0.34, –0.05]	–2.64	.010	84	–0.29
Anxiety	2.12	0.69	1.71	0.64	0.41	0.76	[0.25, 0.57]	4.97	< .001	84	0.54

The results support the hypothesis that there are differences in teachers' motivational and emotional orientations regarding teaching statistics and fractions to the disadvantage of the content area statistics. In our study, teachers reported lower self-efficacy, lower joy, and higher anxiety regarding teaching statistics compared to teaching fractions. In the following, we will examine whether these poorer orientations regarding teaching statistics relate to teachers' statistical content knowledge.

3.2. RELATIONSHIP BETWEEN TEACHERS' ORIENTATIONS AND CONTENT KNOWLEDGE IN STATISTICS

To examine relationships between in-service teachers' orientations regarding teaching statistics and their statistical content knowledge (see Research Question 2), we performed an automated model selection. With this approach, we identified relevant facets of teachers' motivational and emotional orientations as well as relevant demographic teacher characteristics (as control variables) that best estimated teachers' performance in a statistical content knowledge test. More specifically, the model selection process chose fixed effects (i.e., main and interaction effects) that best estimated teachers' probability of correctly solving a task of average difficulty in the statistical content knowledge test, based on the mixed-effects logistic global regression model including teachers' *self-efficacy*, *joy*, and *anxiety* regarding teaching statistics, their *gender*, *age*, and *teaching experience*, and the interactions of *gender* and *self-efficacy*, *joy*, and *anxiety* as fixed effects, as well as *teacher* and *item* as random effects. The resulting model is displayed in Table 3. It reveals relationships between teachers' motivational and emotional orientations regarding teaching statistics and their statistical content knowledge.

Table 3. Mixed-effects logistic regression model resulting from the model selection process estimating in-service teachers' statistical content knowledge

Fixed Effects	Odds Ratio	*SE*	95% *CI*	*p*
Intercept	2.92	0.94	[1.55, 5.50]	< .001
Self-efficacy	1.44	0.16	[1.16, 1.79]	.001
Anxiety	0.95	0.11	[0.76, 1.19]	.639
Age	0.78	0.08	[0.65, 0.95]	.014
Gender (male is baseline)	0.66	0.13	[0.44, 0.98]	.039
Gender (male is baseline) × Anxiety	1.60	0.37	[1.01, 2.53]	.044
Random Effects	**Variance**	***PCV***		
Teacher	0.38	38.3%		
Item	1.41	-		
Model Characteristics				
Observations (Teachers/Items/Total)	81/16/1296			
Marginal R^2	4.4%			
Conditional R^2	38.1%			

Note: SE = Standard Error; *CI* = Confidence Interval; *PCV* = Proportion Change in Variance

Table 3 shows that the resulting model accounted for teachers' *self-efficacy* (motivational orientation) and *anxiety* (emotional orientation) regarding teaching statistics but not for *joy* (emotional orientation). The model further identified teachers' *age* and *gender* as control variables, whereas *teaching experience* was not included. Among the interaction effects, only the interaction of *gender* and *anxiety* was selected by the resulting model, whereas the interactions of *gender* and *self-efficacy*, as well as *gender* and *joy* were not selected (additional analyses did not reveal general gender differences with regard to the three orientation facets examined; see Table 8 in the Appendix). Except for the main effect of *anxiety*, all *p*-values were low, indicating these variables are related to teachers' content knowledge, which will be discussed in more detail below. The *Proportion Change in Variance* (PCV; see Section 2.3) of the random intercept *teacher* indicates that 38.3% of the variance in teachers' content knowledge that can be attributed to their individual characteristics is explained by the fixed effects of the model.

The intercept of the model represents the reference group: male teachers of average age, average self-efficacy, and average anxiety. The corresponding estimate of 2.92 (Table 3) refers to the odds, indicating that for this group, the probability of success (i.e., correctly solving a task of average difficulty in the statistical content knowledge test) was estimated nearly three times as high as the probability of failure (i.e., not solving it correctly). Accordingly, this group's predicted probability of success can be calculated as $\frac{\text{odds}}{\text{odds}+1} = \frac{2.92}{2.92+1} = 0.74$. Therefore, the probability that a male teacher of average age, average self-efficacy, and average anxiety correctly solves a task of average difficulty in the statistical content knowledge test was estimated at 74%, 95% *CI* [61, 85].

Recall that the focus was on relationships between in-service teachers' statistical content knowledge and their orientations regarding teaching statistics. The estimate of the predictor *self-efficacy* (see Table 3) refers to its odds ratio and indicates that male teachers of average age, with average anxiety, and a high self-efficacy (specifically, one standard deviation above the male teachers' sample mean) had an odds of success that were 1.44 times higher than the odds of the reference group (i.e., male teachers of average age, average anxiety, and average self-efficacy; represented by the intercept). Accordingly, the odds for this rather self-effective group can be calculated as $2.92 * 1.44 = 4.20$ and their predicted probability of success as 81% since $\frac{\text{odds}}{\text{odds} + 1} = \frac{4.20}{4.20+1} = 0.81$. Thus, teachers who reported higher *self-efficacy* showed higher statistical content knowledge, controlling for the negative effect of increasing *age* and the *gender* effect in favor of male teachers. Higher self-efficacy in statistics was, therefore, positively related to teachers' statistical content knowledge.

The data in Table 3 also show that teachers' *anxiety* had no unique effect on teachers' statistical content knowledge, but the interaction of *gender* and *anxiety* did. Female teachers who reported higher anxiety showed higher statistical content knowledge than less anxious female teachers, again controlling for the negative effect of increasing *age*. To illustrate this effect, Figure 2 shows the estimated probabilities of success of male and female teachers with average self-efficacy and average age for different levels of their anxiety. For women, higher anxiety regarding teaching statistics was positively related to their statistical content knowledge. This result is quite counterintuitive. It is discussed in Section 4.2.

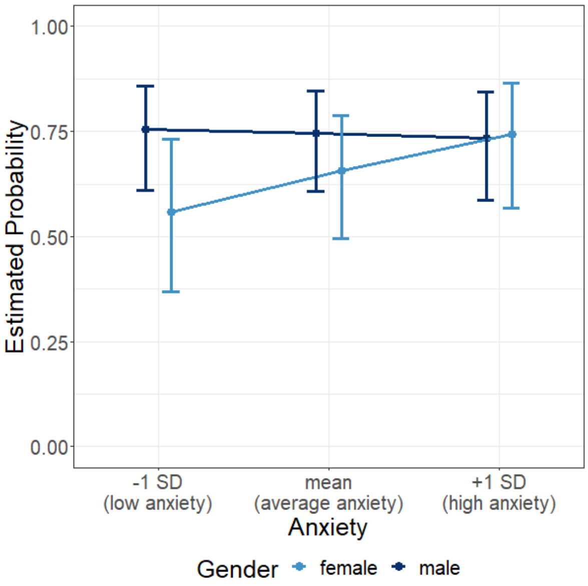

Figure 2. Line diagram visualizing the interaction effect of gender and anxiety for teachers with average self-efficacy and average age. Error bars represent 95% confidence intervals.

In summary, the model that best approximated teachers' statistical content knowledge on an exploratory basis revealed substantial relationships with both teachers' motivational (here: self-efficacy) and emotional (here: anxiety) orientations but did not include the predictor joy as a second aspect of teachers' emotional orientation.

4. DISCUSSION

This study addressed in-service teachers' motivational and emotional orientations regarding teaching statistics and their relationship with teachers' statistical content knowledge. The results contribute to our understanding of teachers' professional competence, which has been largely unclear among in-service teachers in the content area of statistics.

4.1. TEACHERS' ORIENTATIONS REGARDING STATISTICS

The results of our study with in-service teachers largely align with results from studies with prospective teachers. Regarding the content area statistics, teachers generally hold positive (i.e., above the scale mean) orientations (see Table 2). The findings, however, also showed teachers' motivational and emotional orientations regarding teaching statistics are poorer than their orientations regarding teaching fractions. Specifically, our participants reported lower self-efficacy, lower joy, and more anxiety when teaching statistics than teaching fractions. This comparison with another mathematical content area allows a better interpretation of previous findings about teachers' "positive" orientations in statistics. Earlier studies should be re-examined in light of these new findings. In general, it is reasonable to assume that teachers tend to have rather positive orientations regarding the content they teach in their subject, not least because of their career choice. Our results, however, showed that despite this generally positive level, teachers' affective characteristics depend on the specific content they teach. In order to clarify that the comparison with fractions is not an exception, further studies should compare orientations regarding teaching statistics with orientations regarding teaching other content areas of mathematics or even regarding teaching mathematics as a whole.

Our findings underscore the need to investigate the reasons for teachers' poorer orientations in statistics and to find ways to improve the situation. To recall, in a study with prospective teachers, 89% cited their lack of content knowledge as a reason for their low self-efficacy when teaching certain statistical items (Lovett & Lee, 2017). In the knowledge test of our study, the overall solution rate was only 65%. Since all test items addressed exclusively school-relevant knowledge, this low solution rate confirms previous conclusions that statistics teachers have insufficient understanding of statistical content (Lovett & Lee, 2017; Ruz et al., 2021; Schumacher, 2017) and face similar problems with statistical concepts as their students (Garfield & Ben-Zvi, 2008; Shaughnessy, 2007). Thus, providing more content knowledge in teacher training might be beneficial to promote positive motivational orientations in statistics.

We are unaware of previous research on the reasons for teachers' comparatively low joy or high anxiety regarding teaching statistics. Since emotional experiences are closely linked to motivational aspects (Hascher & Krapp, 2014), it stands to reason that teachers' statistical knowledge could also be one possible reason for their poorer emotions regarding teaching statistics. Indeed, in each content area of mathematics, particular types of knowledge are critical to effectively teaching that content (Shaughnessy, 2007). For example, a particular challenge in statistics is that there are hardly any objects of illustration or possibilities for active elaboration to grasp data and data structures, making the concepts abstract and formal (Kaun, 2008). Moreover, the increasing awareness of the importance of statistics has led to significant curriculum changes in many countries. Accordingly, teachers often face the challenge of teaching new curricula using new teaching approaches. In addition, there are numerous other challenges regarding teaching statistics (Ben-Zvi & Makar, 2016). We hypothesize that many of these challenges might be responsible for teachers' poorer orientations regarding their teaching of statistics.

In summary, we conclude that in-service teachers' affective characteristics differ when teaching different content areas. Our findings align with previous research suggesting that orientations are domain-specific (Eichler & Erens, 2015; Törner, 2002). Regarding their teaching of statistics, teachers' orientations were positive but poorer than regarding teaching fractions. Perhaps the content area statistics is not emphasized enough in teacher education compared to other mathematical content areas. Spending more time addressing different teaching strategies for statistics instruction might be useful since, in other fields, such courses also contribute to developing positive orientations (Schwarzer & Warner, 2014). Moreover, as argued before, it might be beneficial to provide more knowledge in order to promote positive orientations in statistics.

4.2. RELATIONSHIP BETWEEN TEACHERS' ORIENTATIONS AND KNOWLEDGE IN STATISTICS

Our study showed that in-service teachers' motivational and emotional orientations regarding teaching statistics are related to their statistical content knowledge, indicating relationships between affective and cognitive aspects of teachers' professional competence in statistics. We used the model selection as an exploratory approach to finding the model that best estimates teachers' ability to correctly solve an item of average difficulty in the statistical content knowledge test. The resulting model considered teachers' *self-efficacy* (motivational orientation) and *anxiety* (emotional orientation) as explaining variables while controlling for their *age* and *gender* (see Table 3). The model did not consider j*oy* as the second facet of teachers' emotional orientation regarding teaching statistics. One possible reason for the latter finding is that the variance of *joy* was smaller than that of *self-efficacy* and *anxiety* (see Table 2), making it a rather homogeneous and only slightly differentiating predictor. Furthermore, by omitting the predictor *joy*, possible multicollinearity in the regression model can be avoided because teachers' *joy* correlates highly with their *self-efficacy* for the content area statistics (see Table 7 in the Appendix).

For teachers' *self-efficacy* regarding statistics, the regression model identified a positive relationship with teachers' statistical content knowledge. This result is consistent with the findings of Schumacher (2017), who showed a moderate positive correlative relationship between in-service teachers' *self-efficacy* and *content knowledge*. In our study, s*elf-efficacy* was the only orientation scale that focused on the content of statistics rather than its teaching. The question arises as to whether the two constructs *self-efficacy regarding statistics* and *self-efficacy regarding teaching statistics* are at all distinguishable in practice. Based on a study by Lovett and Lee (2017), where most of the prospective teachers cited their lack of statistical content knowledge as a reason for their low self-efficacy regarding teaching statistics, we propose that self-efficacy regarding statistics could be a confounding variable. In any case, in our study, we empirically confirmed the relationship between teachers' self-efficacy regarding statistical content and their content knowledge, and the effect was strong (see odds ratio in Table 3). Accordingly, we consider it important to understand relationships in teachers' professional competence in statistics.

Teachers' *anxiety* regarding teaching statistics is related to their statistical *content knowledge*, but only for women; female teachers with higher anxiety showed higher statistical content knowledge. This result cannot be attributed to higher variability in women's anxiety (see Table 8 in the Appendix). Considering the fact that we assessed anxiety regarding *teaching* statistics (instead of anxiety regarding statistics), women's high anxiety, combined with rather high statistical content knowledge, might suggest that female teachers may perceive a gap between their content knowledge and their pedagogical content knowledge. Teacher training in statistics does probably not sufficiently address how to *teach* statistics in schools, but this issue requires further investigation. For highly anxious females, the model predicted similar solution rates to those of highly anxious male teachers, while the predicted solution rates of low anxious female teachers were lower than those of anxious male teachers (see Figure 2). One could also take this result to mean that less anxious women are unaware of their lower statistical content knowledge and, therefore, are not anxious when teaching. This interpretation, however, is inconsistent with Schumacher's (2017) results, where in-service teachers' anxiety was negatively correlated with their content knowledge in the overall sample. Schumacher's sample differed from ours in that females were generally more anxious than males. Such a general gender difference in anxiety did not appear in our study (see Table 8 in the Appendix). Thus, teachers' anxiety regarding teaching statistics as an aspect of their emotional orientation and its relationship with statistical content knowledge raises some questions that should be addressed in future research.

Furthermore, there are still unresolved questions regarding the relationship between teachers' *gender* and their knowledge in general. In other research, Schumacher (2017) found a gender difference in teachers' statistical knowledge, with males performing better, while Haroun et al. (2016) showed a gender difference in favor of female teachers in teachers' mathematical knowledge. Including affect variables, as done in our study, could provide deeper insights into the additional influencing factors concerning relationships between teachers' gender and knowledge and thus clarify these contradictions.

We also found that higher *age* was negatively related to the teachers' statistical content knowledge. One possible reason for this outcome could be that younger teachers have received more statistical training during their teacher education. Since statistics has only recently been incorporated into school curricula, and its proportion has steadily increased, teachers of higher ages may have received less statistics education during their teacher training. Research suggests that the proportion of untrained statistics teachers is remarkable. For example, in a survey with in-service teachers, more than half indicated that they had not covered descriptive statistics in their studies, and 76% had not taken any professional teacher development in descriptive statistics (Schumacher, 2017). The varying amounts of statistics training received due to teachers' age seem to reflect teachers' content knowledge.

Teaching experience, however, was not selected as a control variable in the model selection by the algorithm. This might be to avoid multicollinearity in the model because teachers' age and teaching experience were (obviously) highly correlated ($r = .67$, $p < .001$, 95% CI [.54, .78]). The non-selection of the facet teaching experience and its high correlation with age, combined with the finding that teachers' statistical knowledge decreases with increasing age, suggests that increasing teaching experience is unlikely to have a decisive positive impact on teachers' statistical content knowledge. Thus, teachers acquire statistical content knowledge predominantly at university rather than on the job. Other studies, such as the *COACTIV* study (Baumert et al., 2013), support this conclusion; mathematics teachers' content knowledge develops primarily in university and then stagnates or decreases as they enter the teaching profession (Kleickmann et al., 2013). On the one hand, this result underscores the importance of teacher education as preparation for a teacher's career. On the other hand, it suggests intensifying the training of in-service teachers according to the school curricula and the changing needs of society.

Together, the selected predictors *self-efficacy*, *anxiety*, *age*, *gender*, and the interaction effect of *gender* and *anxiety* explained about 38% of the variance in teachers' knowledge that is attributable to differences in teacher characteristics (see Table 3). Thus, in statistics, teachers' motivational and emotional orientations are related to their content knowledge. In other words, affective and cognitive aspects of competence are associated. Our study, however, does not provide information about causal relationships, therefore, future studies should investigate such effects. For example, studies should explore whether teacher training that focuses on teaching statistics in school addresses teachers' anxieties regarding teaching statistics, or, whether adapting teacher educators' teaching methods contributes to increasing teachers' motivation to learn statistical content, which again could increase teachers' self-efficacy.

4.3. LIMITATIONS AND FUTURE DIRECTIONS

Our sample of Colombian in-service teachers was heterogeneous, not only in terms of age, teaching experience, and motivational and emotional orientations but also in terms of the classes these teachers were used to teaching, which included all grade levels in private and public schools, in urban and rural areas. Unfortunately, based on Schumacher's (2017) findings, where school type did not correlate with teachers' knowledge, we did not systematically record the teachers' school types or locations in our study but assume our sample covers a broad spectrum of teachers based on personal conversations with the participants. Future studies should consider including such variables. For instance, researchers could investigate whether differences in teachers' knowledge can be attributed to their gender or school type, as there may be gender differences across different school types. In addition, participants' cultural backgrounds may be related to their response styles to Likert-type response formats (e.g., rather agreeing or rather disagreeing; Johnson et al., 2005). Although Likert scales are a typical approach for studying orientations and are particularly appropriate for adults (Hannula, 2019), we cannot rule out such a bias in our participants' responses. Therefore, future studies in other countries could consider both teachers' orientations and knowledge in statistics to see if our results are replicable in other cultures.

Our study did not determine whether different groups of teachers share comparable orientations regarding teaching statistics. For example, a group of teachers with high anxiety and low self-efficacy versus a group with low anxiety and high self-efficacy is conceivable, but also groups with opposite affective characteristics (e.g., a group with high self-efficacy and high anxiety), as indicated by our results where, for females, higher anxiety in teaching statistics was associated with higher statistical content knowledge. For identifying these teacher groups, cluster analyses could be utilized. Teachers' assignment to clusters may serve, for example, for predicting the effectiveness of teacher training for specific teacher groups (see Reinhold et al., 2021) or for providing more tailored support to promote teachers' learning (see Pedder & Opfer, 2013).

Moreover, our study covers only a selection of orientation facets regarding teaching statistics, although other facets might also play a role. Schumacher (2017), for example, detected a negative relationship between teachers' anger (emotional orientation) regarding teaching statistics and their content knowledge. When considering further facets, however, a challenge is that facets of orientations are usually highly correlated and, therefore, difficult to separate (Hannula, 2019). In addition, teachers' orientations regarding teaching statistics (i.e., self-efficacy, joy, and anxiety) also seem to be related to teachers' *pedagogical* content knowledge (Schumacher, 2017). Including this knowledge facet in studies might shed more light on teachers' comparatively poorer orientations regarding their *teaching* of statistics. The *BeSt Teacher* framework can remedy both issues. It offers additional scales to assess orientations regarding teaching statistics and items to test pedagogical content knowledge (Schumacher, 2017). As a validated framework (currently only available in German) for jointly assessing teachers' orientations and knowledge in statistics, it forms a solid basis for further studies investigating teachers' professional competence in descriptive statistics. In addition, exploring teachers' orientations regarding teaching statistics concerning their (pedagogical) content knowledge in *inferential* statistics could provide new insights into teachers' professional competence, as teachers face many problems in this area as well (de Vetten et al., 2019; Harradine et al., 2011). Investigations into relationships between teachers' orientation and knowledge would also be interesting and desirable for mathematics overall. Similar to statistics, the number of studies on this topic is limited, and the existing ones mainly focus on prospective teachers and orientations regarding the subject, but not the teaching of the subject. Initial findings have shown relationships among teachers' orientations and knowledge in mathematics (Wilkins, 2008), including the facets of self-efficacy (A. B. Bates et al., 2011) and anxiety (Gleason, 2007; Rayner et al., 2009), which align with the results of our study regarding teachers' orientations and knowledge regarding statistics.

5. CONCLUSION

This study addressed in-service teachers' professional competence in statistics, considering both affective and cognitive aspects. First, teachers' motivational and emotional orientations regarding teaching statistics were rather positive but poorer than those regarding teaching fractions. Second, the results revealed relationships between teachers' orientations and knowledge in statistics. We suggest that statistics should play a greater role in teacher education and professional development. Educational programs should ensure the teaching of statistical content is extensive and detailed. They should also focus on developing positive orientations in teachers, which could be accomplished, for example, by providing a variety of strategies for teaching statistics.

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APPENDIX: COMPLETE LISTING AND DESCRIPTIVE RESULTS OF ORIENTATION SCALES

Table 4. Items of the scale "self-efficacy" as part of teachers' motivational orientations

	Content area statistics	Content area fractions
Stimulus	How confident do you feel in statistics? Please estimate how confident you are in being able to solve tasks on the following topics.	How confident do you feel in fractions? Please estimate how confident you are in being able to solve tasks on the following topics.
Response options	unconfident rather unconfident rather confident confident	unconfident rather unconfident rather confident confident
Item 1	Tasks on absolute and relative frequencies	Tasks on graphical representations of fractions
Item 2	Tasks on cumulative frequencies	Tasks on expanding and reducing fractions
Item 3	Tasks on the arithmetic mean	Tasks on comparing the size of fractions
Item 4	Tasks on the median	Tasks on the addition and subtraction of fractions
Item 5	Tasks on the range	Tasks on the multiplication of fractions
Item 6	Tasks on critical handling of graphical representations	Tasks on the division of fractions
Item 7	Tasks on the topic chance	Tasks on fractions with real Context

Table 5. Items of the scale "joy" as part of teachers' emotional orientations

	Content area statistics	Content area fractions
Stimulus	How do you feel about teaching statistics? Please indicate how much you agree with the following statements.	How do you feel about teaching fractions? Please indicate how much you agree with the following statements.
Response options	I totally disagree I rather disagree I rather agree I totally agree	I totally disagree I rather disagree I rather agree I totally agree
Item 1	In general, I enjoy teaching statistics.	In general, I enjoy teaching fractions.
Item 2	In general, I enjoy teaching statistics so much that I enjoy preparing these lessons.	In general, I enjoy teaching fractions so much that I enjoy preparing these lessons.
Item 3	In general, I teach statistics with enthusiasm.	In general, I teach fractions with enthusiasm.
Item 4	While teaching statistics, I often have reason to be happy.	While teaching fractions, I often have reason to be happy.

Table 6. Items of the scale "anxiety" as part of teachers' emotional orientations

	Content area statistics	Content area fractions
Stimulus	How do you feel about teaching statistics? Please indicate how much you agree with the following statements.	How do you feel about teaching fractions? Please indicate how much you agree with the following statements.
Response options	I totally disagree I rather disagree I rather agree I totally agree	I totally disagree I rather disagree I rather agree I totally agree
Item 1	When teaching statistics, I am tense and nervous in general.	When teaching fractions, I am tense and nervous in general.
Item 2	I am worried that teaching statistics isn't really working out.	I am worried that teaching fractions isn't really working out.
Item 3	I am worried about the preparation of statistics classes.	I am worried about the preparation of fractions classes.
Item 4	I feel helpless while teaching statistics.	I feel helpless while teaching fractions.

Table 7. Intercorrelations (and corresponding p-values) of motivational and emotional orientations regarding teaching statistics and fractions

		Motivational orientation	Emotional orientation	
		Self-efficacy	Joy	Anxiety
Motivational orientation	Self-efficacy	0.47 (< .001)	0.54 (< .001)	–0.61 (< .001)
Emotional orientation	Joy	0.53 (< .001)	0.31 (.003)	–0.44 (< .001)
	Anxiety	–0.33 (.002)	–0.19 (.078)	0.35 (.001)

Note. Intercorrelations of orientations regarding teaching *statistics* are shown below the diagonal. Intercorrelations of orientations regarding teaching *fractions* are shown above the diagonal. Intercorrelations of equal orientation facets regarding teaching *statistics* and *fractions* are shown on the diagonal.

Table 8. Gender differences in teachers' demographic characteristics and motivational and emotional orientations regarding teaching statistics

	Gender				*t*-test			*F*-test		
	Male		Female		(to compare group means)			(to compare group variances)		
	M	*SD*	*M*	*SD*	*t*	*p*	95% *CI*	*F*	*p*	95% *CI*
Age	39.1	7.7	36.5	9.4	–1.31	.195	[–6.6, 1.4]	0.67	.202	[0.3, 1.2]
Teaching Experience	11.4	7.3	9.2	5.9	–1.55	.125	[–5.1, 0.6]	1.54	.188	[0.8, 2.8]
Self-efficacy	3.05	0.60	2.85	0.58	–1.50	.137	[–0.45, 0.06]	1.06	.873	[0.55, 1.94]
Joy	3.14	0.54	3.27	0.64	0.92	.362	[–0.14, 0.39]	0.72	.282	[0.37, 1.32]
Anxiety	2.09	0.77	2.15	0.57	0.43	.671	[–0.23, 0.35]	1.83	.070	[0.95, 3.37]