EDITORIAL: REASONING ABOUT MODELS AND MODELLING IN THE CONTEXT OF INFORMAL STATISTICAL INFERENCE

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All models are wrong, but some are useful.
George Box (1979, p. 202)

Welcome to this special issue of Statistics Education Research Journal (SERJ) on Reasoning about Models and Modelling in the Context of Informal Statistical Inference. The papers included in this special issue are elaborated from presentations at the Ninth International Research Forum on Statistical Reasoning, Thinking and Literacy (SRTL) held in Paderborn (Germany) at from 26 July – 1 August 2015 and organized by the guest editors of this special issue in collaboration with Dani Ben-Zvi and Katie Makar as the chairs of the international SRTL – 9 committee. The SRTL-9 Forum expanded the work discussed at previous SRTL forums (www.srtl.info), which have focused on reasoning about fundamental statistical ideas such as data, variability, distribution, informal inferential reasoning, etc. Building on the SRTL-7 forum (New Approaches to Developing Reasoning about Samples and Sampling in Informal Statistical Inference) and the SRTL-8 forum (Reasoning about Uncertainty in the Context of Making Informal Statistical Inferences), the SRTL-9 forum with its theme of reasoning about models and modelling had the aim of discussing pedagogical approaches to building bridges between the data and the probabilistic perspective in the context of informal statistical inferences (ISI). Recent digital tools like TinkerPlots 2.0 (Konold & Miller, 2011) provide powerful features such as the sampler to help learners build their own models to produce and generate data and therefore help to build a bridge between the data and the probabilistic perspective on modelling. Ideas, research, meaningful tasks and learning environments are needed to effectively enhance reasoning about models and modelling. Modelling is not only fundamental in statistics education but also in mathematics education in general. Furthermore modelling is relevant across all age and education levels (from early reasoning in primary school to tertiary education and adult education). However, one also has to mention that "one of the most overworked words in statistics education and mathematics education is 'model'. Appearing in a variety of dissimilar contexts, its usage is at best unclear, and at worst, inappropriate" (Graham, 2006, p. 194). This special issue is supposed to provide a variety of interpretations and applications of modelling in statistics education.

Statistics Education Research Journal, 16(2), 8-12, http://iase-web.org/Publications.php?p=SERJ
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In the SRTL-9 forum we excluded cognitive issues such as “mental models” and focused on contributions aligned with one or more of the following themes in relation to statistics education, which we have taken from the SRTL-9 preliminary announcement:

- Why bring models and modelling into the research and practical arenas at all? (e.g., what are philosophical, historical, epistemological and/or practical reasons for introducing models and modelling in statistics education research? how is mathematical modelling the same and/or different than statistical modelling?)
- According to G. Box some models are useful: What are the utilities and purposes of models and modelling? What is a model/modelling for?
- How does reasoning about models and modelling develop in the context of learning to make ISIs from data?
- What are rudimentary ideas of models and modelling and how are they expressed among young students? (e.g., what is a model? what does it mean to model?)
- How are ideas related to models and modelling understood and used by students in making ISIs? (e.g., what ideas are needed to understand and use models? what does it mean “to understand a model?”)
- What are innovative tasks, tools, or sequences of instructional activities that may be used to help these ideas emerge?
- How can technology help to develop students’ reasoning about models and modelling in the context of making ISI?
- What are ways to assess reasoning about models and modelling?
- What new approaches can be used to help teachers develop students’ reasoning about models and modelling?
- What new ideas and considerations regarding models and modelling have or will emerge as a result of prevailing trends in the discipline of statistics (e.g., computation, exceedingly large data sets, Bayesian analysis, etc.)?

In this special issue, we find a broad variety of research studies about learners’ reasoning about modelling across all ages. We structure the issue according to the levels of students the papers refer to: primary, secondary and tertiary level.

This special issue starts with five papers (Ainley & Pratt; Aridor & Ben-Zvi; Lehrer; Doerr, delMas & Makar; Manor-Braham & Ben-Zvi) on **primary school students´ reasoning about modelling**.

Ainley and Pratt investigated children´s expressions of signal and noise when creating computational models with the dynamic software tool TinkerPlots. For their investigation, they conducted clinical interviews with 11-year-old students working on the Angry Emus task and the 101 Dalmatians task. The Angry Emus task is about producing and exploring repeated measures, situated in a fictive context of developing a computer game and strikes the question whether children use explanations of causal factors or of random effects. The 101 Dalmatians task is about analysing a small Dalmatians dataset for creating new, realistic-looking Dalmatians based on the given data with a data factory with the TinkerPlots sampler. In their paper Ainley and Pratt explain the design of their tasks and point out how students engage with a tool like TinkerPlots within the context of rich modelling tasks to consider opportunities and constraints of what they call “purposeful computational modelling.”

Aridor and Ben-Zvi add to the work of Ainley and Pratt and investigate the co-emergence of aggregate and modelling reasoning of primary school students via the 101 Dalmatians task. They report on a case study with two fifth grade students analyzing statistical data, doing informal inference and modelling activities with TinkerPlots. In their analysis of the case study, Aridor and Ben-Zvi distinguish two processes: reasoning with statistical modelling of a real phenomenon vs. aggregate reasoning. In addition, they identify nine phases of students´ articulations of aggregate and modelling reasoning towards data, variability and models.

The metaphor of signal-and-noise is fundamental to Lehrer's paper about modelling variability. Lehrer discusses the invention and use of models by sixth graders to grasp the concept of variability. Most students in his study modelled variability with an approach of signal-and-noise constituted through their understanding of variability. Modelling is seen here as the generation of sampling distributions to support the inferences of young students. Simple random devices were used by students to invent and revise models of measurement and production processes. Working with these models enabled students to understand sampling variability.

Doerr, delMas, and Makar developed a sequence for teaching modelling for primary school in Australia in their paper on the basis of informal inferential reasoning. They developed a sequence of activities based on model eliciting activities (MEAs) enriched by model exploration activities (MXAs) and model application activities (MAAs). The final aim was to analyze how grade 5 students generalized models for drawing informal inferences when comparing two distributions. The authors report on five milestones that occurred during the three-day-teaching sequence. This paper is an important contribution on introducing statistical models and a modelling perspective to young students in form of a teaching sequence about modelling.

The paper of Braham-Manor and Ben-Zvi investigates students´ emergent articulations of statistical model and modelling in making informal statistical inferences. Their *Integrated Modeling Approach* (IMA) was used to help students understand the relationship between sample and population and to support students reasoning with models and modelling. In their qualitative study, Braham-Manor and Ben-Zvi focus on the reasoning of a pair of primary school students and investigate how primary school students articulate ideas of statistical models and modelling using TinkerPlots. On this basis, the authors suggest an emergent conceptual framework for reasoning with statistical models and modelling.

The next three papers from Büscher and Schnell, Gil and Gibbs, and Konold, Finzer and Kreetong deal with **middle and high school students' reasoning about modelling**.

The focus of Büscher and Schnell´s work is on how German middle school students interpret (informal) statistical measures to summarize and compare frequency distributions. Büscher and Schnell use the emergent modelling perspective, where measures are understood as models. On the one hand this modelling perspective can be seen as a theoretical framework for describing the conceptual development of their students in the frame of a qualitative analysis in their design experiment and on the other hand as a design heuristic for their teaching-learning arrangements. The authors emphasize the important role of the emergent modelling perspective for design issues as well as for the interpretation of students reasoning when describing and comparing distributions.

Gil and Gibbs explore how secondary students develop an understanding of modelling covariation in the context of big data. In their article they present a three-week unit that supports 12th grade mathematics students in analyzing big and mid-size data and describing relationships between two numerical variables using concepts like trend and scatter. Their learning trajectory includes computer-supported collaborative and inquiry-based approaches, the use of visualization tools and statistical software and presentations where students can present their findings. In their study Gil and Gibbs found that students´ reasoning and modelling of covariation improved and they highlight features of the learning trajectory which may have contributed to this.

Konold, Finzer and Kreetong have focused on modelling as a core component of structuring data. They provided their participants (middle school and high school students), in the frame of a qualitative study, with complex diagrams in form of snapshots showing traffic on two road segments taking into account several attributes like type of vehicle, speed of vehicle, direction and width of the road. The authors investigate how their participants record and organize data in such a complex situation and whether they use a hierarchy of cases structure or a "flat" case-by-attribute structure. The paper of Konold, Finzer and Kreetong makes an interesting contribution to our understanding of how middle school and high school students structure data and conceive of cases.

The papers of Noll and Kirin, Biehler, Frischemeier and Podworny, Kazak and Pratt and Gould, Bargagliotti and Johnson are focused on **preservice teachers and teachers´ reasoning about modelling**.

The work of Noll and Kirin is akin, in some ways, to the work of Doerr, DelMas and Makar, who looked at primary school students' comparisons of two distributions. Noll and Kirin analyze students' reasoning from introductory university courses. The authors use and discuss the framework of Biehler, Frischemeier and Podworny (2015) to take a deeper look at students' thinking about modelling in the context of comparing two groups. They analyze students' modelling of a null hypothesis via a null model expressed in the TinkerPlots sampler and how TinkerPlots supports this modelling. In the center of their analysis are four groups of students in introductory statistics classes working on the dolphin therapy task. Several issues about the reasoning of these students about modelling a randomization test with TinkerPlots are presented in the paper. They highlight the role of TinkerPlots' sampler to serve as a visualization tool for a null hypothesis in a randomization test and how the sampler supports the reasoning process of students.

Like Noll and Kirin, Biehler, Frischemeier and Podworny used the TinkerPlots sampler to build bridges between the data world and the chance world to support learners reasoning in expressive modelling. They conducted a research study with elementary preservice teachers and asked them to set up and evaluate their own models with TinkerPlots by using a real and open dataset. The reasoning processes of the elementary preservice teachers were analyzed with qualitative content analysis methods and typical structures of the modelling cycle were identified when the participants worked on the task. In addition, the authors investigated and interpreted the processes of setting up and evaluating models in TinkerPlots.

Kazak and Pratt report on a study of preservice teachers' reasoning about probability models for the sum of two dice. The reasoning of the participants was influenced by the nature of the data, the modelling assumptions and the simulation capacity of TinkerPlots. The authors identified key moments of insight about students' informal inferential reasoning and analyzed them with content analysis. In accord with other studies, these preservice teachers had difficulties correctly identifying the sample space for the sum of two dice, ending up with only 21 possibilities. One main insight of this study is that playing the game physically helped the participants revise their initial models and facilitated model-based reasoning. The reported task makes a contribution to the enterprise of fostering students' reasoning between an empirical situation and its mathematical model.

The paper of Gould, Bargagliotti and Johnson finishes this special issue with a focus on teachers' reasoning with big data. Gould et al. argue for the use of participatory sensing data as a kind of big data, and computational thinking in addition to statistical thinking. The authors examine teachers' reasoning as part of the *Mobilize Introduction to Data Science (IDS)* curriculum. Like Doerr, delMas and Makar they used a model eliciting activity for investigations about data. A qualitative analysis was conducted on the working process of two pairs of teachers working on the MEA about humans acting as "trash" sensors. In this case mobile phones were used for collecting data on trash. Later this data was analyzed by the pairs of teachers. In contrast to all other papers of this special issue, 'R' is used as software in this study. The authors present a *data cycle* as a framework for analyzing their participants' work. For the transitions between stages of this data cycle the model of discrete Markov chains is used as a means for describing the participants' progress through the data cycle. The variety of pathways through the data cycle was one surprising result of this study. Less surprising, but in accord with other studies (e. g. Arnold & Franklin 2017), was the result that formulating statistical questions is a key component of the investigation process.

ACKNOWLEDGEMENTS

Within the process of editing this special issue many people have supported us in a great way. At first we are deeply grateful to Randall Groth who has supported us tremendously as the assigned SERJ associate editor in setting up the associate editor reports and improving the quality of the papers with his very helpful, constructive and intensive feedback. In addition we thank Maxine Pfannkuch as the SERJ chief editor, and Dani Ben-Zvi and Katie Makar as current leaders of the SRTL community for giving us the honor and the opportunity to be guest editors of this special issue. We thank all reviewers for their support in writing constructive and helpful reviews to improve the quality and writing of the papers in this special issue. Last but not least we are very grateful to all authors who submitted a paper to this special issue on modelling. We wish our readers a pleasant journey into students' reasoning with models and modelling.

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