AN EXPLORATION OF STUDENTS' STATISTICAL THINKING WITH GIVEN DATA[1]

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SUMMARY

This paper examines how two twelve-year-old students built up their recognition and understanding of relationships in a set of data. Using a small multivariate dataset created by Watson, Collis, Callingham and Moritz (1995), the students conducted an investigation of their choice in a pencil-and-paper environment. The students' thinking across the three representations of cards, tables and graphs is analysed from the perspectives of transnumeration, consideration of variation, reasoning with statistical models, and integrating the statistical with the contextual, which were identified as fundamental statistical thinking elements in empirical enquiry in the framework of Wild and Pfannkuch (1999). The ways of thinking within each element across the representations are identified. In the analysis, references are also made to the types of statistical thinking present in the other ten students in the study. From the analysis we identified five issues that should be considered for determining how students construct meanings from data. They are: prior contextual and statistical knowledge; thinking at a higher level than constructed representations; actively representing and construing; the intertwinement of local and global thinking; and the changing statistical thinking dialogue across the representations.

Keywords: Statistical thinking; Statistical investigation; Middle-school students; Constructing meanings from data.

1. INTRODUCTION

Applied statistics is part of an information gathering and learning process which is undertaken to inform decisions and actions. Industry, medicine, government agencies and many other knowledge-based endeavours are increasingly relying on data for decision making and thus statistics and statistical thinking are becoming an integral part of a societal way of thinking. A statistical investigation is conducted in order to learn more in the context sphere. This learning is much more than collecting information, it involves synthesising new ideas and information with existing ideas and information into an improved understanding. The question is then raised as to how students develop this type of learning.

A statistical investigation requires learners to take on a data-detective role. The learners' thinking activates a constant dialogue between the data and themselves. Of course this dialogue is an internal conversation that resides within the learners as an "understanding" is built up of the real situation. To build up this "understanding" of the context reality, the statistical models that have been constructed, and the learners' statistical knowledge are "interrogated" to provide information that will feed into their mental models (Wild & Pfannkuch, 1999). Furthermore, as the learner engages in a dialogue with the different representations of the real system the dialogue necessarily changes as new meanings and understanding are conveyed about the problem under consideration. The resultant new learning about the context does not reside within one representation but rather

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is built up through engagement with a number of representations.

The overall aim of this exploratory study was to examine how twelve Year 7 and 8 (11-13 year-old) students reasoned with data within each representation and how they built up their recognition and understanding of relationships in a given set of data over the course of an investigation. The main purpose, however, was to establish a baseline position in terms of the students' statistical thinking using a theoretical framework. Therefore the first interest lay with what the students could construct for themselves without having experienced any teaching in how to deal with and reason with multivariate data. The second interest lay with the framework itself and the degree to which its use could help in uncovering and illuminating the statistical thinking going on in the students.

A four-way analysis was performed which entailed examining the data from the perspective of four of the five fundamental elements of statistical thinking – recognition of the need for data, transnumeration, consideration of variation, reasoning with statistical models, and integrating the statistical and the contextual. These five fundamental thinking types are now elaborated upon.

Recognition of the need for data. The foundations of statistical enquiry rest on the assumption that many real situations cannot be judged without the gathering and analysis of properly collected data. Anecdotal evidence or one's own experience may be unreliable and misleading for judgements and decision-making. Therefore properly collected data are considered a prime requirement for reliable judgements about real situations.

Transnumeration. For this type of thinking the word transnumeration was coined which means "changing representations to engender understanding". Transnumeration occurs in three specific instances. If the real system and statistical system are thought of from a modelling perspective then transnumeration thinking occurs when (1) measures that "capture" qualities or characteristics of the real situation are found, (2) the data that have been collected are transformed from raw data into multiple graphical representations, statistical summaries, and so forth, in a search to obtain meaning from the data, and (3) the meaning from the data, the judgement, has to be communicated in a form that can be understood in terms of the real situation by others.

Consideration of variation. Adequate data collection and the making of sound judgements from data are required for an understanding of how variation arises and is transmitted through data, and the uncertainty caused by unexplained variation. It is a type of thinking that is started from noticing variation in a real situation, and then influences the strategies that are adopted through every stage of the investigative cycle. Applied statistics is about making predictions, seeking explanations, finding causes, and learning in the context sphere. Therefore patterns in the data are sought and characterised in an attempt to understand them in terms of the context of the real situation.

Reasoning with statistical models. The predominant statistical models are those developed for the analysis of data. Most people interpret the term "statistical models" as meaning, for example, regression models or time series models. Even much simpler tools, however, such as statistical graphs can also be thought of as statistical models since they are statistical ways of representing and thinking about reality. When statistical models are employed to reason with, the focus is more on aggregate-based rather than individual-based reasoning, although both types of reasoning are used. A dialogue is set up between the data and statistical models. The models may allow patterns in the data to be found, group propensities to be found, and variation to be seen about these patterns via the idea of distribution. The variety of models available enable data to be explored in multiple ways dependent upon the nature of the data. For example, graphs, centres, spreads, clusters, outliers, residuals, confidence intervals, p-values are read, interpreted and reasoned with in an attempt to find evidence on which to base a judgement.

Integrating the statistical and contextual. Although the above types of thinking are linked to contextual knowledge, the integration of statistical knowledge and contextual knowledge is an identifiable fundamental element of statistical thinking. The statistical model must capture elements of the real situation and thus the resultant data will carry their own literature base (Cobb & Moore, 1997), or more generally, their own body of context knowledge. Information about the real situation is contained in the statistical summaries and therefore a synthesis of statistical and contextual knowledge must operate to draw out what can be learned from the data about the context sphere.

Although the student interviews could be analysed from the perspective of each of these elements there was nothing in the given task that really encouraged "the need for data" so that element is not part of this particular analysis.

2. OTHER RESEARCH

2.1. REASONING FROM DATA

Lesh (2000) believed that teaching should aim to help students develop conceptual technologies by focussing on mathematics-as-communication for constructing and making sense of complex systems. For statistics this might mean students should be actively engaged in empirical enquiry from the formulation of questions from a real situation to the drawing of conclusions, and be focussing on multivariate data rather than univariate data (Shaughnessy, Watson, Moritz, & Reading, 1999).

A number of research studies from different theoretical perspectives (e.g., Watson, Collis, Callingham, & Moritz, 1995; Konold, Pollatsek, Well & Gagnon, 1997; Ben-Zvi & Friedlander, 1997) have analysed how students reason with and handle data, and recognise relationships within a multivariate data-set. Konold et al. (1997, p.165) concluded that the students they interviewed had not made *"the transition from thinking about and comparing properties of individual cases, or properties of collections of homogeneous cases, to thinking about and comparing group propensities*". Ben-Zvi and Friedlander (1997) and Watson et al. (1995) focussed on developing a hierarchy of statistical thinking that ranged from using selected pieces of information to a full synthesis of information.

During statistical empirical enquiry, when students are given a multivariate data-set, they must undergo a process of understanding the data, posing questions to investigate, determining appropriate analysis methods, interpreting the data and drawing conclusions. How students interconnect each phase of the enquiry cycle and how reasoning at one phase impinges on the next phase is not known. If representational fluency, which according to Lesh (2000, p. 80), "*is at the heart of what it means to "understand" most mathematical constructs*" then more attention should be paid to elaborating on how students reason with graphs and other models at every stage of empirical enquiry. Kaput (2000, p. 8) stated "*knowledge is co-constituted by the means through which it is represented and used – it does not exist independently of its representation*". Therefore following the chain of representations that are used by students in statistical empirical enquiry might build up a picture of how statistical knowledge is constructed.

2.2. UNDERSTANDING THE GIVEN DATA

Statistical questions arise in real situations and are usually prompted by noticing an incident or phenomenon and wondering "Why?". Before the actual statistical questions are formulated, however, there is a long phase of understanding the real situation system and all the phenomena that impinge on it. Even if the data are given, there should still be a phase of "noticing and wondering" (Shaughnessy, 1997) during the process of understanding the data before there is a transition to defining statistical questions. During the process of understanding given multivariate data sets it has been found by Watson et al. (1995) and Biehler (1997) that students make sense of such information by conjecturing linear causal chains acting on individual cases. It is unclear how or if students take a group propensity view at this stage. Biehler (1997) observed that questions were chosen with regard to the variables and he thinks this might act as a constraint on the types of questions posed. This raises issues about how the posing of questions is linked to the prior perceptions and understandings of raw data, and linked to constructions and interpretations of statistical summaries.

2.3. STUDENTS' BUILDING AND INTERPRETATION OF TABLES

The construction of tables from raw data involves sorting and classifying data into groups. Hancock, Kaput, and Goldsmith (1992) found that students lacked awareness of the need to represent implicit information in the data, such as gender from the given names, which they attributed to a lack of knowledge about data structures. Recognising, developing and implementing criteria for an effective classification procedure is not easy for students (Lehrer & Schauble, 2000).

Little research appears to have been conducted on students' construction and interpretation of statistical data tables. This raises questions about how students perceive tables. For instance Biehler (1997) observed that students frequently dealt with messy tabular data rather than thinking that they would see more structure in the data through graphical representations. It seems, however, that students find it a struggle to re-represent data into another grouping thereby perhaps demonstrating this tension between focussing on individual cases and focussing on variables that may describe group behaviour. Bright and Friel (1998, p. 67) conjectured that *"tables* *may play an important role as an intervening representation that can smooth the transition between representing raw and reduced data*". We think, however, that the representations that re-aggregate or reclassify the raw data, which may pertain to the future analysis or to the question posed, play a role in table representations and are another transition point.

2.4. STUDENTS' BUILDING AND INTERPRETATION OF GRAPHS

Students' construction of graphical representations from verbal descriptions has been probed by Moritz (2000) and Mevarech and Kramarsky (1997). Moritz (2000) studied how 9-12 year-olds represented bivariate and multivariate data given verbal descriptions. Although the forms of the graphs were not conventional there were distinguishable responses of single comparison, double comparison, and series comparison. Construction of graphs is considered to require a different set of cognitive skills to interpretation. When tertiary students were given a multivariate data table Chick (2000) was surprised at their limited use of basic statistical techniques and use of inappropriate techniques.

Much research attention has focussed on the interpretation of graphs. Students' ability to recognise a trend or to compare variables when analysing data is an important part of determining relationships among variables. The ability of teaching programmes to shift attention from individual cases to group propensities is described by Ainley, Nardi & Pratt (2000) and Cobb (1999) in their teaching experiments. According to Ainley et al. (2000) their active graphing method using computing technology enabled 8-9 year-old students to construct meanings for trend. Their method encouraged students to plot their data during collection. Students learning to perceive a trend in a scatterplot started from a pointwise interpretation, and then moved through a series of transitions to being able to look through individual points to identify a trend. Cobb's (1999) teaching approach is based around structuring and organising data. Activities were designed where the purpose was to compare two or more data sets to make a judgement. Cobb concluded that the students were analysing data from a mathematical point of view since the focus was on the rate of occurrence of some set of data values within a range of values which he believes is at the heart of a statistical perspective. Also the notion of distributions rather than collections of data-points emerged for the students, pointing to this shift from an individual case to a group propensity perspective. These teaching approaches clarified how students can learn to reason with statistical models within a structured situation.

Curcio (1987) identified three levels of graphicacy which relate to the kinds of questions graphs can be used to address. These levels are: reading the data, reading between the data and reading beyond the data. We believe, however, that if the purpose of drawing a graph is to glean information about the real situation problem, to interrogate the data, then more levels need to be added such as 'looking behind the data' (Shaughnessy, Garfield & Greer, 1996). Most research on the interpretation of graphs seems to have functioned at a statistical analysis level and has yet to include a full statistical enquiry perspective. As Biehler (1997) noted students' interpretation of their graphs seemed to reflect what they had learned in the elementary classroom and this did not include upgrading their statistical-causal modelling capability. When studies such as Ben-Zvi (2000) did not include set questions but rather an exploration of data then it is found that students interpret data through using comparison methods and their contextual knowledge about the situation to explain interesting phenomena in the data. Many studies have set the statistical tasks and questions. It would be interesting to find out more on what students can do and why within an exploratory data-based environment.

2.5. DRAWING CONCLUSIONS FROM DATA

Research has found a range of responses for drawing conclusions from data. At one end of the continuum students will: give their personal opinion without reference to the data (Hancock et al. 1992; Chick, 1999); derive their conclusion from a single case (Konold et al., 1997); or use their graph as an illustration rather than a means of analysing the data (Ben-Zvi & Friedlander, 1997). At the other end of the continuum are students who are involved in an "ongoing search for meaning and interpretation to achieve sensible results" (Ben-Zvi & Friedlander, 1997, p. 50). Ben-Zvi and Arcavi (1998), Lesh, Amit and Schorr (1997), and Cobb (1999) in their teaching experiment research describe how students can be pushed to high levels of discussion on statistical information when they have to defend their interpretation. Cobb (1999) used Toulmin's theoretical scheme to describe how students argued for their interpretation and conclusions. This raises the issue as to whether there are other theoretical schemes for drawing conclusions from statistical data.

The research literature available indicated, that while much has been learnt about the interpretation of graphs there is still little research concerning how students reason about data when engaged in empirical enquiry. Also little research has been conducted in statistics on how students think about and reason with variation (Shaughnessy, 1997; Torok & Watson, 2000), think with a contextual knowledge base or think transnumeratively. In our opinion there is much more work to be done on understanding students' reasoning with data during statistical investigation.

Therefore our analysis will be not only interpreting students' thinking from the perspective of the statistical thinking elements but also suggesting how the elements are manifested in the thinking of middle school students who have had no experience with tools for interpreting multivariate data. Furthermore, research has not been conducted on the thinking of students using a framework that is specific to statistical thinking in empirical enquiry. This study might provide some insights into the thinking pathways and thinking behind the artifacts that students produce.

3. METHOD

In schools statistical investigations occur that range from open investigations in which students pose their own question and collect data to closed investigations in which data are given with specific questions. The investigation given to the students was one in which the data were given and the students chose what to explore. Although the data-set could be considered unrealistic and too small a sample it is nevertheless a useful school activity to introduce students to EDA (exploratory data analysis) within the constraints of a pencil-and-paper environment. Students in New Zealand begin statistics in Year 1 and by Year 8 have experienced conducting investigations, displaying discrete and continuous data, calculating averages, and comparing data. The focus is entirely on univariate data.

The second author chose a group of students where she was able to obtain the permission from the school and parents to interview them. The twelve Year 7 and 8 students (11-13 year olds) were randomly selected from the mathematics-extension students in a New Zealand intermediate school. Based on her knowledge of the students they were put into pairs so that each pair was the same year level, same gender and was known to be able to work together. Using an investigation and protocol created by Watson et al. (1995), the students were given sixteen cards. Each card contained information about one person: the name, age, weight, eye colour, favourite activity and number of fast food meals eaten per week (Fig.1). The information on these cards was all that they were given. Such a data set can be easily understood by students as the contextual knowledge required is within their own life experiences and hence questions and relationships between variables can be generated.

Name: Simon Kahn
Age: 18

Favourite activity: TV
Eye colour: Brown
Weight (kg): 74
Fast food meals per week: 12

Figure 1: Example of a Data-card

The students had not worked with multivariate data sets before and were not familiar with scatterplots. The students were first required to read and understand the information on the cards before thinking about what they could investigate. A pencil-and-paper environment was provided with materials such as calculators and graph paper being available for the students. They were interviewed by the second author and audio-taped for approximately one hour while they were conducting an investigation of their choice. The students were asked and prompted to think aloud as well as explain their actions as they progressed through the investigation. At times the interviewer pushed the students to think more deeply about what they were saying. However, the investigator did not suggest techniques to use rather used phrases such as "Can you tell me what your graph has shown you?" or "Anything else?".

Three people separately analysed the interviews qualitatively. Rubick (2000) analysed all the interviews, Yoon (2001) three interviews, and the first author four interviews. All the interviews had at least two independent analyses. Each interview incurred a four-way qualitative analysis. For example, interview data would be analysed for possible instances of variation-type thinking and this was recorded first as an annotation on the script, and after consideration, in a table format with the data and description of the thinking. Since it was not known what variation-type thinking to expect the data was used to suggest possible instances. Therefore the analysis and findings are explorative and tentative.

4. RESULTS

There was no uniformity of reasoning across all the students. Hence for this paper the dialogue of two Year 7 twelve year-old boys, Andy and Ron, was selected because it covered a good range of the thinking displayed across the set of students. The results are presented in an unconventional format since an excerpt from a transcript may convey all four ways of thinking and since the focus of the study is on the chain of reasoning that these students used to build up an understanding of the information contained in the data. In this particular statistical investigation three types of representations were used: the cards, tables of data, and graphs.

The dialogue summary given below was chosen on the basis of demonstrating the progression of the students' thinking to the next representation and of highlighting potentially interesting stages in their thinking. After the dialogue summary an analysis is conducted from the four perspectives of transnumeration, consideration of variation, reasoning with statistical models, and integrating the statistical with the contextual, which were identified by Wild and Pfannkuch (1999) as being fundamental types of statistical thinking. References will be made to the other five interviews in the analysis to highlight aspects that could be considered about the statistical thinking of students.

4.1. THE DIALOGUE SUMMARY

The numbers, for example, (1) or (5), in the dialogue summary act as a cross-reference for the dialogue analysis that follows this summary.

(1). Andy and Ron were introduced to the information on the cards. They immediately focussed on the number of fast foods eaten per week by each person. In particular Ron suggested they find out who ate the most. On finding out that Simon Kahn ate the most and being aghast that he ate 12, Andy remarked "*he is a bit of a fatso isn't he?*".

(2). Ron was then asked to choose a card (Fig. 1) and to say what information was on it. "*Okay his name is Simon Kahn and he is age of 18, he weighs 74 which is pretty big for an 18-year-old I guess. And his eye colour is brown … favourite activity is TV. He probably sits down and does nothing … that's why his weight is 74kgs. And he takes 12 fast food meals per week which is pretty incredible. And by eye colour being brown it seems as if he could be … but he doesn't look like he's a native of Australia or New Zealand*".

(3). On being asked what sort of questions they thought they could answer using these cards, Andy said: "*Quite a lot of statistics. Like you could compare favourite activity with each other … like how much people weigh in accordance with their ages, like that might have something to do with … (searches for card to support his argument) … David Jones he weighs 30kgs and he's only 8 whereas (searches for another card) … whereas Andrew Williams is 14, he weighs 60kgs. So that might be something to do with age and weight*".

(4). Later on Ron remarked about the Simon Kahn card "*74kgs. My Mum doesn't weigh that much*".

(5). Andy: '*We could do something with age and weight like compare them like two 17 [year-old] people at 56 and 65 [kgs]. There's sort of a difference … . Fast food meals per week and weight like compare that. If there is more fast food and if they weigh more there could be a connection there. Eye colour - probably won't be much to do because it doesn't compare with anything else*".

(6). The interviewer then asked what the connection was, to which Ron replied: "*the more fast foods you eat the more weight you get [put on]*".

(7). Then Andy thought: "*there might be something to do with the sex of … like David Jones is like a male and since he develops more muscle as he grows that might contribute … and like Rosemary is a girl so she might be less heavier … so that might be something*" .

(8). Ron immediately thought of a girl in their school who disproved this conjecture: *"Girls nowadays are not like girls in the olden days. Like I mean look at [name of girl]".*

(9).The interviewer intervened at this stage and stated that they have now mentioned four factors. She asked what the fourth factor would be called to which Andy responded with “gender” and “we could do two [factors] each may be … we could split it up”.

(10). Andy decided: “*first we will have to analyse the cards and write down on a piece of paper the things what everyone has. Like I could do the age groups … like from youngest to oldest … probably that would be the best idea and then we could transfer them on to graph paper … we could do two [graphs] on each [piece of] graph paper”.*

(11). Andy took half the cards and gave the other half to Ron. He decided that they would each record their card information on their separate tables. He recorded age, weight, and fast food in approximate columns (Fig. 2) whereas Ron abbreviated the card layout (Fig.3). Throughout the transfer of the card data to the tables they continued to compare two people at a time and suggest reasons for the weight difference. Ron decided to do fast food and age, because the difference in the measurements was not so great, and Andy decided to do gender and weight.

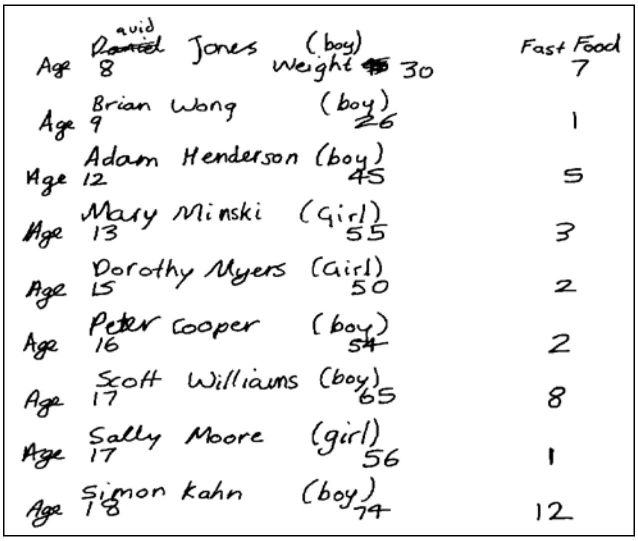

Figure 2: Andy’s table

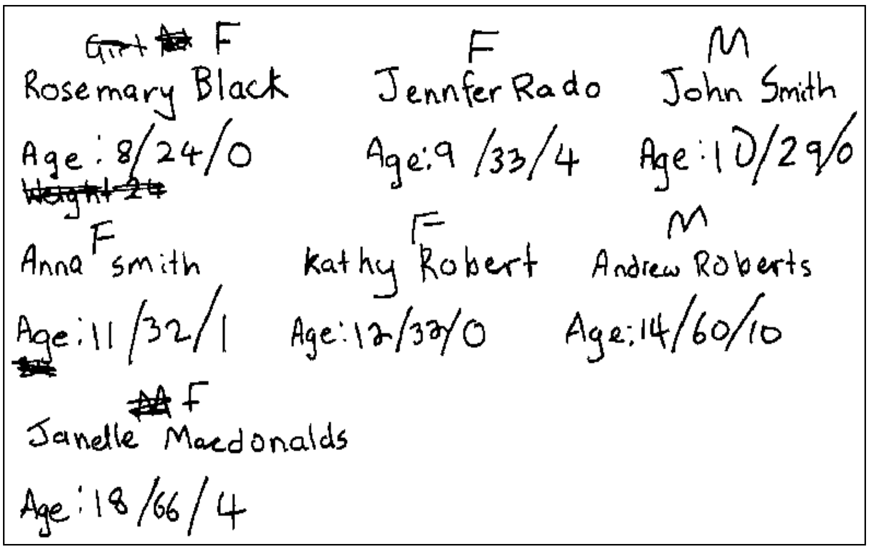

Figure 3: Ron’s table

(12). Andy then asked Ron whether they should do a bar graph. Ron quickly agreed.

(13). After Andy had written the initials along the x-axis he decided that he could actually do gender on the same graph because he could use a separate colour for each. Andy started to interpret his graph (Fig. 4) as he was constructing it: "*Okay, I've noticed that from 6 people I've already done they're actually about the same … between 33 and 26…24 [kgs]. So there's actually quite small [differences] between them and that's between the ages of 11 and 8, so it's quite even at the moment. … We would expect that".*

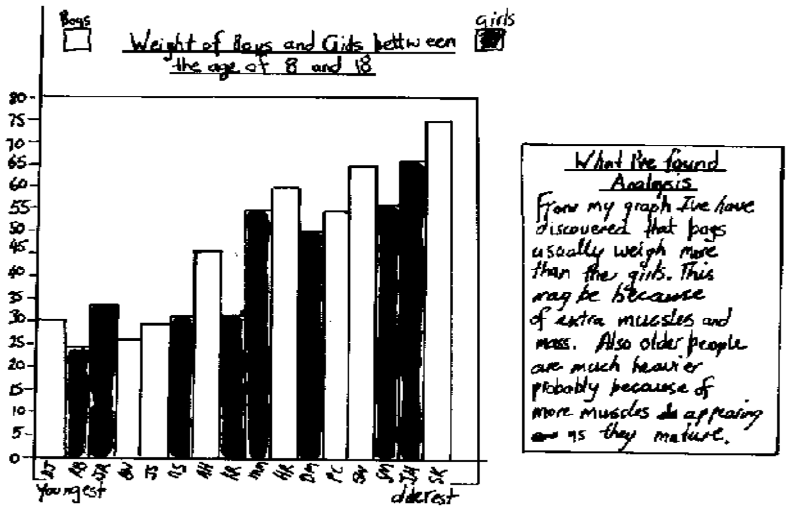

Figure 4: Andy's graph display for weight, age, and gender

(14).They continued graphing and discussing the people on the cards. Meanwhile Ron was constructing all the green bars for age first on his graph (Fig. 5). He noticed that the bars were climbing and he remarked: *"weeee there goes the beanstalk"* and if he "*attached a line to each end of this it would look like a mountain*".

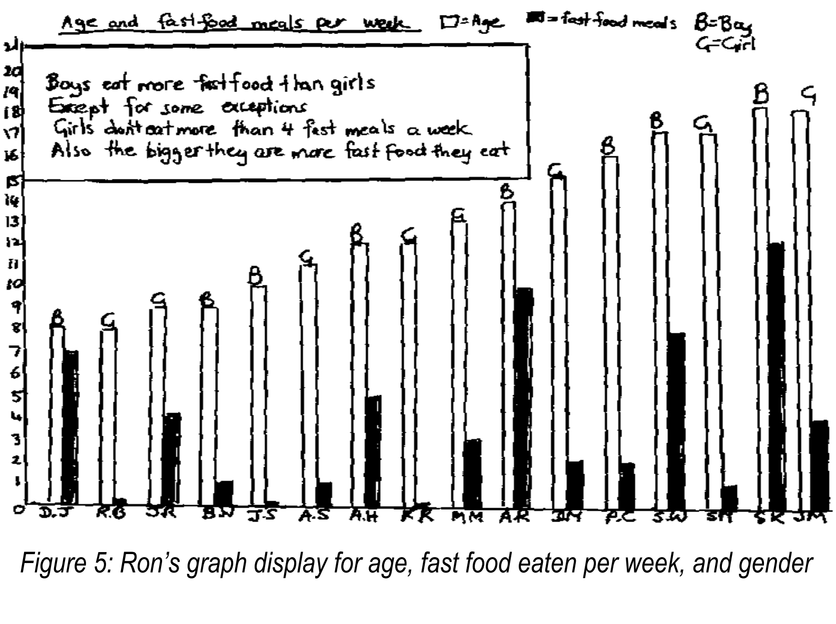

Figure 5: Ron's graph display for age, fast food eaten per week, and gender

(15). Andy announced he would write down what "*the graph tells me" like "how as the age gets more it's increased gradually…it [weight] keeps going up*".

(16). Ron objected to him mentioning age since he was doing that factor. Andy agreed "*but I did it in order of age so it sort of looks like that*". When Andy was asked if the weights followed a rule he replied: "*probably not, there's just a few exceptions where they go down … but that's because of like gender differences, but mainly they go up.*"

(17). Ron chimed in with: "*Obviously the boys weigh more than the girls*". The interviewer objected to Ron's statement since he had not looked at the graph. But Ron was confident of his ground since: "*Janelle MacDonald and Simon Kahn…they are the same age but obviously he, Simon, weighs more. Maybe it's because of diet (looks at list). Yeah diet - the fast food - he takes 12, Janelle takes 4*".

(18). Finally Andy read out his written conclusion: "*From my graph I have discovered that boys usually weigh more than girls. This may be because of extra muscles and mass. Also older people are much heavier probably because of more muscles appearing as they mature*"

(19). Ron in discussion with Andy started to interpret his graph, "*Hey, this is so easy to tell – boys eat way more than girls … fast food. If you look here* (points to graph) *one of the tallest* [bars] *is this* (points to first bar – David Jones with 7 fast foods) *he's a boy. Boy, boy, boy, boy and boy*" (points to the 5 tallest bars).

(20). After further discussion they noticed that the spread or range of fast food eaten was much smaller for girls than boys. Suddenly Andy noticed something about his graph: "*It appears that here (points near the middle of his graph) - as people get about 12 (age) they gradually get … they expand … they sort of weigh much more. Because it's quite even along here (points to the first 8-9 bars) until it gets to KR and MM. Then it goes right up and like keeps going above like another 20kgs (points to remainder of bars) and it keeps sort of staying the same. So it might be something there".*

(21). The interviewer asked for some possible reasons. "*Ah I don't know … it could be something to do with changing schools maybe. … intermediate to high school*".

(22). The interviewer reflected on the fact that they were about 12 years old and whether they would expect to have an increase in weight within the next few years. Andy agreed: *"Yeah we are 12. But … Actually it might be something to do with hormones too …"*

(23). Ron then finally interpreted his graph: "*Boys eat more fast foods than girls except for some exceptions. Girls don't eat more than 4 fast food meals a week. Also the bigger [older] they [boys] are the more fast food they eat".*

4.2. THE DIALOGUE ANALYSIS

Transnumeration

Transnumeration of the data could involve thinking about reclassifying the data, calculating averages for data grouped on a particular variable, or representing the raw or transformed data in tables or graphs. It is thinking about how to change the current representation to another representation.

Cards

Andy and Ron suggested that favourite activities played a role in weight (2) but they were unable to use this information since they did not think to transnumerate or reclassify the activities as sedentary and non-sedentary. Andy thought that gender might be a factor (7). This insight allowed transnumeration to occur since the names of people could now be reclassified as male or female (9). The reclassification of data into new variable groupings was not a prevalent feature of students' work. For instance Andy and Ron recognised age as a factor then sorted the data on age. Other students went further and classified data into age subgroups but only one pair of students recognised and dealt with age group as a variable. Before they moved on to recording the data Andy thought they could do a graph representation involving two factors (9). The identified factors determined the information to be recorded in the tables. The names, however, were still included, since these acted as an identifying marker to the original data which is equivalent to normal statistical practice. Andy transnumerated his data into a recognisable table format (Fig. 2) whereas Ron did not (Fig. 3). Similar practices were observed in the other students.

Table

From the cards to the table the dialogue should shift from one that is rich in context to one that is more reliant on statistical knowledge. There should be a focus on abstracting quantitative and qualitative variables from the context to produce a table. Most students were observed attempting this process. The tabulation of data is a skill that requires determining how to present the data with clarity and unambiguity. It means losing some of the information on the cards and using variable descriptors. This was a skill that was being developed by all the students although it appeared to be a more sophisticated skill than we had realised.

Ideally the dialogue with the table should include both noticing patterns in the table data, and determining graph construction. Both dialogues were observed but such a dual dialogue was not, perhaps indicating that this might be beyond the students at this stage. For most of the students the table was used as an organising tool for graphing (10). It was seen as a method of transferring data to the graph not as a method of also allowing a fuller inspection of the data for possible relationships and trends that might have been missed by looking at the cards. For some students the table was engaged with at the level of deciding scales on axes and/or what factors to graph together. One pair of students engaged with their tables and data fully to the extent that they did not see a need for a graph until they were prompted by the interviewer. Some students were aware that colour coding of factors would enable them to 'see' differences (13) whereas other students used colour for presentation purposes.

Graph

The dialogue with the graph could consider whether another representation might convey more information. This was not part of Andy and Ron's conversation. The graph-representation transnumeration for both Andy and Ron's graphs were in the form of a series comparison of case values (Figs. 4 & 5). Half the students represented multivariate data in series comparison graphs. For Andy and Ron it was only when the graph was being constructed that the idea emerged on how to delineate the two factors (13). Basically students seemed to be actively building on their current experience and knowledge. All students continued the dialogue with cards during this phase (11). It might be a necessary part of the conversation that needed to be continued for convincing each other that the investigation was on track. Even though some students constructed multiple representations, transnumeration thinking was not a feature of the dialogue with the graph(s) perhaps because of the pencil-and-paper environment that was provided and time constraints.

The communication of findings involves visually describing and/or writing a text to convey the information that has been gleaned from the data. Students demonstrated a range of methods from text only, to graph and text, and graph only. Andy and Ron were the best communicators in this respect.

Consideration of Variation

Cards

Initially there were two ways of noticing variation. The first way was noticing anomalies in the data which were beyond what they would normally expect (1). The second way was noticing differences between two people on one variable, say age, then finding another variable where there was a difference such as weight (3). They took datum as absolutes and did not seem to be aware of or did not argue about variation of weight within an age group, nor, when they compared different-aged people, did they argue from a perspective of variation in weights between age groups. An interviewer prompt caused Andy to think of another difference between two cases, that is gender (7). Ron thought of an anomaly outside the dataset (8) to demonstrate that gender and weight might not be related. Anomalies and noticing differences between two individual cases appeared to help shape many of the students' arguments about possible interconnections of variables.

All of the students were aware of what 'norms' to expect in data from their contextual knowledge to which they compared the card data (2, 4). The discussion was based around these contextual 'norms' but as yet the discussion did not include argumentation about variation about those 'norms' or statistical 'norms'. The type of argumentation was to find out whether the data on the cards was representative of the population or would confirm or refute what they already knew about the population (2).

Table

The variation patterns in the table did not appear to be noticed or engaged with, instead the variation dialogue continued with the cards.

Graph

When considering a graph representation the noticing of variation between two individual cases is no longer appropriate in the argumentation; it is now about noticing the variation between groups. There also has to be a distinction made between possibly real (may be explained) and random (cannot be explained) variation. Andy was able to make this distinction in when he 'saw' the jump between the 12 and 13 year olds (possibly real variation) (20) and 'saw through' the random variation for those under 12 (13). Both Andy and Ron 'saw through' the random variation with comments such as "just a few exceptions" (16, 23) but mainly the trend was increasing. Ron reverted to using two individual cases, however, to argue for a trend (17). Differences or variations among the heights of bars were noticed (19) which seemed to help Andy and Ron in picking up the pattern between the girls and boys. These comparisons suggested that they were also *informally quantifying variation* within a group since they noticed that the range for the amount of fast food eaten by the boys was larger than the range for the girls (20). Only some of the students could be said to be 'quantifying' or 'seeing through' the variation.

Noticing variation and arguing for a particular interpretation of the graphs was demonstrated on one or more of these levels by the students through: (a) using the cards or graph and comparing two individual cases, (b) checking a preconceived relationship which determined what variation to notice between two groups of data, and (c) the variation noticed in the graph between two groups of data determining a new possible relationship.

Reasoning with Statistical Models

Cards

Reasoning with raw data means making sense of the information and thinking of possible relationships that could exist. A feature of Andy's thinking was the use of the words 'compare' and 'connection' (3, 5). He seemed to realise that to reason statistically two variables were compared to see whether there was a connection between them. Ron used the word 'compare' on only two occasions – it was not a feature of his thinking – rather he was in a transition between thinking of descriptive questions such as how many people eat 0, 1, 2, etc. fast food meals and thinking of comparison questions such as "fast food and weight". But he did have a contextual group propensity perspective (6). On the one hand, Andy and Ron's argumentation followed the pattern that if something general about the population was proposed then they would confirm or refute it based on one specific example from their experience or from a card (4, 8). On the other hand, if they knew something general about the population, then they would give a specific example from the cards in order for the other person to understand (3). Only Andy and Ron demonstrated an argumentation that determined what questions were feasible or not feasible with the data.

Table

Andy and Ron's dialogue with the table involved the type of graph to be drawn and determining what was possible to graph based on their statistical knowledge (11, 12). Their statistical knowledge actually determined the factors to be compared. It was noteworthy that Andy did not know beforehand how he would display the factor gender. It was only in the action of graphing that he understood how he could communicate that factor in his representation (13). The placing of the names along the x-axis could be due to (a) not wanting to lose that information, (b) not knowing about scatterplots, (c) knowing that the x-scale should quantitatively increase and that it was not possible to have 8, 8, 9, 9, etc. along the axis. A similar feature was present in all the students' work apart from one pair of students who, for example, grouped the data into age groups and plotted average weights. Whatever the explanation might be, the decision to draw such a graph, was based on their current statistical knowledge.

Graph

When students reason with a graph the dialogue should change to focus on 'seeing' patterns in the whole of the dataset, and to talking about central tendencies, the trend (15), the way the data are distributed, and the spread of the data. The discussion should no longer be centred on reading individual cases, which could still be done, but on the whole group. Andy started reasoning with his model during construction by noticing the horizontal trend of the first six bars (13). He did not, however, allude to this trend in his conclusion. It was not until much later when he continued to interact with his graph that he articulated this possible pattern (20). Perhaps the action of graphing helped him to 'see' this particular pattern or perhaps he knew that he should be 'noticing' trends. This has highlighted the need for students to engage with the graph in two ways. The first way should be, based on their question, deliberately looking for a relationship to 'see' if there was a pattern (18). The second way should be to notice (20) a pattern in the data, and see what relationship if any, the pattern might convey.

This type of reasoning was not prevalent in all the students. Some continued to read values from the graph.

For example, Ron reverted to using the cards to argue that boys weigh more than the girls which was based on noticing differences between two people (17). This was a typical dialogue for the other students when interpreting their graphs and for some this was their conclusion. Ron, however, seemed to be in a transition since he seemed to be at the beginning of noticing a trend when constructing his graph (14). Then, with the help of Andy, he wrote a conclusion that communicated two patterns or trends and an upper limit (23). When Ron was reasoning about a trend he did so by comparing the heights of the bars (19) and since more male bars were higher he was arguing that "boys eat more fast food than girls' (23). This aspect was observed in some of the other students. One pair of students actually compared two bars at a time. Therefore a value comparison among bars was the method of argumentation for a trend.

Three pairs of students reasoned about the models they had produced. Their decisions included determining the model was valid, not valid or could not be validated with such a small sample. They then accepted the model, or rejected the model on the grounds that it was incorrect, or decided they could not draw valid conclusions from the model about the data (Rubick, 2000; Yoon, 2001).

The students' argumentation with a bar graph or other statistical model showed they were possibly operating on one or several of these four levels: (a) comparing two individual cases to argue that one individual variable value was higher than the other; (b) comparing two or several groups of data, individual case or averages, to argue that one group had higher values on the whole than the other; (c) 'seeing' and comparing the slope trends of one group with another group to argue that one group was different from the other; and (d) questioning, challenging or dismissing the model itself.

Even though their bar graphs were not easy to reason with for comparison of variables, a third of the students demonstrated a remarkable facility to reason with such 'poor' tools or models. By concentrating on the information contained in the data, these students formed quite sophisticated interpretations. The other students did not tend to do this, but when the interviewer asked two students to look at their three bar graphs and to explain to her if there was anything that was meaningful, one of the students was able to detect a non-preconceived pattern. This suggested that students could 'see' patterns if their attention was drawn to notice what graphs were telling them rather than looking for preconceived patterns or using the graph as an illustration.

Integrating the statistical and the contextual

Cards

Story telling about the observations seemed to be a precursor for thinking about variable relationships, for making sense of and rationalising the information on the cards. Causal thinking was prominent and involved the students using their imagination as well as being sceptical, whereas their statistical knowledge appeared to have little influence at this stage. Contextual knowledge influenced the positing of alternative explanations for causes-and-effects and what variables would be considered or not considered. Causes outside the data gathered, however, tended not to be considered; rather the thinking was mostly within the confines of the given data. From the very beginning, in the case of Andy and Ron, a causal relation was posited between fast food and weight (1). This causality was brought up a number of times (5) yet it was never challenged. Causal relationships between eye colour and name, and gender and weight, were challenged (5, 8). Their own contextual knowledge about population "norms" was drawn upon often to support their statements about whether the datum was an extreme value (2, 4) or to support cause and effect arguments such as weight and favourite activity (2, 3, 5, 6, 7).

Most of the students focussed on the weight variable. This required them to think what they knew about weight and the other variables from their general knowledge. From their dialogue with the cards Andy and Ron suggested the following about weight for the age group 8 to 18 years: as you got older you weighed more (3); eating fast food made you weigh more (6); sedentary activities made you weigh more (2); boys were heavier than girls (7); eye colour did not affect weight (5). They did not evaluate which explanations were more likely but did know that several variables influenced weight.

Table and graph

From the table to the graph the dialogue should ideally shift from one that is focussed on statistical knowledge to one of statistical and contextual interpretation. Hence the integration of statistical and contextual ideas, and conceptions should shape the main dialogue. This dual dialogue was present in the students' thinking in varying degrees. Causal inference was intertwined with the information divulged by the graph and with the contextual knowledge of the students. When Andy noticed the first six bars of his graph were 'even', he stated that this was to be expected (13). This suggested that he was using some contextual knowledge to reassure himself that the pattern was not unusual. When he was asked for possible reasons for the pattern (21) he thought of differences between younger and older people but did not evaluate whether they were ‘sensible’ reasons. Such a response was typical for the other students. The interviewer then asked him to think whether he would have such a weight gain within the next few years. This prompt produced another reason (22). The use of contextual knowledge to explain the variation or patterns is considered vital for determining whether the patterns could be ‘real’ and for the interpretation of the patterns for the real situation. Andy justified his written conclusion (18) by suggesting reasons for the trends he had noticed whereas Ron did not (23). Nor in their conversation were any reasons posited for the relationships discovered by Ron. Andy thought of explanations outside the realm of the data given (21, 22) for his graph. Ron tended not to think of explanations outside the given data which was typical of the responses from the other students. Andy, however, suggested that the factors were probable (18).

Even though the students knew there could be several possible explanations for a relationship between variables they displayed a range of interpretations: (a) did not think of an explanation, (b) gave one definitive explanation, (c) gave one probable explanation, (d) thought of several explanations within the confines of the given data, and much less evident (e) thought of alternative explanations outside the realm of the data. Evaluating which explanations would be more likely, likely or not possible was not a feature in the dialogue with the graph. Causal inference seemed to be a natural way of thinking for the students.

5. DISCUSSION AND CONCLUSIONS

From the analysis of the students’ dialogue, we have identified five issues within the four statistical thinking elements that should be considered when determining how students construct meanings from data. They are: prior contextual and statistical knowledge; thinking at a higher level than constructed representations; actively representing and construing; the intertwinement of local and global thinking; and the changing statistical thinking dialogue across the representations.

5.1. PRIOR CONTEXTUAL AND STATISTICAL KNOWLEDGE

Firstly, lack of prior statistical knowledge was not an insurmountable obstacle for these students when dealing with multivariate data. Prior contextual knowledge was essential for conjecturing possible relationships within the set of data. Both contextual and statistical knowledge influenced students’ understanding and interpretation of the data. Students were building on their own knowledge or folklore of relationships among variables to incorporate new learning about the dataset. Their interaction with the raw data suggested they reasoned from the population to the sample to help in the identification of factors to consider; that is, they used their existing contextual knowledge or folklore about expected norms, patterns or group propensities in the population. The strength of these preconceptions about cause-and-effect affected the way the data were focussed upon and the variables that were compared, and infiltrated the conclusions drawn. It would appear that such preconceived ideas should not be underestimated since they have a strong effect on the meaning that is obtained from data (Watson et al., 1995). This prior contextual knowledge was essential, however, for making sense of the data before posing a question and again when interpreting patterns in the variation (Cobb & Moore, 1997).

Statistical knowledge influenced the types of investigations undertaken, how the data were transnumerated, and how the representations were engaged with. For some students the first barrier was an inability to pose a question with the given data, for others it was an inability to engage with a representation other than at a reading level. Therefore, on the one hand, statistical knowledge was a limiting factor in their ability to construct meaning from the data, but on the other hand, having only a very basic knowledge did not prevent some students from gleaning quite sophisticated insights into the meaning of the data.

5.2. THINKING AT A HIGHER LEVEL THAN CONSTRUCTED REPRESENTATIONS

Secondly, the tables and graphs produced by the students did not indicate the level and depth of their thinking. Students are able to think at a higher level than their constructed representations might suggest. Most representations that the students constructed were not the conventional statistical representations (cf. Moritz, 2000); rather they were their own creation or way of dealing with data in table and graph form. Most students’ graphs, however, retained identification of the individual case, which has been highlighted by other research (Friel & Bright, 1996). Their graphical inventions for dealing with bivariate or multivariate data were based on the individual-case bar graph. They spent a lot of time thinking, conjecturing, and learning about the data before and during the construction of representations which, according to Tukey (1977), is a necessary prerequisite for data interpretation. That is, they seemed to be concentrating on the information contained in the data or in the variation. Perhaps this concentration, coupled with understanding their own invented graphs, enabled them to recognise the relationships within the dataset. They had the capacity to think at a considerably higher level than their tools would indicate. At least half of these students had this capacity (cf. Chick & Watson, 2001).

5.3. ACTIVELY REPRESENTING AND CONSTRUING

Thirdly, the physical actions of tabulating and graphing together with the activation of a constant dialogue between the data and themselves helped the students to gain information from the data. Their continuous imaginings and their problems with representations seemed to place them into an enquiry mode of thinking. Thus, the strong feature of all the students' actions and dialogue was that they were actively representing and construing and that interpretation was an ongoing process. As soon as they picked up a card their imaginations were prompted as they interpreted what they were seeing or experiencing. For the two students focussed upon in this paper the table and the graph were not fully visualised before proceeding. Their method was to start their representation and then to solve problems of representation during construction. At the same time as they were actively constructing their graphs they were actively interpreting any perceived patterns or anomalies through focussing on the height of the bars. The physical action of graphing data is highly prized in quality management (Hare, Hoerl, Hromi & Snee, 1995). It is also interesting to note that Ainley et al. (2000) have developed the notion of active graphing when students use technology. It would seem that the actions of graphing and tabulating coupled with ongoing interpretation helped these two students gain insights into the data. Half of the students did not articulate such thinking *during* the construction of the tables and graphs and therefore we could not make any inferences about whether or not they were actively construing during this stage.

5.4. THE INTERTWINEMENT OF LOCAL AND GLOBAL THINKING

Fourthly, the intertwinement of local and global thinking was observed as a feature of the ways in which one thinks with and constructs meanings from data. This observation led us to consider that this continual shuttling between local and global thinking underpins the development of the four statistical thinking elements. Research on teaching statistics suggested that students first focussed on individual cases and then made the *statistical* transition to focus on group propensities (Konold et al., 1997; Ben-Zvi & Arcarvi, 1998; Cobb, 1999; Ainley et al., 2000). We observed, however, that when the students first focussed on individual cases they interpreted them by referring to 'known' group propensities or expected 'norms' and by conjecturing possibilities about deviations from the norm for that person. They compared individual cases and thus thought locally at the statistical level, but at the same time, at the contextual level, they thought locally and globally. The interchange of perspectives occurred within all the elements. For example, for the transnumeration element, when students are presented with raw data, one individual-case entry, say age, is considered at a local level. From this starting point: data are sorted on age and age becomes one global entity; data are split into local age subgroups; and then data are globally redefined as age-group and perceived as one entity again and so forth (Pfannkuch, Rubick, & Yoon, 2002a). For the variation element, at a statistical level, students notice variation by comparing two quantities and eventually shift to comparing patterns qualitatively, whereas, at the contextual level, they deal with variation by hypothesising relationships between individual cases and then shift to hypothesising relationships between variables (Pfannkuch, Rubick, & Yoon, 2002b). The focus on individual cases at the beginning of the investigation seemed to help in drawing out possible global relationships while at the conclusion stage a global relationship could be confirmed or discovered and the explanations for the relationship could range from none to one or several possibilities. Therefore the students seemed to build up their recognition of the relationships in the data through the dual use of local and global thinking, both statistically and contextually.

5.5. CHANGING STATISTICAL THINKING DIALOGUE ACROSS THE REPRESENTATIONS

Fifthly, for the development of statistical thinking across the elements it was observed that each element requires different ways of thinking with different representations. For example, variation thinking with raw data is different from variation thinking with graphs. The students' narratives highlighted awareness of how statistical thinking changed across the representations. Unless prompted, the thinking did not change for some of the students and continued to be the same for the graph as it was for the raw data. Even when the students engaged with the graph, their previous thinking with the other representations continued. It seemed that as the students moved from one representation to another their statistical thinking moved on a continuum back and forth among the representations as they gradually built up their understanding of the data. The continuum started with attention on one individual case and shifted towards noticing patterns in the whole data set. This raises questions about how to shift and draw students' attention to noticing that their thinking should change with changing representations. Cobb (1999) and Ainley et al. (2000) in their teaching experiments seemed to accomplish this subtle shift in students' focus with informed teacher questioning and by concentrating students' attention on looking within representations.

This statistical thinking continuum could be described in terms of the fundamental elements. Reasoning with statistical models changed from attending to isolated pieces of information, to answering the questions and discovering new relationships about global features within the graphical patterns (cf. Lesh et al., 1997). The integration of the contextual and the statistical changed from making up causal stories to fit one or two people, to causal inferences about perceived regularities in relationships between two or three variables. The variation dialogue changed from noticing exact differences between two individual people or noticing extreme values, to noticing systematic differences between grouped data. Transnumeration thinking changed from how to reclassify or order data, to how to represent the variables and data graphically and textually for communication purposes.

5.6. IMPLICATIONS FOR RESEARCH AND TEACHING

This small study suggested there was a big transition required in thinking from anomaly and pairwise variation to global variation. Transnumeration thinking was prevalent but at this stage seemed haphazard which is not surprising as these students were untutored. The students' structuring of data into different representations, be it changing classifications of data, tabulation of data, average, or graph showed different degrees of success. Representational fluency should mean not only interacting with a representation but also connecting that representation to other representations, and being able to change that representation to another representation. The integration of contextual and statistical information, knowledge and conceptions is necessary for making sense of complex systems. Students were arguing from a limited contextual and statistical knowledge base but if prompted sufficiently revealed a rich store on which to build and develop reasoning about data.

Furthermore, this study has led us to consider that reasoning with a statistical model seemed to have two distinct parts. The first part involved reasoning about a group propensity (Konold et al., 1997; Biehler, 1997) (and individual cases sometimes) either by looking for preconceived patterns or by seeing new patterns through a confirmatory or discovery approach. The second part involved decision-making under uncertainty. This decision-making was observed in the evaluation of the statistical model itself (cf. Lesh et al., 1997) whereby a judgement was made on the validity of the model and on whether another model was necessary. Other aspects of decision-making that were observed in the very initial stages of development were: probabilistic judgements which were quantitative (cf. Cobb, 1999) or qualitative (cf. Ainley et al., 2000); causal judgement on whether the variation was real or random and on possible contextual explanations; inference space judgement for generalisation of findings; evaluation of whether the results corroborated with the real situation; and conclusion judgement about what information could be communicated.

Through concentrating the four different lenses of "transnumeration", "consideration of variation", "reasoning with statistical models", and "integrating the statistical with the contextual", on students' thinking we believe it is possible to identify and describe the ways students are thinking within each element and identify the points where their thinking could be scaffolded to another perspective. For transnumeration, a scaffolding point may be the reclassification of raw data into new or transformed quantitative or qualitative variables. For variation, teaching strategies could be developed to turn students' attention from reading a datum for an individual case to thinking of that datum as belonging to a set of theoretical data; and from comparing values on a graph to looking at the range of the data for groups and informally quantifying the variation. When students reason with models, teaching strategies could involve predicting the pattern they might see including the type of variation expected, and drawing their attention to noticing other possible patterns with full awareness about the types of decision-making under uncertainty that are required. Teaching experiments should also consider how to shift students' thinking from their intuitive series comparison graphs for multivariate data to the conventional scatterplot graphs. The integration of the statistical with the contextual may mean scaffolding students' argumentation through attention to evaluation of causal reasoning when responding to statistical data. Overall, teaching strategies may need to focus first on comparing two individual cases and then gradually shift the focus to more and more cases.

This initial exploration into identifying and describing students' thinking from a theoretical framework has now opened up possibilities to do further research at a micro-level on students' thinking and to develop more explicit models for each element of thinking. Further exploration, however, should be continued at the macro-level since such research raises more awareness of the elements of thinking that should be considered overall in developing students' thinking. The study has also highlighted how the use of the Wild and Pfannkuch (1999) framework, helped dissect the student dialogue and reveal insights about the ways in which students are thinking. The model suggested to us to look for certain aspects which we probably would not have noticed without such a prompt. Educators awareness of these elements might give them a greater understanding of how statistical thinking could be developed in students. Reasoning with data is complex and requires fostering students' imagination and producing a web of connections between contextual and statistical knowledge. This integration enables students to construct meanings from data through a constant dialogue with a chain of statistical representations. Questions remain, however, about how such findings could be utilised to effectively develop learning from data in the classroom setting.

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