CAN KINDERGARTEN CHILDREN BE SUCCESSFULLY INVOLVED IN PROBABILISTIC TASKS?

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SUMMARY

This paper describes a classroom teaching experiment, concerning the concept of probability, with children aged 5 in a kindergarten school. The teaching experiment was based on constructivist and interactionist theories about the learning of school mathematics and lasted one month. The collection of the information was based on the tape-recorded interviews with the children (each child was interviewed prior to the research program, at the end of the program and one month later) and the videotaped teaching sessions. During the program, we identified three critical steps in the development of the children's probabilistic thinking: a) the interpretation of the "different" outcomes in a two stage experiment, b) the acceptance of the realization of the experiment for resolving their conflicting viewpoints, and c) estimating the outcomes in a problem. At the end of the program the majority of the children managed to overcome their subjective interpretations and seemed to develop a primitive quantitative reasoning in probabilistic tasks.

Keywords: Statistics education research; Probabilistic thinking; Kindergarten; Instructional activities

1. INTRODUCTION

Recently, the interest of mathematics educators in stochastic concepts has increased, as a consequence of the recognition of the role of these concepts in our daily life. The use of data and graphs to communicate information, as well as the ability to take decisions under uncertainty, is increasing in our society. Moreover, this interest becomes even greater due to the fact that students have much difficulty with these concepts, as many researchers have shown (cf. Fischbein & Schnarch, 1997; Kapadia & Borovcnik, 1991, pp. 73-105; Shaughnessy, 1992).

In order to cope with students' problems concerning the concept of probability, it has recently been proposed that students could be engaged in probabilistic tasks at an early age (cf. National Council of Teachers of Mathematics, 1989). Although a lot of research has been done on young children's probabilistic thinking (cf. Acredolo et al., 1989; English, 1993; Fischbein & Gazit, 1984; Piaget & Inhelder, 197; Schroeder, 1988), the results sometimes seem to be contradictory, as the researchers start from different theoretical viewpoints. These differences are mainly related to the analysis of this concept and to the role of the instruction towards its acquisition. As is well known, the nature of this concept has been connected with different interpretations (classical, frequentist, subjective) and the comparison of them is not always an easy task. Moreover, the work of Piaget and his colleagues and the work of Fischbein seem to have crucial differences with respect to their educational implications (cf. Greer, 2001).

However, substantial research in the classroom is still necessary on the development of children's thinking on probabilistic tasks. Towards this effort, this paper provides focus on the development of children's probabilistic thinking in a kindergarten classroom. More specifically, we will present the learning opportunities that occurred in the social setting of the classroom, as the children tried to

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overcome their subjective interpretations and develop a primitive quantitative reasoning when they were engaged in probabilistic tasks.

2. THEORETICAL BACKGROUND

The research program described in this paper is based on the cognitive framework that Jones and his colleagues (1997, 1999) have constructed in connection with the development of children's thinking in probability. According to this model, four key constructs have been used for the understanding of the probability concept: sample space, probability of an event, probability comparisons and conditional probability. Furthermore, young children's probabilistic thinking has been described across four levels for each of the four constructs: the subjective level, the transitional level, the informal quantitative level and the numerical level. For the purpose of this paper only the first and the second levels of their theoretical framework are presented in full here.

At the first level, the children:

- *Can list an incomplete set of outcomes for a one-stage experiment.*
- *Predict most/least likely events based on subjective judgments.*
- *Recognize certain and impossible events.*
- *Compare the probability of an event in two different sample spaces, usually based on various subjective or numeric judgments.*
- *Cannot distinguish "fair" probability situations from "unfair" ones.*
- *Following one trial of a one-stage experiment do not give a complete list of outcomes even though the complete list was given prior to the first trial.*
- *Recognize when certain and impossible events arise in a non-replacement situation.*

(p. 111, Jones et al., 1997).

At the second level, the children:

- *List a complete set of outcomes for a one-stage experiment and sometimes list a complete set of outcomes for a two-stage experiment using limited and unsystematic strategies.*
- *Predict most/least likely events based on quantitative judgments, but may revert to subjective judgments.*
- *Make probability comparisons based on quantitative judgments (may not quantify correctly and may have limitations when non-contiguous events are involved).*
- *Begin to distinguish "fair" probability situations from "unfair" ones.*
- *Recognize that the probability of some events changes in a non-replacement situation, however recognition is incomplete and is usually restricted only to events that have previously occurred.*

(p. 111, Jones et al., 1997).

The above findings were used as broad guidelines for the organization and the development of the instructional activities given to the children as well as the analysis of the children's activity and the interpretation of the results. The development of the instructional activities given to the children was related to their abilities according to these levels. Therefore, the instructional activities were concerned with the listing of all the possible outcomes for a one and a two-stage experiment, the prediction of the most/least likely event in a random experiment, the probability comparisons, the listing of the possible events after the realization of one trial in a random experiment and the distinction between fair and unfair games.

Moreover, the classroom teaching experiment was based on the constructivist and the interactionist views about mathematical learning. According to these approaches, mathematical learning is characterized as both an individual and collective process. Learning of school mathematics is a process in which students reorganize their mathematical activity to resolve situations that they find personally problematic as well as a process of enculturation into the mathematical practices of wider society (Cobb & Bauersfeld, 1995). Learning opportunities can arise for students from their personal engagement with the mathematical activities as well as from their interaction with the other members of the classroom. Teaching of school mathematics can be characterized as a process in which the students and the teacher negotiate their mathematical meanings and interactively constitute the truth about a "taken-as-shared mathematical reality" (Cobb, Wood, Yackel & McNeal, 1992).

3. METHODOLOGY

All fifteen children that attended the kindergarten classroom in a typical state school of Athens participated in the classroom teaching experiment. The research project commenced in November 2001 and it lasted one month. The children were engaged in mathematical activities related to probability, three times per week. Every lesson lasted 30 minutes. The program was implemented in collaboration with an experienced and well-informed kindergarten teacher.

In the course of the program, we developed a set of instructional activities on the concept of probability concerned with the following themes: sample space, probability of an event, probability comparisons and conditional probability. The instructional activities used in the classroom were developed before and during the progress of the program, according to the evolution of the children's ideas, so that the activities could enhance problematic situations for them.

A typical lesson can be described as follows. The teacher introduced the instructional activity to the whole class, the children made their predictions about the concrete problem by explaining their thinking and the teacher recorded the different ideas of the children. The children checked their predictions through the realization of the experiment and they recorded the results. Then, they compared their initial predictions to the experimental results and discussed the solution of the problem.

The children had not received any previous instruction in probability theory. Moreover, they had not yet received any instruction in whole numbers. Each child was interviewed for one hour prior to the instructional program, at the end of the program and one month later. The interview was realized by the researcher and included 14 tasks. The children were asked:

a) in 5 tasks to report all the possible outcomes for a probability situation (for example, "what may happen if a die with different colors is thrown?" or "In a transparent bag we put three balls (1 red, 1 green and 1 blue). Imagine that you close your eyes and you draw two balls from the bag. Which colored balls could come out?"),
b) in 4 tasks to predict the most likely outcome in a random experiment (for example, to predict which color is "the easiest" to be drawn from a box with colored balls or to come out if we turn a colored spinner),
c) in 2 tasks to choose the probability situation for the most likely realization of an event, that is, to predict among different sample spaces, the one for the most likely realization of an event (for example, we put in a transparent box 2 red balls and 1 white ball and in another transparent box 1 red ball and 1 white ball and asked the children which box they would choose, if they wanted to draw a white ball), and
d) in 3 tasks to report the outcomes of an experiment, after they had seen the outcome of one trial (for example, in a box we put 6 balloons (2 yellow, 2 blue and 2 green). The children closed their eyes and they drew one balloon. They got the balloon out of the box. Then they were asked to predict what could happen in the next trial).

All the items were presented with the use of manipulative materials.

All the teaching sessions were videotaped and the interviews were tape-recorded. The transcripts of the interviews and the teaching sessions provided the data for the analysis of the students' learning. Analytical descriptive narrative was the method used for the analysis of the results (Erickson, 1986). The choice of this method was considered the most appropriate as:

> Analytic narrative is the foundation of an effective report of fieldwork research. The narrative vignette is a vivid portrayal of the conduct of an event of everyday life, in which the sights and sounds of what was being said and done are described in the natural sequence of their occurrence in real time (Erickson, 1986, pp. 149-150).

4. THE SEQUENCE OF THE INSTRUCTIONAL ACTIVITIES

All the instructional activities were related to the interests and the experiences that children have at this age (5 years old). They were presented through small stories in puppet shows (where the basic persons were a grandfather, a grandmother and a squirrel), dramatic metaphors (teacher in role and children in role play) and games. All fifteen children participated together in the realization of every activity. Four categories of activities were discussed with the children:

In the first category of the activities the children were asked to recognize and distinguish events that always, sometimes or never happen in their daily life as well as in random experiments. The children were asked to make certain movements, according to the instructions of the teacher, when they heard these words and they were also asked to talk about events that always, never or sometimes happen at school. Then, they were asked to comment on different pictures from everyday life, by discussing which of these events take place always, sometimes or never and to construct a poster with the title “never”, “sometimes” and “always”. These activities gave the opportunity to the children to begin to estimate that an event could be certain, possible or impossible.

Afterwards, they had the opportunity to talk about the same words in random experiments. The random experiments were concerned with two or three possible outcomes. For example, the two heroes from the puppet show, the grandmother and the grandfather, were in complete disagreement about the color that could occur if they tossed up a “piece” with two colors (a “piece” is used in the Greek game of backgammon and in our activity it was presented like a coin with two different colored sides).The grandfather said that the red color would always occur and the grandmother said that sometimes the red color would occur and sometimes the yellow. The children had to express their opinion by discussing which of the heroes was right and why, and then to realize the experiment.

When the children had been engaged in this category of activities, they had the opportunity to concentrate on the colors that existed in front of them in the experiments. This was a significant step in their thinking. As we had observed from the first interview, the children sometimes mentioned colors that they desired but which did not exist in the task. Furthermore, we had the opportunity to initiate them into the process of the experiment. However, we should note that, at this phase, the realization of the experiment did not always influence the children’s answers. This means that some children insisted that a specific color could always come out, even though the results of the experiment did not support this opinion.

The second category of the instructional activities was concerned more systematically with the sample space. The children had to discuss all the possible events in a one or a two stage random experiment. Especially for the two stage experiments, the instructional activities were concerned with ordered and unordered pairs. For example, there were activities like the following: a) They had to design Christmas cards with two flowers, in order to help Father Christmas give the cards together with his presents to the children. The children had to pick one color from a box with three colors to paint the first flower and then they had to put it back. Then they had to pick one color for the second flower. After painting the flowers, they would discuss how many different cards had been made by the class. b) The squirrel wanted to make a curtain for the window in his house. He went in a shop that sells curtain material and the shop assistant showed him three plain materials with different colors. He would like the curtain to be made with two different materials. What colors could the curtain have? (The children were shown the materials).

The third category of the activities was concerned with the probability of an event and the probability comparisons. The tasks were related to situations that all the possible outcomes were equally likely. We used boxes with balls, dice and spinners. The children were asked to predict the color that was “the easiest” to come out or the color that could come out “more often”. For example, we discussed with the children activities like the following: a) The children were asked to help the squirrel to solve a problem he had: “Every morning my mother puts fruits in the basket for my lunch at school. Today, she gave me one apple and three oranges. I will close my eyes and take a fruit at random, because I like both fruits. Which fruit is the easiest to come out?” b) We showed the children one spinner with two colors, red and green, the proportions of the colors were 3/4 and 1/4 respectively. The children were separated into two groups: the “red” and the “green”. They played the following game: “One child from every group turns the spinner. According to the color that comes out, the respective group gains one marble. The winner is the team that has gained the most marbles after 20 turns.” After the end of the game the children had to discuss the fairness of the game.

The last category of the activities was related to conditional probability. The children were asked to find all the possible outcomes in a random experiment after having completed one trial in a one-stage experiment. Moreover, they had the opportunity to discuss if an event becomes more or less likely to appear according to the results of previous trials, in non-replacement experiments. For example, the children participated in activities like the following. The squirrel would like to decorate the Christmas tree with colored balloons. He went in a shop and he found a box with red and yellow balloons (5 red and 3 yellow). He decided to pick a balloon at random. The squirrel picked one balloon out of the box and showed it to the children. The children were asked to predict the color that was “the easiest” to come out as the second balloon. Then, the squirrel picked a second balloon out of the box and showed it to the children. The children were then asked to predict the color that was “the easiest” to come out as the third balloon.

5. RESULTS

In sections 5.1 to 5.3 we present the children’s answers at the interviews prior to the teaching experiment, the critical moments on the development of the children’s probabilistic thinking during the teaching experiment and their solutions in the interview tasks after the teaching experiment.

5.1. BEFORE THE TEACHING EXPERIMENT

Prior to the teaching experiment, all the participating children seemed to interpret the tasks in probability in a subjective manner. However, we could identify some qualitative differences in their thinking. More specifically, the children’s responses could be classified in two general categories.

In the first category (4 children), the children gave answers that were strongly influenced by their favorite color for all the tasks. Ada was a representative for this category. She answered the red color as the only one that could appear in all the tasks. So, she reported only one outcome in a one or two stage experiment (e.g. she gave only one pair in a two-stage experiment including the red color) and she insisted that the red color was the most likely (“the easiest”) to come out in all the experiments. Having already seen the results of one trial in an experiment without replacement, when she was asked to describe the possible outcomes on the next trial, she repeated the color that she had got in the first trial.

In the second category, the children could give all the outcomes for a one-stage experiment. However, they had great difficulty in a two-stage experiment. For example, in the following task: “In a transparent bag we have put three balls (1 red, 1 green and 1 blue). Imagine that you close your eyes and you draw two balls from the bag. Which colored balls could come out?” the children gave only one combination. These children considered that they could not give another combination, because they had only three colors in the bag and so “it is only one that is left over”. The children answered in a similar way in tasks designed in terms of ordered pairs (for example in the same context of the above task, we asked them to imagine that they wanted to draw two balls, one ball for themselves and one for a friend). That means that it was very difficult for them to understand the context of these tasks, as they could not imagine repetitions of the two-stage experiment so as to think that for every trial all the balls could be inside the bag again. In the above tasks they considered that one trial of the experiment was identical with the solution of the problem. We could say that the children could propose a number of pairs according to the number of the balls inside the bag. Then they stopped when they could not make other pairs with the rest of the colors. This tendency was also recorded in a similar task, where we had put two boxes with two balls in each of them. The children were asked to predict the colors that could come out if they drew a ball from the first box and a ball from the second box. They proposed two different pairs of balls (one ball from each box), thinking that they had exhausted all the colors.

Furthermore, these children did not have a consistent way of thinking about their answer to the question that this task brought out, regarding the probability of an event and the comparison of probabilities. Sometimes they gave a right answer and sometimes a wrong answer. However, in all cases it was very difficult for them to provide an explanation for their answer. In the case when they presented their arguments, they seemed to be influenced by the position of the materials (especially in the experiments with the balls) or their favorite color (especially in the experiments with the spinners), that is they based their judgments on subjective beliefs. In the comparison probability tasks, they usually gave the right answer for the probability situation related to the spinners and the wrong answer for the one related to the boxes with the balls. This difference in their answers shows that the comparison tasks were easier for them when they had to compare sizes rather than numbers of objects.

After the completion of a trial in an experiment, they did not usually mention the color that they had picked, but they gave all the other outcomes at the next trial.

We could remark, at this point, that according to the theoretical framework of Jones et al. (1997), all the children attained the first level for the probability of an event, the probability comparisons and the conditional probability. However, in relation to the sample space, except for the children in the first category, their problems were with the two-stage experiments, as they could find the possible outcomes in the one -stage experiments.

5.2. DURING THE TEACHING EXPERIMENT

We could identify three critical steps in the development of the children's probabilistic thinking during the classroom teaching experiment. These steps will be presented through representative episodes that took place in the classroom. The common characteristic in all these incidents was that the children found a way to resolve their problems that gave them the chance to *overcome their subjective interpretations on the probabilistic tasks***.** These solutions influenced the development of the children's probabilistic thinking during the program, with the help of the teacher who always referred to them when the same problems appeared again.

These steps are described in the following way: a) the interpretation of the "different" outcomes in a two stage experiment; b) the acceptance of the realization of the experiment for resolving conflicting viewpoints; and c) estimating the outcomes in a problem.

The interpretation of the different outcomes in a two-stage experiment

In order to introduce the children to the idea of searching for all the possible outcomes in a two-stage experiment, we asked them to participate in the *Christmas cards* activity that was described earlier (section 4). This activity seemed to be very fruitful, as it gave them the opportunity to discuss what it means to have different outcomes in a two-stage experiment with ordered pairs. We could identify two important moments in the discussion during the engagement of the children with this task.

The first instance took place when it was Thomas's turn to pick the two colors. After his picking the first color (it was the red one) and replacing it, he picked the second one and it was also red. Thomas told the teacher with surprise that this could not happen, because "we have to choose two different colors". As the teacher tried to pose Thomas' s thoughts as a topic of discussion with the rest of the class, many children agreed with Thomas's thought. However, some children argued that they had to accept this outcome making comments like **"**But this color came out". This argument showed that some of the children had already begun to accept the randomness of the outcomes in an experiment. Although, this acceptance was also a consequence of the previous activities concerning the one-stage experiments, the interesting point at this moment was that this acceptance of the randomness of the colors that could come out during the realization of an experiment allowed them to accept as a “normal pair” the pair with the same color twice. This way, they managed to overcome this difficulty.

When the teacher asked them to find the cards that were same, the children began to put together the cards having the same colors. As they began to put together the cards with the red flower on the left and the green flower on the right, on the next card, the flower on the left was green and the other one was red. The following discussion between the teacher and the children took place in the classroom.

Teacher: Where will you put this card?

Thomas: Here. *(Together with the other cards)*

Anna: No, this is different. The green goes first and then the red.

Socrates: This is wrong. They are not different. They are the same.

Thomas: It’ s a *little* different!

Socrates: Yes, it’s a *little* different.

Teacher: OK. They are not the same, however they are a little different, so can we put this card alone, not with the others?

Thomas: Yes, here. *(He is showing a place near the other cards).*

Teacher: Can we make other different cards?

Anna: Yellow, red.

Teacher: OK. We have the red-yellow card and we can make the yellow-red one.

Marina: Two green.

Teacher: Very nice.

Thomas: Two blue.

Teacher: Bravo, Thomas.

On the above episode, as the children tried to negotiate their different interpretations about the word “different” in the concrete context, they managed to resolve their conflict and construct an acceptable characterization for the difference of the cards. The acceptable interpretation of the word “different” gave them the possibility to find all the different ordered pairs in subsequent similar activities. For example, in another activity, the children had to construct different Christmas trees with two balls by the same manner. When the children checked if they had made all the different trees, they used the same words (“little different”) to describe their results.

Two days later, the children had to find all the different combinations with two colors for a curtain, when they had to choose among four colors. The children found some of the combinations with the process of the experiment. When the teacher asked them if they had found all the different ways for the colors of the curtain, Marina said that there was one more as “we have red and green, red and blue, we don’t have red and yellow”. The way that she managed to justify her answer showed that she had made a first step towards the generation of a strategy for the listing of the possible outcomes in a two-stage experiment. This does not mean that the children managed to construct a systematic strategy to find all the possible outcomes in a two-stage experiment. However, it was a major advance for them to develop such arguments concerning these probabilistic situations.

The acceptance of the realization of the experiment for resolving their conflicting viewpoints

A second critical step was the acceptance of the realization of the experiment for resolving conflicting viewpoints. At the beginning, there were children that insisted on their solutions even though they knew the results of the experiment. When they legitimated the way to resolve their disagreement by respecting the results of the experiment and began to change their opinion according to them, they were based more often on quantitative judgments, by using the words “more” or “less” and sometimes numbers to justify their answers. The children seemed to recognize that there was a solution to the problem independent of their desire and they were trying to find a logical explanation for their answer.

Moreover, one type of argument that some children used to justify their opinion in the probabilistic situations was related to the process of executing the experiment. This was expressed as follows: “we close our eyes and we pick a ball”, or “we mixed the balls” and it seemed to constitute a fruitful contribution to the development of the children’s thinking. We could maintain that this type of argument was a different expression of the acceptance of the notion of randomness during the realization of the experiment. More specifically, these statements served as a counter argument to the solutions that were based on subjective judgments like: “this one, because it is higher up in the bag”, or “it is the blue one, because the two green are below”, which referred to the position of the objects.

The representative episode described below took place when children were engaged in the following activity concerning the probability of an event.

The grandmother and the grandfather of the squirrel would like to make a scarf for him. The grandmother found a bag with balls of wool with different colors (1 blue and 3 green). As the grandmother and the grandfather were in complete disagreement about the color of the scarf, they decided to pick a ball at random. The children were asked to predict the color that was “the easiest” to come out. (The bag was painted on cardboard as in diagram 1. The black ball represents the blue color and the grey balls represent the green color.)

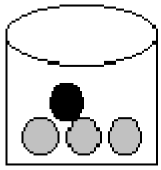

Diagram 1

The following dialogue took place among the members of the classroom.

Anna: The green color, because there are 3 green balls.

Teacher: Is there another opinion?

Socrates: The blue color, because this is higher up in the bag.

Teacher: What do you want to say Paul?

Paul: I say the green ...we close our eyes and we pick a ball.

Teacher: Paul said that we close our eyes when we pick a ball. So, we cannot see where it is. As we are picking a ball, we can mix the balls. *(He is doing the movement.)*

Socrates: It is the blue one, because the three green are below.

Anna: It is the green, because there are more. Let’s do it.

Socrates: Yes!

Teacher: OK! Let’s do it.

Anna, Paul and Socrates were three of the children whose responses in the interview could be classified in the second category. However, in the progress of the program, Anna’s responses were based on quantitative judgments, as she often used the words “more” or “less” to justify her thinking. It was the first time that she compared numbers to justify her answer. On the other hand, Socrates was still influenced by subjective judgments, when he tried to justify his answer. The level of Paul’s thinking about the probability of an event could be characterized as transitional, between subjective and naive quantitative thinking. In this episode, as he tried to justify his answer, he was thinking about the process that the children used to make the experiment and he used an argument which could upset Socrates’ argument. This argument seemed to be a fruitful contribution to the development of Socrates’ thinking. Although Socrates insisted on his answer, when Anna told him that they could find the solution to their problem by executing the experiment, he willingly accepted her idea. This was the first time that the children legitimated a way to resolve their disagreement by respecting the results of the experiment. After the realization of the experiment, Socrates changed his opinion. This episode served as a catalyst for the following lessons, as the teacher used it as a point of reference when the children expressed their different opinions about the solution of a problem. In this sense, this was a critical moment in the development of children’s probabilistic thinking.

Estimating the outcomes in a problem

A third critical step was attained when the children could give the solution in a probabilistic situation by estimating the outcomes in a problem without the need of the realization of the experiment. This step arose from children's attempts to pose their own problems about the probability of an event, when they had to discuss probability situations with equally likely events.

The following episodes illustrate the previous remarks.

Paul: I want to give a notebook to Kostas.

Children: What color does Kostas like?

Paul: The red. *(He is putting in the box, 2 red, 2 green and 2 blue notebooks).*

Teacher: Which color is the easiest to come out?

Anna: The red.

Paul: No. It' s equally easy to get the red and the green and the blue.

Teacher: *(To the class)* What do you say about Paul's opinion?

Thanasis: Yes. We have two red, two blue and two green notebooks.

Anna: Paul, put more red notebooks in the box!

Although the teacher had not talked with the children about probability situations with equally likely events until this episode, this problem arose from the children's attempts to make their own problems. In this way they provided us with the opportunity to discuss similar activities with them, like fair and unfair games.

The episode described below is representative of the dialogues that evolved later in the classroom. The children were engaged in the following activity.

The grandmother and the grandfather of the squirrel would like to make a jumper for him. The grandmother found a bag that contained balls of wool with different colors (1 blue, 2 white and 3 red). As the grandmother and the grandfather were in complete disagreement about the color of the jumper, they decided to pick a ball at random. The children were asked to answer which color was "the easiest to come out". The bag was painted on cardboard as in diagram 2. The black ball represents the blue color and the grey balls represent the red color.)

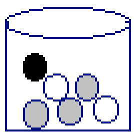

Diagram 2

Andreas: The red color, because there are three red balls.

Teacher: Basil, what do you want to say?

Basil: The blue color, because it is higher.

Andreas: If we mix the balls, we do not know which is higher up in the bag.

Teacher: Andreas said that we can mix the balls and then we cannot know which is higher up in the bag, the red, the white or the blue. What do you say now, Basil?

Basil: Can we do it? (*He means the experiment*).

The teacher asked the opinion of the other children and then they realized the experiment. After having reached the conclusion that the red color was the solution of the problem, the teacher asked the children the following question.

Teacher: If we would like all the colors to be "easy" to come out, the red and the blue and the white, what do we have to put in the bag?

Basil: 3 blue, 3 white and 3 red.

Teacher: Do you agree with Basil?

Anna: No.

Teacher: What do you believe? What must we do?

Anna: We will put 2 blue, 2 white and 2 red.

Teacher: Is there another opinion?

Thodoris: 4 blue, 4 red and 4 white.

Teacher: Socrates?

Socrates: 1 blue, 1 red and 1 white.

Teacher: OK. Finally, which of you is right?

Thodoris: They are all the same.

Teacher: Very good. All of you are right. If we would like all the colors to be "easy" to get out, the red and the blue and the white, then we have to put an equal number of red, blue and white balls in the bag.

5.3. AFTER THE TEACHING EXPERIMENT

The children were interviewed at the end of the teaching experiment and one month later. After both interviews, the results were the same, as described below.

In relation to the sample space, all the children could find the outcomes in a one-stage experiment and the majority (8 children out of 15) could report all the outcomes in a two-stage experiment, although without a systematic strategy.

In probability situations concerning the probability of an event and the probability comparisons, apart from two children, the rest of them used quantitative arguments to justify their answers. Eight children used both the words "more" or "less" and numbers. Five children used exclusively numbers for their justification.

Following one trial of a one stage experiment, all the children could give the right answer, that is a complete list of outcomes without being influenced by the outcome of the first trial of the experiment. Moreover, they recognized the changes that happened in an event (that is, if it was easier or more difficult to appear), according to the results of previous trials, in an experiment without replacement

So, we could infer that all the children had made real progress in their probabilistic thinking as a result of the teaching experiment. They had all developed a naïve quantitative thinking responding to probability tasks and they had approached the second level according to the cognitive model that Jones and his colleagues (1997) have constructed.

6. CONCLUSIONS

The results of the research presented in this document exhibit that kindergarten children are able to make considerable progress in their probabilistic thinking when they are involved in simple probabilistic tasks. They can overcome their subjective interpretations and develop a primitive quantitative thinking. The classroom teaching experiment reveals many critical steps in children's development of probabilistic thinking. The steps that the children made in order to proceed from a subjective towards a naïve quantitative level of thinking are connected with the discussion of the notion of different outcomes in a two-stage experiment, the acceptance of the realization of the experiment for the solution of a probabilistic problem, and the estimation of the data in a probabilistic situation with equally likely events. It should also be noted that the acceptance of the notion of randomness, expressed in different ways from the children, in their various activities, constituted a hidden driving force in the children's development of probabilistic thinking. However, more evidence from different cultural kindergarten classrooms' settings is needed to investigate the critical steps that enhance the development of children's probabilistic thinking at this age.

Moreover, we should mention the fruitfulness of the sequence of the activities that were used in the classroom teaching experiment. This manner of the organization of the instructional activities seemed to offer a lot of learning opportunities to the children. Thus, we could infer that this is a possible path for the incorporation of probabilistic activities in the kindergarten school.

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