PREPARING PRESERVICE TEACHERS TO TEACH PROBABILITY AND STATISTICS TO EARLY LEARNERS: A SYSTEMATIC REVIEW

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ABSTRACT

Various sources suggest preparing teachers of early- or primary-age students to teach probability and statistics involves various challenges. Some of the approaches researchers take for resolving these challenges include developing preservice teacher content knowledge, pedagogical knowledge, as well as providing other opportunities to learn. Various sources also suggest that research in this area is missing or underemphasizing some components. The systematic review undertaken here considers this question by comparing extant literature to teacher preparation standards. Results show that studies emphasize development of probability and statistics concepts and procedures, and an underrepresentation of development of pedagogical knowledge, learning from school experiences, such as student teaching, and reflection on practice.

Keywords: *Statistics education research; Early childhood; Elementary; Primary; Content knowledge; Pedagogical knowledge; Opportunities to learn*

1. INTRODUCTION

1.1. CHALLENGES IN PREPARING PRESERVICE TEACHERS

There is no shortage of omissions in various areas of education research (National Research Council, 2010). This is true of studies dealing with preservice teachers of early- or primary-age students (PTEP) who teach probability and statistics (PS). For example, Leavy et al. (2018) noted an immediate need for studies in early childhood learning standards, teacher content knowledge and pedagogical knowledge, and comprehensiveness and influence of research literature, among other issues.

One reason for these gaps is that mathematics teacher education is complex. It requires a profound understanding of fundamental mathematics, coursework that involves reasoning and explaining, along with ongoing opportunities for professional development (Conference Board of the Mathematical Sciences, 2012; Ma, 1999). It also requires different kinds of opportunities to learn, such as coursework and field experiences that promote productive habits of mind (e.g., modeling, seeing structure, generalizing; Conference Board of the Mathematical Sciences, 2012). Furthermore, mathematics teacher education requires collaboration between various stakeholders, such as teacher education faculty, mathematics faculty, and Pre-K–12 teachers (Conference Board of the Mathematical Sciences, 2012).

Mathematics teacher preparation programs that attend to these requirements are more likely to produce effective PTEPs. There are disagreements, however, about the similarities and differences between mathematics and PS, which have implications for teacher preparation. For example, many contend that PS is distinct from mathematics (Browning & Smith, 2015; Capaldi, 2019; Leavy, 2015). Cobb and Moore (1997) summarized the situation in this way:

> Although mathematicians often rely on applied context both for motivation and as a source of problems … the ultimate focus in mathematical thinking is on abstract patterns: the context is part of the irrelevant detail that must be boiled off … like mathematicians, data analysts also look for patterns, but ultimately, in data analysis, whether the patterns have meaning, and whether

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> they have any value, depends on how the threads of those patterns interweave with the complementary threads of the storyline. In data analysis, context provides meaning. (p. 803)

Furthermore, Rossman and Chance (2012) stated that PS is the science of reasoning from data, while Moore (2004) noted that many core statistical concepts are not mathematical in nature. Capaldi (2019) provided a useful anecdote for illustrating differences, noting that it is not unusual for students who are learning mathematics to ask, "What is this useful for?" while it is difficult to imagine students asking this question in a statistics class that involves analysis of authentic data and context (p. 150).

Reports by various organizations further illustrate distinctions between mathematics and PS. For example, *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) was produced to address shortages in PS standards provided by the National Council for Teachers of Mathematics (Bargagliotti et al., 2020). Similarly, the *Statistical Education of Teachers* (Franklin et al., 2015) was produced to address several areas, such as articulation of statistical problem-solving across grade levels that were left undeveloped by the Conference Board of the Mathematical Sciences (2012). In both cases, reports intended to clarify mathematics teaching and learning necessitated additional reports for clarifying PS teaching and learning.

One thing PTEPs of mathematics and PS have in common, however, is difficulty in teaching these subjects. For example, Tatto and Senk (2011) showed that 97% of teachers of primary-age students (up to Grade 6) in the United States are successful with some school-level mathematics such as performing basic computations with whole numbers, understanding properties of operations with whole numbers, and solving problems involving simple expressions and equations. Alternatively, this group of teachers also tended to overgeneralize problem-solving strategies, and they had difficulty solving abstract problems, along with difficulty solving problems with multiple steps. In relation to these results, Tatto and Senk reported that teachers in the United States, both primary and secondary, studied mathematics less often both at the university level and in school settings compared to teachers in high achieving countries. Tatto and Senk reported this finding by measuring preservice teachers' opportunity to learn mathematics in relationship to their amount of mathematics content knowledge.

Findings reported by Tatto and Senk (2011) coincide with decades of research on mathematics achievement in the United States. According to the National Research Council (2001), 30 years of data has shown that students are reasonably proficient at computational procedures, though they misunderstand mathematical concepts, and they are deficient at solving mathematical problems. As prelude to these trends, Ball (1988) observed similar difficulties, noting that teachers were able to set up and conduct various mathematical procedures, while they were unable to explain or apply concepts to unfamiliar situations. The combination of procedural proficiency, shallow conceptual understanding, and inability to solve abstract problems with multiple steps is obviously an encumbrance for teaching mathematics, and even more so for teaching PS. One reason for this is that PS involves different kinds of problem solving. For example, Wild and Pfannkuch (1999) suggested problem solving in terms of statistical inquiry is a process beginning with problem identification, which moves on to generating a plan to investigate, collecting and analyzing data, and drawing conclusions based on evidence.

Unfortunately, research suggests that both teachers and students in the United States have many gaps in these areas and that these gaps have persisted for decades. As it might be expected, these difficulties appear in literature dealing with PTEP preparation to teach PS. For example, Heaton and Mickelson (2002) noted that PTEPs focus on procedures, such as making graphs, when engaging students in statistical inquiry. Authors de Vetten et al. (2019) reported PTEPs fail to recognize that representative samples can be used to make inferences and Leavy (2004) showed that PTEPs focus on mean as a measure of representativeness regardless of distributional shape. Furthermore, Heaton and Mickelson (2002) and Leavy (2010) observed PTEPs having difficulty formulating questions for statistical inquiry. Still more, Park and Lee (2019) noted PTEPs struggle to identify instructional strategies for resolving student statistical misconceptions. Researchers have taken different approaches to resolve these problems. One of the most common is to improve PTEP content knowledge.

1.2. APPROACHES TO RESOLVING PROBLEMS IN PREPARATION

Content knowledge. Scholars define content knowledge in different ways. According to Shulman (1986), content knowledge is the amount and organization of knowledge in the teacher's mind. Others label content knowledge as subject matter knowledge, which is likewise defined as facts, concepts, and the organizing frameworks of a discipline (Leavy, 2015). Ball et al. (2008) specified that one component of content knowledge is common knowledge, which is the information or content taught to students, such as whole numbers, fractions, decimals, measurement, patterns, formulas, and so on. Two specific examples of common knowledge include the ability to identify a number that lies between 1.1 and 1.11, and to explain that a square is a rectangle (Ball et al., 2008, p. 399).

Groth and Bergner (2006) suggested that content knowledge is conceptual and procedural. According to Hiebert and Lefevre (1986) conceptual knowledge is a network of relationships linking discrete pieces of information. Procedural knowledge, again according to Hiebert and Lefevre, is comprised of two parts. The first is the symbol representation system, for example, recognizing the symbols in the equation $6 + 2 = 8$, along with recognizing that the syntax of the equation is correct. The second part is the set of algorithms for completing tasks, for example, the steps applied to solve the equation $x + 2 = 8$.

An important study by Ma (1999) emphasized conceptual and procedural knowledge as components of content knowledge. Ma explained that a sample of Chinese preservice teachers possessed more conceptual and procedural knowledge in comparison to a sample of practicing United States teachers. Ma further noted that the gap in knowledge between these two groups of teachers imitates the gap between students in their respective countries. One conclusion of Ma's research was that raising student achievement requires improving teacher content knowledge, though addressing content knowledge independent of other efforts is insufficient.

Improving pedagogical knowledge. That improving student learning requires more than focus on teacher content knowledge is evident in the models used to explain teacher competence. Shulman (1986) paired content knowledge with pedagogical knowledge, defined as subject matter knowledge for teaching. Examples include scope and sequence of topics, useful representations of key ideas, analogies, usual misconceptions, and all other approaches used by teachers for making subject matter comprehensible to students. Likewise, Ball et al. (2008) paired common knowledge with pedagogical content knowledge, adding that the latter is a "kind of amalgam of knowledge of content and pedagogy that is … needed for teaching" (p. 392).

Unlike Shulman (1986), Ball et al. (2008) noted that teachers also possess specialized knowledge, which they defined as mathematical knowledge and skill unique to teaching and not necessarily needed in other contexts. Teacher activity representative of specialized knowledge includes judging the usefulness of alternative procedures, defining key terms, analyzing errors, and recognizing the limits of algorithms for solving various problems (Leavy, 2015).

An astute observation by Ma (1999), however, is that improving teacher content knowledge occurs as teachers attend to teaching. Ma emphasized this relationship by reporting experienced United States elementary school teachers in the study did not possess more content knowledge compared to new teachers. One implication of this finding is that experience teaching elementary mathematics in the United States does not change a teacher's depth of mathematical understanding. Alternatively, Ma reported that Chinese teachers did improve their content knowledge while teaching. Along with attending to teaching, Ma suggested the following steps for making improvements: i) peer collaboration focused on lessons and curriculum, ii) individual study of curricular materials that teach teachers during implementation, iii) emphasis on mathematics during teacher preparation, and iv) classroom interactions that focus on substantive subject matter.

Just as Ma (1999) emphasized the relationship between content knowledge and teaching, others assert the same relationship exists for preservice teachers. Improving preservice teacher content knowledge occurs as preservice teachers attend to teaching. With respect to programming generally, Canada (2006) noted that teacher preparation is concerned with developing PTEP content and pedagogical knowledge. In terms of readiness to teach, Leavy (2015) stated that assessing PTEP content knowledge alone, though helpful in resolving gaps and misconceptions, is an insufficient indicator of readiness. As for coursework, Green and Blankenship (2013) have noted that an introductory statistics course, while helpful for improving PTEP knowledge, may not necessarily provide opportunities to develop PTEP pedagogical or specialized knowledge.

Another way to illustrate the relationship between improved content knowledge and teaching appears in some studies that implemented a variety of pedagogical techniques to improve PTEP competence to teach PS. For example, Hourigan and Leavy (2020) showed that improving PTEP probabilistic understanding requires a variety of instructional practices such as "hands-on experiences, experiments with repeated trials and simulations supported by opportunities for reflection on results through small group and whole-class discussion" (p. 1001). Alkas Ulusoy and Kayhan Altay (2017) noted that improving PTEP statistical reasoning involves engaging PTEPs with case studies, authentic tasks and authentic data for analysis. Similarly, Cross Francis et al. (2014) used a statistical investigation cycle (composed of generating a plan to investigate a problem, collecting data, analyzing data, and drawing conclusions) to improve PTEP knowledge of PS. That research supports the claim that improving PTEP content knowledge is in some ways dependent on pedagogical skill.

The statistical education of preservice teachers. The way in which content and pedagogy are prioritized, apportioned, and implemented to form a program that effectively prepares PTEPs to teach PS is unknown. Nevertheless, Franklin et al. (2015) provided an outline for starting. According to Franklin et al., the first two goals of PTEP preparation are to develop content knowledge, followed by developing an understanding of how PS concepts build from elementary to middle school. Franklin et al. recommended different coursework formats for achieving these goals, including an introductory course in statistics covering content and instructional strategies, another course dedicated to learning statistics content, and greater emphasis of PS topics in existing mathematics courses. Franklin et al. further described content standards for PTEPs, such as ability to carry out steps of the statistical problem-solving process, using appropriate data analysis methods, and examining patterns in data to answer questions.

While Franklin et al. (2015) focused mostly on PTEP content knowledge, they also recognized the importance of improving pedagogical knowledge by identifying this as a third goal of preparation. According to Franklin et al., preparation programs should "model effective pedagogy by emphasizing statistical thinking and conceptual understanding, relying on active learning and exploration of real data, and making effective use of technology and assessment" (p. 2). Nevertheless, Franklin et al. primarily addressed pedagogical knowledge in the context of classroom assessment and use of technology.

Opportunities to learn. The summary of these studies and reports suggest that current PTEP programs are missing or underemphasizing some components. Use of content knowledge and pedagogical knowledge, though conceptually helpful, may not represent the necessary array of *opportunities to learn* (Tatto & Senk, 2011) for effective PTEP preparation. According to Tatto and Senk (2011) opportunities to learn include i) university level mathematics (e.g., post-secondary coursework), ii) school level mathematics (e.g., learning from curricula), iii) mathematics-specific pedagogy, iv) general pedagogy, v) classroom diversity, vi) reflection on practice, and vii) school experiences (e.g., student teaching). The concept of opportunities to learn includes content knowledge, as represented in items i and ii, as well as pedagogical knowledge, as represented in items iii and iv. In addition, opportunities to learn includes elements alluded to by Ma (1999), especially vi and vii.

One model for directing the preparation of PTEPs to teach PS. There are many sources that provide standards and recommendations for preparing PTEPs. Examples include reports by the Conference Board of the Mathematical Sciences, (2012), Franklin et al. (2015), and the National Research Council (2010). Nevertheless, *Standards for Preparing Teachers of Mathematics*, authored by the Association of Mathematics Teacher Educators ([AMTE], 2017) provides information that specifically develops a range of competencies. For example, AMTE Standard C.1 indicates, "well-prepared beginning teachers of mathematics possess robust knowledge of mathematical and statistical concepts that underlie what they encounter in teaching" (p. 8). More specific competencies for PTEPs identified as part of this standard include the ability to "solve problems in more than one way, explain the meanings of key concepts, and explain the mathematical rationales underlying key procedures" (Indicator C.1.1; AMTE 2017, p. 8). In another example, standards P.3 notes that programs arrange "practice-based experiences …" (AMTE, 2017, p. 33). Specific characteristics described within this standard include opportunities for PTEPs to analyze how Pre-K-12 students think and learn about mathematics, implement high-level tasks, and generally engage in the everyday work of teaching

(Indicators P.3.1, P.3.2, P.3.4; AMTE, 2017). The Appendix shows AMTE headings for understanding the organization of chapters, standards, and indicators.

As noted, many contend that PS is a distinct subject from mathematics (Capaldi, 2019; Cobb & Moore, 1997; Leavy, 2015, Rossman & Chance, 2012). This position may imply that program recommendations and standards for preparing teachers of mathematics are deficient for directing programs as they prepare PTEPs to teach PS. The standards and recommendations produced by AMTE (2017), however, focus on candidate knowledge, skills, and dispositions, along with description of program characteristics for promoting competencies within each of these areas. For example, standard C.2 is addressed to "beginning teachers of mathematics," though the specific competencies fit PTEPs teaching PS, such as ability to gather information on students' prior knowledge, clarify objectives, plan detailed lessons, and select meaningful tasks (Indicator C.2.2; AMTE, 2017, p. 12, 14).

2. LITERATURE REVIEW

2.1. STUDIES THAT EMPHASIZED DEVELOPMENT OF CONTENT KNOWLEDGE

As might be expected, a prevalent category of literature focused on improving PTEP content knowledge to teach PS. One early article by Batanero et al. (2004) examined two activities for teaching probability to primary and secondary preservice teachers. The first activity involved subjective perception of randomness using the following problem:

> Some children were each told to toss a coin 40 times. Some did it properly. Others just made it up. They put H for Heads and T for tails. These are Daniel and Diana's results: Daniel: H T H T T H H T H T H H T T H T T H H …. Diana: H T T T H T T H T H T T T H T T T T H H T T T H T T H T T …. Did either or both Daniel or Diana make it up? How can you tell? (Batanero et al., 2004, p. 4)

Preservice teachers were presented with the problem and asked, "What type of people do you think are interested in problems similar to Item 1?" Next, preservice teachers were presented with responses to the problem from 14 and 18 year-old students. For example, one response was, "The sequence pattern is too regular to be random, results almost alternate" (Batanero et al., 2004, p. 6). This was followed by discussion of more questions, such as asking why the 14 and 18 year-olds answered differently. A notable feature of this activity is how preservice teachers discussed the responses of secondary students, rather than posing answers to the activity themselves.

A more recent article by Hourigan and Leavy (2020) examined how preservice teachers of primary aged students developed and understood probabilistically fair and unfair tasks. The study began with preservice teachers completing four sessions of instruction on probability, focused on specific topics such as the nature of probability, describing likelihoods, and sampling. Sessions included different kinds of activities: i) discussion of questions, such as, "What is probability?" and "Why is it important?" ii) making predictions about a video clip showing a race between a car and a student, iii) analyzing the work of primary age students based on the same clip, and iv) describing the chance of picking counters from a bag.

At the conclusion of sessions, Hourigan and Leavy (2020) asked preservice teachers to "Select 3 different materials and for each material demonstrate how it can be used to create both a fair and unfair chance activity [and in] each case, briefly describe the rules associated with the activity" (p. 1003). Preservice teachers generated qualitative data in response to this prompt, which Hourigan and Leavy analyzed. One result of the analysis showed that some preservice teachers misinterpreted concepts such as confusing *fair* with *certain* and *unfair* with *impossible*. Another result showed that some preservice teachers misunderstood representativeness by assuming the characteristics of a random generator (such as a die) will manifest after a few trials. Still another result suggested preservice teachers misunderstood the potential for bias in games.

Another recent study by de Vetten et al. (2019) examined the informal statistical inferencing abilities of PTEPs. These researchers defined informal statistical inference as "generalizations based on sample data, expressed with uncertainty, and without the use of formal statistical tests" (p. 640). Unlike studies by Batanero et al. (2004) and Hourigan and Leavy (2020), which involved participants enrolled in courses, de Vetten et al. assessed hundreds of preservice teachers of primary age students from multiple institutions.

The assessment used by de Vetten et al. (2019) consisted of five tasks combining different kinds of items. For example, one of the tasks required comparing and generalizing from data from 15 boys and 15 girls, who responded to the item "I enjoy doing math" (p. 646). Preservice teachers first responded to this open-ended task with an answer and justification. Next, participants read the answer of a fictitious peer and judged whether it was correct using true-false items. The authors reported several results, including the following: approximately one-third of preservice teachers attended to sample size when responding to open-ended questions, such as believing that unequal samples were ineligible for comparison. In addition, most preservice teachers recognized the uncertainty of generalizations, though less than half of participants recognized that generalization is possible with small samples.

2.2. STUDIES THAT EMPHASIZED DEVELOPMENT OF COURSES AND PROGRAMS

A second response to challenges in teacher preparation is for studies to prioritize course and program development. A study by Metz (2010), who revised an undergraduate course for PTEPs in probability and statistics, provides an example. The aim of the revision was integration of the GAISE framework and NCTM standards. Metz noted that revision emphasized statistical problem solving across topics. For example, the original course listed "quantitative literacy descriptive statistics activities" as a major topic, with "statistical studies, surveys and experiments" as one of its subordinate topics (Metz, 2010, p. 6). Alternatively, the new course listed "statistical studies and connections to the real world" as a major topic, with "formulating questions and collecting data" as a subordinate topic (Metz, 2010, p. 7).

The study also elaborated on a course task, used in the first session, which involved PTEPs in the statistical problem-solving process, beginning with formulation of a question (i.e., "How would we describe the typical person taking this class?" [Metz, 2010, p. 9]), and ending with interpretation of results. The study further described two class assignments. The first involved PTEPs conducting a statistical inquiry investigation, again beginning with a question, and ending with interpretation. The second assignment involved PTEPs teaching a lesson, designed for elementary age students, to peers. One of the results reported by Metz (2010) was that PTEPs enrolled in the revised course provided different kinds of comments in course evaluations compared to PTEPs from the previous course such as, "We used examples applicable to the elementary classroom," and "We got to practice things we will someday have to teach" (p. 18).

A study by Bilgin et al. (2017) provides an example in program development. Bilgin et al. reported the creation of an online module in statistics for PTEPs. The module was part of a larger project called *Opening Real Science*, which was funded by Australian Government to improve the quality of preservice teacher programs. The statistics module was organized around five topics (e.g., using statistics in the real world), requiring at least 30 hours to complete. One of the learning objectives for the module was to calculate descriptive statistics and interpret results using an authentic data set. The module included different activities such as asynchronous discussion, readings, and videos. An example assessment included analysis of a data set and reporting results in a brief paper. Bilgin et al. reported several outcomes after pilot testing the module. Though responses varied, many PTEPs indicated learning activities promoted exploration of topics. In addition, a majority of the PTEPs noted that their understanding of statistics had changed after completing the module. Nevertheless, a minority of PTEPs indicated they felt unengaged with module activities and that they would not recommend the module to colleagues.

2.3. STUDIES THAT INCLUDED ASPECTS OF PEDAGOGICAL KNOWLEDGE

Another category of literature includes studies that addressed aspects of PTEP pedagogical knowledge to teach PS. An early study by Heaton and Mickelson (2002) examined PTEP use of statistical inquiry through two projects. The first project involved posing three research questions that enabled PTEPs to collect, analyze, and interpret data from their practicum sites. Example inquiry questions included, "How much time does the teacher spend helping individual children during math class? ... How often did the teacher use the whiteboard or the chalkboard?" (Heaton & Mickelson, 2002, p. 42). The products of the project included a report and reflection.

Heaton and Mickelson (2002) noted that PTEPs often posed trivial questions, such as asking how often a cooperating teacher uses the dry board during a lesson, or they posed questions that prohibited collection of quantitative data, such as what a mathematics class is like. Furthermore, Heaton and Mickelson reported that while preservice teachers were able to reach meaningful recommendations about the teaching phenomenon, many were unable to use data collected as part of the project to support recommendations. Related to this, when Heaton and Mickelson conducted interviews with preservice teachers to ask what they had learned, several reported that the project was not a good use of time, and that it was unnecessary to collect data to improve teaching.

The second project implemented by Heaton and Mickelson (2002) required PTEPs to engage primary students at their practicum sites in a statistical inquiry project, with the idea that PTEPs would apply knowledge and skills gained during participation in the first project. Heaton and Mickelson directed PTEPs to identify topics with cooperating teachers, develop questions and identify variables with K–6 students, then work with students to collect, analyze, and summarize data and reach conclusions. For example, a Kindergarten class surveyed relatives on pet ownership.

As was the case in the first project, Heaton and Mickelson (2002) reported preservice teachers had difficulty forming questions for inquiry, tending to be too simplistic such as "What is your favorite meat?" or "How many teeth do you have?" (p. 46). Relatedly, Heaton and Mickelson noted that knowledge and skills acquired from the first inquiry project did not transfer to the second project. Nevertheless, Heaton and Mickelson tried again and implemented the second inquiry project with a different group of PTEPs. During this second attempt, the researchers took more active roles in the development of inquiry questions, intervening with PTEPs at different points and meeting with pairs of PTEPs who were interning at the same school site. Heaton and Mickelson assisted PTEPs in planning and implementation of inquiry projects, though with similar results to previous efforts. For example, one PTEP noted, "The hardest part about this assignment was getting started with the children … they were having a hard time forming questions …" (Heaton & Mickelson, 2002, p. 48).

Based on these outcomes, Heaton and Mickelson (2002) astutely observed parallels "between telling preservice teachers to do something, which does not happen, and the preservice teachers telling children to do essentially the same task, which also does not happen" (p. 48). In summary, PTEPs lacked sufficient knowledge and skill to implement statistical inquiry with their students. As one PTEP reported,

> I didn't know what I wanted the children to get out of the inquiry. I didn't know enough about reasoning of the inquiry … so it was hard to make a discussion about something I wasn't too knowledgeable about …. (Heaton & Mickelson, 2002, p. 49–50)

At the same time, Heaton and Mickelson (2002) recognized that their expertise with statistical inquiry did not transfer in ways that assisted preservice teachers in implementing the same processes with their students.

A later study by Leavy (2010) examined the use of lesson study by PTEPs to design and teach lessons about informal inference. The study began by organizing PTEPs into small groups and then implementing a lesson study project in three phases. The first phase involved introducing PTEPs to lesson study and inferential reasoning through a variety of activities such as presentation, readings, discussion, and lesson modeling. Elements of lesson modeling included collecting data from preservice teachers on family size, making predictions about the population represented in the data and then simulating with the sample data to the population (Leavy, 2010).

The second phase involved PTEPs in small groups over several weeks. Groups met to research, design, and implement a lesson on informal inferential reasoning. Initial meetings to plan lessons were attended by Leavy (2010), who provided guidance on lesson design and informal inferential reasoning. One example question devised by a lesson study group was,

> A friend of mine, Liam, who is the same age as you, says his favourite cereal is Frosties, but his parents are always [complaining] that he eats way too much sugar. They say that all cereals have way too much sugar. We are going to see today if they are right. (Leavy, 2010, p. 53)

After planning, one PTEP from the group taught the lesson to primary age students. After teaching the lesson, the group revised it, whereupon it was taught by another PTEP a second time, but to different students. After teaching the lesson a second time, the group reflected and revised the lesson again. The final phase included small group interviews on statistical and pedagogical aspects of lessons, reflective papers on a range of topics such as pedagogical aspects relating to the teaching of statistics, and presentations by groups to peers. Presentations provided an overview of lessons, identified obstacles to planning and teaching lessons, provided opportunity for trying lesson activities, and displayed primary age student work.

One result noted by Leavy (2010) was that PTEPs had not been instructed in inferential reasoning prior to the study. Relatedly, PTEPs indicated that lesson modeling at the beginning of the study contributed to their comprehension. Likewise, PTEPs noted that analyzing the ways primary-age students understood the content aided in their own understanding. For example, one PTEP reported,

> When I taught the first lesson my understanding developed further as the children came up with some reasons that we didn't even think of when writing the lesson plan. Teaching the first lesson was a good way of finding out the children's point of view on inferential reasoning and then from there thinking a little more about my own understanding …. (Leavy, 2010, p. 59)

Leavy further noted difficulties PTEPs had in developing inquiry questions during the planning phase, and the tendency for PTEPs to focus on procedures while teaching lessons during implementation. Another result was that PTEPs did not appear to recognize the importance of data collection and analysis in the development of informal inferential reasoning.

2.4. STUDIES THAT EMPHASIZED USE OF TECHNOLOGY

As noted by Franklin et al. (2015), an aspect of PTEP pedagogical knowledge includes use of technology for improving comprehension of PS. There are several studies in this area, such as one conducted by Casey et al. (2020), where PTEPs designed statistical inquiry tasks using *Common Online Data Analysis Platform* (CODAP) in conjunction with *Enhancing Statistics Teacher Education With E-Modules* (ESTEEM) curriculum. CODAP is free online software for assisting students in learning how to analyze data (https://codap.concord.org/). ESTEEM curriculum provided teacher development for optimizing use of CODAP, such as modules in statistics concepts and how to initiate statistical inquiry with students in online learning environments. Features of ESTEEM include readings, videos of students learning statistics, video interviews with teachers, and data investigations (https://place.fi.ncsu.edu/local/catalog/course.php?id=22).

Part of the Casey et al. (2020) study required preservice teachers to design a statistical inquiry task for Grade 4 to 12 students. Some features of the task included a description of how to initiate the inquiry, inquiry questions posed to students, examples of student work, and one page reflection explaining what preservice teachers learned because of designing the inquiry. Results showed that the preservice teachers designed statistical inquiry tasks that engaged students in analysis of authentic multivariate data sets. In addition, tasks were initiated with context, such as datasets reporting the amount of sugar in soda, or student characteristics influencing college acceptance. About half of tasks included a question for directing the inquiry process, and most tasks required students to create visual representations and numerical measures of data.

2.5. STUDIES THAT EMPHASIZED PRESERVICE TEACHER ATTITUDE

Somewhat distinct from those studies addressing content or pedagogical knowledge, and other opportunities to learn, are those that address preservice teacher attitude. Research by Estrada and Batanero (2020) provides a recent example. That study involved development of a questionnaire to assess the attitudes of preservice teachers of primary age students. Example items on the questionnaire included "Probability is not as valuable as other areas of mathematics" and "I feel worried about being able to reply to my students' questions about probability" (Estrada & Batanero, 2020, p. 14). In their results, Estrada and Batanero noted that female participants showed lower scores on five of the seven questionnaire subscales in comparison to males, and of these five, three were statistically significant. In addition, Estrada and Batanero noted those participants who studied probability closer in time to completion of the questionnaire were more likely to rate highly their ability and willingness to teach probability.

As noted previously, various sources indicated programs missed or underemphasized some components that are needed for preparing PTEPs to teach PS. A question related to this is whether the same omissions appear in research. While there are different criteria useful for answering this question, AMTE (2017) standards include many of the components judged necessary for programs to effectively prepare PTEPs, such as consideration of content and pedagogical knowledge, as well as other opportunities to learn. The study that follows undertakes addressing these issues.

3. METHODOLOGY

The method of this study is characteristic of a literature review, two goals of which are to identify gaps and generate new perspectives (Boote & Beile, 2005). The method of this study may be further categorized as a systematic literature review, which emphasizes procedures for selecting studies for analysis to generate conclusions (Vogt & Johnson, 2015). Steps taken for conducting a systematic literature review include establishing search criteria, developing keywords, searching databases, and conducting additional searches based on results (e.g., Lawson et al., 2015). After assembling studies which met search criteria, they were processed using content analysis steps, as described by Bhattacherjee (2012). Results were then reexamined for adequacy using a few strategies suggested by Williams and Morrow (2008), such as careful description of procedures, searching individual studies for keywords, and contacting some study authors to check interpretations.

3.1. IDENTIFYING SEARCH TERMS

This systematic review began with selection of search terms from studies dealing with primary and early childhood statistics education edited by Leavy et al. (2018). Terms were selected based on grade level relevance (e.g., early childhood, primary, elementary), content (e.g., statistics, probability, data collection), and participants (e.g., preservice, teacher education, teacher preparation).

3.2. CONDUCTING SEARCHES

Forty-three searches of Education Resources Information Center database were conducted for peer reviewed articles using terms from each category as subject guides. Nine of the 43 searches produced 80 studies. The other 34 searches either produced irrelevant studies (e.g., participants were in-service teachers) or the searches produced duplicate results. Forty-nine of these 80 studies matched search criteria. Terms yielding the most studies were *elementary*, *statistics*, *preservice* (24 results), while the combination of terms yielding the fewest studies were *primary*, *statistics*, *teacher education* (1 result). Several search term combinations produced 0 results, such as *elementary*, *probability*, *teacher preparation*, and *preschool*, and *probability*.

3.3. CONTENT ANALYSIS OF STUDIES

Studies were initially examined, with focus on abstracts and results sections, to ensure they included PTEPs as participants and PS as content. After this, AMTE (2017) standards and indicators were analyzed for key words in preparation for their use in coding. For example, key words for C.1.1. (Know Relevant Mathematical Content) included content, concept, and procedure, whereas key words for P.3.4. (Incorporate Practice-Based Experiences) included practice, experience, and teaching (AMTE, 2017). Studies were then reexamined in their entirety.

During the second examination, particular attention was given to the methods section, which often indicated whether the study prioritized content knowledge, pedagogical knowledge, or other opportunities to learn. Studies were coded with one to three AMTE (2017) indicators using both indicator keywords as well as reference to AMTE standards. For example, the study by Chernoff and Russell (2011), coded C.1.1, reported on teaching PTEPs probability. Alternatively, Green and Blankenship (2013), coded P.2.1, reported on development of an introductory statistics course for PTEPs. Most studies collected for analysis addressed more than one indicator. For instance, though the study by Green and Blankenship (2013) emphasized the development of a course, the study also involved PTEPs in writing lesson plans (coded C.2.2.) and analyzing student misconceptions (coded C.3.1.). See the Appendix for a description of indicators, which were used as codes.

One of the difficulties of coding was distinguishing between studies that focused on developing candidate knowledge, skill, and disposition, versus development of program characteristics for achieving these goals. These domains of candidate development and program characteristics are intertwined, though they are somewhat distinct in AMTE (2017) standards. Nevertheless, the first strategy used for managing this difficulty was to search studies for keywords related to program characteristics such as *program*, *course*, and *student teaching*. The second strategy was to conduct member checking with study authors. A random sample of 20 authors, 16 of whom replied, were contacted through email, and asked if their study seemed to match selected codes. In 12 instances authors agreed that the codes represented the study. Nevertheless, authors were at a disadvantage since AMTE standards are not disseminated internationally and are therefore unfamiliar to many. In two instances authors disagreed, and in another two instances authors suggested corrections. Feedback from authors was merged into the results.

A second difficulty was deciding how to order codes to show priority. For example, the study by Odom and Bell (2017) was coded P.2.1, C.2.2, and C.3.1 in order of how the study appeared to emphasize these components. The reliability of code ordering was checked with another rater on a random sample of five studies. Results showed agreement in selection of the first code at 60%, the second code at 40%, and in cases where a third code was applied, at 66%.

A final step in the analysis was to use Google Scholar as an indicator of how the study may have influenced subsequent research efforts. Piotrowski (2013) has noted Google Scholar is suitable for this purpose. One reason for this is that Google Scholar includes an extensive list of journals that includes international and open-source, important resources in PS research. Since the age of a publication may influence the number of citations, rates were calculated using the following formula: *number of citations* ÷ (2021 – *year of publication*). For example, the study by Lavidas et al. (2020) was noted as having five citations in Google Scholar in 2021, for a rate of 5 citations per year.

4. RESULTS

The question addressed in this systematic literature review was whether research on PTEP preparation to teach PS missed or underemphasized PS components in comparison to AMTE (2017) standards. One answer to this question is shown in Table 1. Most studies (61%) prioritized development of PTEP content knowledge [C.1.1]. The remainder of studies prioritized a variety of preparation components such as disposition (6%), use of technology (6%), instructional planning (2%), and school experiences (2%). The percentages with which these components were represented, however, are much smaller compared to prioritizing development of PTEP content knowledge.

Table 1. First code applied to show primary area of emphasis

Code	Area of Emphasis	*n*	Percent
C.1.1	Know Relevant Mathematical Content	30	61.2
P.2.1	Attend to Mathematics Content Relevant to Teaching	5	10.2
C.1.3	Exhibit Productive Mathematical Dispositions	3	6.1
C.1.6	Use Mathematical Tools and Technology	3	6.1
P.3.4	Incorporate Practice-based Experiences	3	6.1
C.1.2	Demonstrate Mathematical Practices and Processes	2	4.1
C.2.2	Plan for Effective Instruction	1	2.0
C.2.4	Analyze Teaching Practice	1	2.0
C.2.5	Enhance Teaching, Collaborate W/Colleagues, Families, Community	1	2.0

Note: Percent adds to < 100 because of rounding.+

As noted, many studies addressed multiple areas, which are summarized in Table 2. There was a noticeable correspondence between prioritizing content knowledge [C.1.1] and developing practices and processes [C.1.2]. Sixteen studies that prioritized C.1.1 also had a secondary emphasis on C.1.2. Another six studies that prioritized content knowledge also had a secondary emphasis on using tools and technology [C.1.6]. Overall, about two-thirds of studies that prioritized content knowledge also emphasized process, and a smaller, but still substantial number, emphasized use of tools and technology. The remaining areas of secondary emphasis mostly addressed pedagogical knowledge such as planning instruction, implementing instruction, and attending to student thinking.

Table 2. Second code applied to show secondary area of emphasis

Code	Area of Emphasis	*n*	Percent
C.1.2	Demonstrate Mathematical Practices and Processes	17	34.7
C.1.6	Use Mathematical Tools and Technology	6	12.2
C.2.2	Plan for Effective Instruction	6	12.2
P.2.2	Build Mathematical Practices and Processes	3	6.1
C.1.5	Analyze Mathematical Thinking	1	2.0
C.2.3	Implement Effective Instruction	1	2.0
C.3.1	Anticipate and Attend to Students' Thinking About Content	1	2.0
P.3.4	Incorporate Practice-based Experiences	1	2.0

Note: Percent adds to < 100 since some studies were not assigned a second code.

A third code was applied in less than half of all studies to indicate a tertiary area of emphasis. Table 3 shows a summary of these areas. In comparison to primary and secondary codes, topics represented with a third code appear distributed across preparation components, such as PTEPs analyzing their own thinking or the thinking of peers [C.1.5]; planning for instruction [C.2.2]; attending to content for teaching [P.2.1]; and analyzing practice [C.2.4].

Table 3. Third code applied to show tertiary area of emphasis

Code	Area of Emphasis	*n*	Percent
C.1.5	Analyze Mathematical Thinking	4	8.2
C.2.2	Plan for Effective Instruction	3	6.1
C.2.3	Implement Effective Instruction	3	6.1
C.3.1	Anticipate and Attend to Students' Thinking About Content	3	6.1
C.1.1	Know Relevant Mathematical Content	2	4.1
C.1.2	Demonstrate Mathematical Practices and Processes	2	4.1
P.2.1	Attend to Mathematics Content Relevant to Teaching	2	4.1
C.2.4	Analyze Teaching Practice	1	2.0
P.3.1	Address Deep and Meaningful Mathematics Content Knowledge	1	2.0

Note. Percent adds to < 100 since some studies were not assigned a third code

Each study was checked for its number of citations according to Google Scholar. The mean number of citations was 24.4 with standard deviation 54.0. Figure 1 shows a dot plot of studies and their rates of citation. Ten studies with the largest citation rates are labeled with the code used to show their primary area of emphasis. Five of the studies prioritized development of content knowledge, C.1.1 and C.1.2. Two studies prioritized school experiences, P.3.4. Two more studies prioritized disposition, C.1.3. One study prioritized development of pedagogical knowledge, C.2.4.

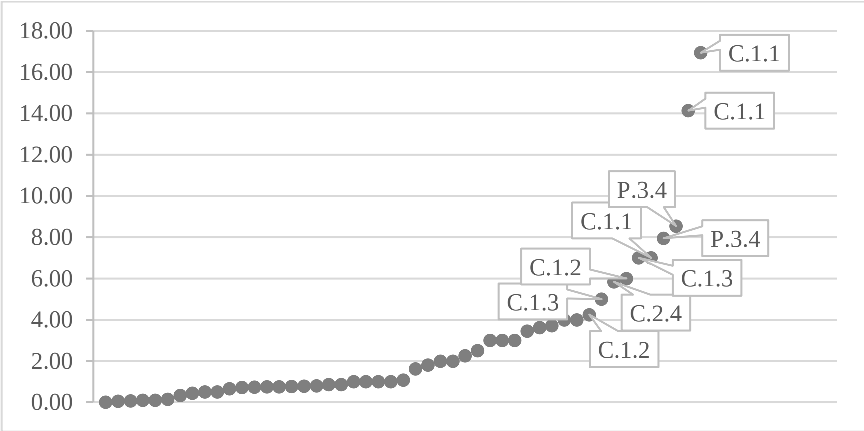

Figure 1. Rate of citation and primary code applied to frequently cited studies

5. DISCUSSION

Decades of research in mathematics teaching and learning, at least in the United States, has shown that there are ongoing problems. One of the obvious solutions for resolving these problems is to focus on improving preservice teacher content knowledge. Nevertheless, some have noted that improving teacher content knowledge alone will not increase achievement (Ma, 1999; Tatto & Senk, 2011). Rather, teacher preparation programs must address development of content knowledge, while simultaneously addressing pedagogical knowledge and opportunities to learn, not the least of which include reflection on practice and learning to teach through school experiences.

The way in which these different components are combined into an effective program for preparing PTEPs to teach PS is, again, unknown, though education research provides indicators about how the field is developing. The study undertaken here considered how research dealing with the preparation of PTEPs to teach PS reflects content knowledge, pedagogical knowledge, and opportunities to learn, in comparison to AMTE (2017) standards.

5.1. FOCUS ON DEVELOPING CONTENT KNOWLEDGE

Results show that there is no shortage of research prioritizing development of PTEP content knowledge. Most often, these studies address both statistical concepts and related procedures (C.1.1 and C.1.2 shown in Tables 1 to 3). An example is the study by Batanero et al. (2004), which involved PTEPs exploring randomness and selecting strategies for winning a game of chance. Another example is the study by Hourigan and Leavy (2020), which involved PTEPs selecting materials for creating fair and unfair chance activities. While these studies prioritized development of PTEP content knowledge, they did not neglect aspects of pedagogy. For instance, Batanero et al. (2004) also had PTEPs analyze the responses of secondary students, and Hourigan and Leavy (2020) had PTEPs analyze the work of primary age students.

5.2. IMPROVING COURSES AND PROGRAMS

A smaller number of studies emphasized course or program development (P.2.1 shown in Table 1 and 3). Studies by Metz (2010), which addressed revision of an undergraduate course using GAISE, and Bilgin et al. (2017), which explained general steps for developing an online statistics module, are examples. An additional study by Groth (2013) provided more information about developing an introductory statistics course that simultaneously considered content and pedagogy. A sample of teaching strategies identified by Groth included having PTEPs speculate about student misconceptions, discuss cases of classroom practice, observe example statistical representations, and construct and administer a class survey. The course matched recommendations by Franklin et al. (2015), namely that PTEPs take an introductory course that covers more than PS content.

5.3. IMPROVING CONTENT KNOWLEDGE AS PRESERVICE TEACHERS ATTEND TO TEACHING

As noted, several studies that prioritized content knowledge also incorporated aspects of pedagogical knowledge as part of their methodology. Besides use of technology for learning, discussed later, two frequently observed approaches related to pedagogy included having PTEPs analyze their thinking or the thinking of a peer [C.1.5], and plan a statistics lesson [C.2.2]. Nevertheless, few studies had PTEPs learn content by attending to teaching through school experiences (P.3.4 shown in Table 1 and 3) that involved interaction with early learners of PS, coupled with collaborative planning with PTEP peers, and reflection on practice. One exception to this is the study by Heaton and Mickelson (2002), which involved PTEPs implementing inquiry projects with primary-age students. Another exception is the study by Leavy (2010), which had PTEPs collaborate with peers in lesson study to plan, teach, and revise statistics lessons after PTEPs taught them to primary-age students. These studies are further notable for their similarities. Both studies involved teaching PTEPs content knowledge, in the case of the first, statistical inquiry, in the case of the second, inferential reasoning. Both incorporated project assignments, implemented in more than one phase, with small groups. Both studies reported on the need for instructors to advise students on some aspect of their project. One notable difference in the study by Leavy was that PTEPs revised and retaught their lessons, while PTEPs in the study by Heaton and Mickelson did not revise and reteach. This may help explain why one PTEP reported, "After meeting with our group following the first lesson, we realized there was a lot more opportunities for inferential reasoning throughout the lesson. I think this is when my concept of inferential reasoning broadened" (Leavy, 2010, p. 59). In comparison, another PTEP reported "I didn't know what I wanted the children to get out of the inquiry" (Heaton & Mickelson, 2002, p. 49).

5.4. USE OF TECHNOLOGY

A substantial number of studies emphasized the use of technology for improving comprehension of PS (C.1.6 in Tables 1 and 2). For example, a study by Frischemeier and Biehler (2018) showed PTEPs learned PS concepts and procedures (e.g., center, spread, data analysis and interpretation) by comparing groups using the exploratory data analysis software, *TinkerPlots*™ (https://www.tinkerplots.com/). Another study by Casey et al. (2020) showed PTEPs capable of designing data analysis tasks for use with Grades 4–12 students using CODAP. These studies support recommendations by Franklin et al. (2015) that technology be used to assist PTEPs in learning and in teaching PS.

5.5. PRESERVICE TEACHER ATTITUDE

Studies that addressed preservice teacher attitude are another area of focus (C.1.3 shown in Table 1). Two examples include Lavidas et al. (2020) and Tasgin and Kaya (2018), which focused on measuring PTEP attitudes in relation to PS content, using subscales related to competence, difficulty value, and anxiety. Another study in this area, by Estrada and Batanero (2020), is distinct, however, since it assessed content (e.g., personal feelings about probability) but also aspects of pedagogy (e.g., perception of ability to teach probability).

5.6. RATE OF CITATION

As might be expected, frequently cited studies are those that focused on teaching PTEPs PS. Table 1 predicts this outcome with approximately 65% of studies attending to content knowledge. In contrast, three frequently cited studies prioritized pedagogy and school experiences (i.e., P.3.4 and C.1.4), although a small proportion of studies in the overall sample analyzed here included this as a component of their methodology. Again, Table 1 and Figure 1 show this gap in correspondence. A similar lack of correspondence was observed for studies that prioritized preservice teacher attitude, which included two frequently cited studies, but only 6% of the sample.

5.7. LIMITATIONS

Application of results shown in this study are limited. One reason is coding studies using the AMTE (2017) standards oversimplified their methods and results. The study by Estrada and Batanero (2020) is an example. Though the study was coded C.1.3 for emphasis on preservice teacher attitude, it also assessed dimensions of pedagogical knowledge, though this was not configured into the results generated here. Another reason is trustworthiness in assignment of codes, including difficulty in distinguishing between studies that focused on developing candidate competence versus program characteristics, and differences in perception based on member checking. A third limitation is AMTE (2017) standards were written by authors from the United States. Experts in other countries and regions likely have alternative conceptions of effective teacher preparation. Last, the reliability of applying codes in priority order was not high.

6. CONCLUSION

There is ample evidence showing that it is difficult to prepare PTEPs to teach PS. Research suggests that there are different approaches for dealing with these difficulties, the most common of which is to prioritize development of PTEP content knowledge. This is predictable and necessary. If PTEPs struggle with fundamental concepts and procedures, such as trouble identifying questions for statistical inquiry, it is hardly reasonable to expect that they will be able to teach their Pre-K-12 students effectively.

Nevertheless, there is also plenty of evidence showing that improving teacher content knowledge does not occur independent of attending to teaching. Preparing effective teachers means recognizing that content and pedagogy are intertwined. Not all research can or should integrate pedagogical knowledge, learning from school experiences, and reflection on practice, but the field would benefit if more studies did. Obviously, researchers must take advantage of their circumstances for making contributions. For many, these circumstances preclude having PTEPs learn from school experiences. At the same time, there appear to be many opportunities for having PTEPs attend to teaching as they learn PS. Notable instances shown in research include when implementing a lesson study, using technology, constructing and administering class surveys, and analyzing student work samples, among many others. Application of these techniques do not require work with Pre-K–12 students, though in many instances this might be best. Rather, researchers simply need to consider how these techniques may be elevated or even prioritized within their current research agendas. Preferably, future research would focus on teaching PTEPs content knowledge simultaneously with teaching them pedagogical knowledge through school experiences and reflection on practice.

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APPENDIX

CHAPTERS, STANDARDS, AND INDICATORS (AMTE, 2017)

Chapter 2 Candidate Knowledge, Skills, and Dispositions

C.1. Mathematics Concepts, Practices, and Curriculum
- C.1.1. Know Relevant Mathematical Content
- C.1.2. Demonstrate Mathematical Practices and Processes
- C.1.3. Exhibit Productive Mathematical Dispositions
- C.1.4. Analyze the Mathematical Content of Curriculum
- C.1.5. Analyze Mathematical Thinking
- C.1.6. Use Mathematical Tools and Technology

C.2. Pedagogical Knowledge and Practices for Teaching Mathematics
- C.2.1. Promote Equitable Teaching
- C.2.2. Plan for Effective Instruction
- C.2.3. Implement Effective Instruction
- C.2.4. Analyze Teaching Practice
- C.2.5. Enhance Teaching, Collaborate W/Colleagues, Families, Community

C.3. Students as Learners of Mathematics
- C.3.1. Anticipate and Attend to Students' Thinking About Mathematics Content
- C.3.2. Understand and Recognize Students' Engagement in Mathematical Practices
- C.3.3. Anticipate and Attend to Students' Mathematical Dispositions

C.4. Social Contexts of Mathematics Teaching and Learning
- C.4.1. Provide Access and Advancement
- C.4.2. Cultivate Positive Mathematical Identities
- C.4.3. Draw on Students' Mathematical Strengths
- C.4.4. Understand Power and Privilege in the History of Mathematics Education
- C.4.5. Enact Ethical Practice for Advocacy

Chapter 3 Program Characteristics to Develop Candidate Knowledge Skill and Disposition

P.1. Partnerships
- P.1.1. Engage All Partners Productively
- P.1.2. Provide Institutional Support

P.2. Opportunities to Learn Mathematics
- P.2.1. Attend to Mathematics Content Relevant to Teaching
- P.2.2. Build Mathematical Practices and Processes
- P.2.3. Provide Sustained, Quality Experiences

P.3. Opportunities to Learn to Teach Mathematics
- P.3.1. Address Deep and Meaningful Mathematics Content Knowledge
- P.3.2. Provide Foundations of Knowledge About Students as Mathematics Learners
- P.3.3. Address the Social Contexts of Teaching and Learning
- P.3.4. Incorporate Practice-Based Experiences
- P.3.5. Provide Effective Mathematics Methods Instructors

P.4. Opportunities to Learn in Clinical Settings
- P.4.1. Collaboratively Develop and Enact Clinical Experiences
- P.4.2. Sequence School-Based Experiences
- P.4.3. Provide Teaching Experiences With Diverse Learners
- P.4.4. Recruit and Support Qualified Mentor Teachers and Supervisors

P.5. Recruitment and Retention of Teacher Candidates
- P.5.1. Recruit Strong Candidates
- P.5.2. Address Diverse Community Needs
- P.5.3. Provide Experiences and Support Structures

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