AN ANALYSIS OF TEACHERS' CONFIDENCE IN TEACHING MATHEMATICS AND STATISTICS

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ABSTRACT

The purpose of this study is to explore the expressions of confidence by a group of South African mathematics teachers about teaching mathematics and statistics concepts from various perspectives. The participants were 75 mathematics teachers who were teaching Grades 4 to 12 in KwaZulu-Natal (KZN) schools. They were asked to express their opinion on their level of confidence in teaching using 17 confidence items on a 5-point Likert scale from very low to very high. The study drew upon factor analysis, Rasch analysis, as well as regression analysis. The findings suggest that teachers' confidence in teaching mathematics concepts is quite different from their confidence in teaching statistics concepts and concepts that require connections across topics. Furthermore, the study found differences in teachers' confidence level by gender during the middle teaching years as well as a significant interaction between phases of teaching and whether teachers completed additional professional qualifications.

***Key words**: Statistics education research; Mathematics teachers' confidence; Rasch model; Principal components; t-test; Factor analysis; Regression model*

1. INTRODUCTION AND BACKGROUND

Over the years, many studies have identified the need for a relevant statistics education curriculum that can help students develop the type of statistical thinking needed to cope with the demands of the real world (Bansilal, 2014; Carver et al., 2016; Gattuso & Ottavani, 2011; Justice et al., 2020; North et al., 2014; Wessels & Niewoudt, 2013). Like most countries, the study of statistics forms part of the mainstream mathematics curriculum in schools and has traditionally been an under-represented strand in the core mathematics curriculum in South Africa. Many statistics education researchers have pointed out differences between the two disciplines (Franklin et al., 2007), where statistics is set apart from mathematics because the discipline is more focused on the contextual space of the problem being investigated, thus relying on explaining and quantifying the variability of data within the context and not just by applying mathematical formulas. Cobb and Moore (1997) argued it is more than just content that distinguishes statistical thinking from mathematics because statistics is dependent on a "different kind of thinking, because data are not just numbers, they are numbers with a context" (p. 801). Carver et al. (2016) articulated that a central purpose of statistics education is to help students develop statistical thinking, which relates to statistical problem solving and decision making around issues arising from the variability of data.

The term "statistical literacy" refers to the set of abilities required to understand statistical information, such as being able to organize and represent data using different representations, understanding and correctly applying the concepts, terminology, and symbols and procedures

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associated with statistics (Ben-Zvi & Garfield, 2004). There has been many calls in recent years to expand the mathematics curriculum so students can develop the statistical literacy skills necessary and teachers have been encouraged to help their students to not only go beyond just reading statistics reported in the media but to also engage in critical debates about the statistical results presented in the media (Carver et al., 2016).

South Africa embarked on the expansion of the traditional school mathematics curriculum to include some statistics concepts in the late 1990s (Department of Education [DoE]; 1997). Curriculum reform is a complex process, however, and there is much research that points to the challenges experienced by teachers in implementing curriculum reform (Handal & Herrington, 2003). With respect to this specific reform of the mathematics curriculum to include statistics concepts, South African teachers have found it difficult to cope with the changes (North et al., 2014; Umugiraneza et al., 2016; Wessels & Nieuwoudt, 2011; 2013).

Teachers' confidence is a key factor in contributing to effective learning and student achievement. Moreover, teachers' confidence and their knowledge are connected to each other (Beswick et al., 2012). This claim rests on the fact that teachers' confidence is a component of teacher knowledge, and they link straightforwardly with other aspects of knowledge (Beswick et al., 2012).These authors support the idea that "teachers' confidence in teaching mathematics is related to both common and specialized content knowledge, and also to pedagogical content knowledge in relation to content and teaching and content and the curriculum" (Beswick et al., 2012, pp. 137, 139). Teacher confidence develops over time and with experience (Tolbert, 2008) and further develops together with an increase in pedagogical content knowledge (Witt et al., 2013).

The purpose of this study is to explore KwaZulu-Natal (KZN) mathematics teachers' expressions of confidence in teaching mathematics and statistics concepts from various perspectives. Firstly, we are interested in whether teachers' confidence in teaching statistics and mathematics topics can be conceived as a single one-dimensional construct or whether these are conceived as different dimensions. We also identify concepts for which teachers were more confident in teaching than others. Finally, we investigate whether some demographic teacher factors are associated with different levels of confidence. The corresponding research questions are listed below:

1. Can mathematics teachers' confidence in teaching different mathematics and statistics concepts be considered as a unidimensional or multidimensional construct?
2. Which mathematics and statistics concepts are teachers most (least) confident about teaching?
3. Are there any differences with respect to teachers' confidence in teaching mathematics and statistics concepts according to gender, teaching experience, teaching phase and whether they received additional professional teaching certification?

This study will add to knowledge about teachers' confidence in teaching mathematics and statistics in school by providing insights about differences in confidence in terms of concepts as well as differences in confidence in terms of demographic factors.

2. LITERATURE REVIEW

With the introduction of democratic reforms in South Africa in 1994, the education system was identified as requiring a complete overhaul. Hence, numerous reforms were carried out to define the new national education system. Prior to the election of a democratic government in the country, there were 18 different departments that administered education in the country, many of which used different curricula, i.e., there was no uniform standard for school education for the country. A first step to bringing about uniformity was to institute a common curriculum, called the Interim Core Syllabus, defined in the early 1990s. For mathematics, this initial common curriculum consisted of a list of common topics to be covered in each year of schooling. It is noteworthy that there were no statistics or probability concepts that were assessed in the Grade 12 mathematics examinations (Western Cape Education Department, 1998), at that time.

The next set of curriculum revisions resulted in the *Curriculum 2005* (C2005) policy, which was introduced in 1998. North and Scheiber (2008) noted that before C2005 was introduced, little was known about teaching and learning statistics at school level in South Africa. The C2005 curriculum documents (DoE, 1997) referred indirectly to statistical reasoning when stating that students must be able to collect, summarise, display, and analyze data. The curriculum was understated, however, and did not include details of specific content and the depth to which these should be covered. It was only in the next set of curriculum revisions, the Revised National Curriculum Statements (RNCS) for the General Education and Training (GET) band, that is, Kindergarten to Grade 9, that Data Handling was stipulated as one of four outcomes (DoE, 2002) of the curriculum.

Another development with respect to statistics, in the RNCS, was for the Further Education and Training (FET) band, that is, Grades 10 to 12, in which there was an increased emphasis on statistics (Data Handling was one of the strands that was to be assessed in the Grade 12 core mathematics examinations). It must be noted, however, that this was only implemented in the classroom in 2006 and culminated in being part of the final Grade 12 examination in 2008. It can thus be seen that statistics is a relatively new addition to the mathematics curriculum in South Africa and only formed part of the Grade 12 core mathematics assessment from 2008 onwards.

There have been many calls for a shift in the mathematics curriculum to reflect a focus on developing statistical literacy skills in learners (Franklin et al., 2007) and the changes in the mathematics curriculum in South Africa are indicative of a global movement towards an increasing emphasis on statistical literacy. Regardless of the discipline, curriculum reform is a complex process, and its success is dependent on many factors (Handal & Herrington, 2003). Developing the policy that underpins new curricula is just one part of the curriculum change process. Chisholm and Leyendecker (2008) note that the implementation of new curriculum policy does not follow the "predictable path of formulation—adoption—implementation—reformulation but is recontextualized through multiple processes and mechanisms" (p. 196).

Much of the research on curriculum reform agrees that teachers need sustained classroom support, over and above training workshops, as they try to negotiate changes in teaching to what they have been accustomed to doing in the past (Avalos, 2011; Bansilal, 2011; Maoto & Wallace, 2006; O'Sullivan, 2002; Webb & James, 2015). Changes in the curriculum must accordingly be accompanied by intensive teacher support as the teachers try to implement the new curricula. In South Africa, the education authorities established workshops for teachers to help them improve their teaching of statistics. The workshops were designed to provide the necessary cross curricular training to promote increased statistical literacy levels of learners (North & Zewotir, 2006).

Some studies focusing on teachers' confidence levels in the teaching of statistics found that teachers' confidence in teaching statistical ideas was not uniform across concepts (Begg & Edwards, 1999; Beswick et al., 2006, 2012; Harrell-Williams et al., 2015). Beswick et al. (2006) conducted a study with 42 middle school mathematics teachers about their confidence in relation to the mathematics topics they teach. Their findings revealed the teachers were most confident about teaching Measurement and Space, and least confident about Pattern and Algebra. Many teachers indicated a lack of confidence in teaching topics related to proportional reasoning. Begg and Edwards (1999) found that the extent of the teachers' confidence was related to their degree of familiarity with the concepts in the curriculum. The teachers lacked confidence in probability for example, which was a new concept in the curriculum.

Harrell-Williams et al. (2015) applied a Rasch model to their middle grade Self-Efficacy to Teach Statistics (SETS) instrument to identify the concepts associated with teachers' high and lower self-confidence in middle grades. Their findings showed that the concepts associated with teachers' lower confidence concern the relationship between two variables, comparison of two groups, and the development of research questions to test a hypothesis. The items associated with teachers' high confidence were those corresponding to the first level of statistical literacy described in the GAISE Report, that of reading data (Carver et al., 2007).

Harrell-Williams et al. (2019) used confirmatory factor analysis and a Rasch model to explore the validity and the reliability of the high school version of SETS. The study was conducted with 290 pre-service mathematics teachers from 20 institutions across the United States. Their instrument was composed of three subscales (Harrell-Williams et al., 2019, p. 197) of "reading data", "reading between the data" and "reading beyond data", which corresponded to the three levels of statistical literacy described in the GAISE report (Franklin, et al., 2007). The results indicated the items in the SETS performed as expected. The statistics also provided strong evidence to support the three dimensions in their model. The notion of dimensionality of instruments measuring teachers' self-efficacy has been investigated by other studies (Beswick et al., 2006; Beswick et al., 2012; Yim et al., 2007). On the one hand, studies showed that teachers' confidence in teaching statistical ideas was not uniform across concepts (Callingham & Watson, 2014). These authors used factor analysis as well as Rasch analysis and found teachers' confidence varied across concepts. On the other hand, Yim et al. (2007), after applying confirmatory factor analysis, found that teachers' confidence in teaching different music items was in fact unidimensional.

Hill and colleagues (2004) examined teachers content knowledge for teaching elementary mathematics. Using factor analysis, their findings revealed that teachers' knowledge for teaching elementary mathematics was multidimensional and included knowledge of various mathematical concepts and domains. The authors explained that the "multidimensionality" emerged because the factors describing statistical content knowledge (SCK), content knowledge (CK) of patterns, functions and algebra accounted for between 21% and 45% of the commonality in items written to represent these areas, while the SCK factor explained 12–23% of the commonality of items written to represent knowledge of content in number and operations.

In South Africa, researchers exploring mathematics teachers' knowledge suggested there is an urgent need to develop the "mathematical knowledge for teaching" of primary mathematics teachers (Venkat & Spaull, 2015). Concerning statistics and data handling, several studies also found limited teacher knowledge (Adu & Gosa, 2014; North & Zewotir, 2006; Wessels & Nieuwoudt, 2011). In this study, we replicated, and hence extended the work of Callingham and Watson (2014) and Beswick et al. (2006) to the South African context by investigating the expression of confidence by a group of mathematics teachers who teach mathematics and statistics at the FET and GET phases.

3. THE PARTIAL CREDIT RASCH MODEL

Rasch models fall within the group of Item Response Theory models (Stocking, 1999). A Rasch measurement model approach permits joint scaling of person abilities and assessment item difficulties by mapping the relationship between latent traits and responses to test items (Linacre, 2008). Based on the interaction between persons and items, an estimate of the probabilities of the response of each person on each item is derived, and then of each item to each person.

Rasch models assume the probability of a given respondent affirming an item is a logistic function of the relative distance between the item location and the respondent location on a linear scale (Tennant & Conaghan, 2007). The data can be dichotomous (two response options) or polytomous (more than two response options). Since the response data in this study is polytomous, we use the Masters (1982) Partial Credit Model (PCM), which is a unidimensional model that analyses the responses recorded in two or more ordered categories (Wright & Masters, 1982). The equation of the model for the x^{th} step in item i is

$$\varphi_{nix} = \frac{\exp(\beta_n - \delta_{ix})}{1+\exp(\beta_n - \delta_{ix})}, \ x = 1, 2, \ldots, m_i$$

and gives the probability, φ_{nix} of a person n, with ability β_n, scoring x rather than $x-1$ in item i, as a function of the ability β_n of person n, and the difficulty δ_{ix} of the x^{th} step in item i, which has m_i steps (Masters, 1982). Although there have been some extensions to Rasch models that take a multidimensional approach (for more details, see Briggs & Wilson, 2003), within the PCM Rasch model, measurement is assumed to be unidimensional. In this study, we start off with the assumption of one-dimensionality. This implies that a scale measuring a single construct can be built to represent varying item difficulties along that measurement scale. This assumption is tested in Section 5.1.

The items in the research instrument in our study represent 17 concepts in teaching mathematics and statistics, hence the differences in the location of items are not related to item difficulty but can be interpreted in terms of teacher confidence. In the setting of this study, the Rasch analysis allows us "to see how items act on a continuum" of "easiest" to be confident (score 0) to "hardest" to be fully confident (score 4) of the teaching in the mentioned item (Harrell-Williams et al., 2015; Donovan, 2018). Then easiest to be confident about refers to high confidence and hardest to be confident about refers to low confidence. Hence, items located at lower levels of the scale indicate the teachers were more confident about teaching these concepts, while items located at the higher end of the scale were those that teachers expressed low confidence about teaching those concepts. Hence, items with high difficulty level are equivalent to less confidence while low difficulty is equivalent to high confidence.

4. METHODOLOGY

This study was conducted with 75 mathematics teachers who were part of a group of teachers who attended a series of five teacher professional development workshops focused on improving their knowledge and skills in statistics. Those teachers, who taught from Grade 4 upwards in KwaZulu-Natal (KZN) schools, were selected purposively by the provincial educational department, according to the schools that were in most need of assistance of training in mathematics (North et al., 2010). Details of the participants are provided in Table 1.

Table 1. Details of participants

Factors	Categories	Frequency
Gender	Male	38
	Female	37
Teaching Experience	≤ 10 years	45
	11–20 years	19
	> 20 years	11
Phases	GET	30
	FET	45
Received additional professional certification	Yes	57
	No	18

A questionnaire was used to probe various aspects of teachers' knowledge, beliefs, and confidence related to the teaching and learning of mathematics and statistics. The questionnaire included a confidence section, which was based on a revised version of an instrument developed by Beswick et al. (2006). The original questionnaire focused mostly on teachers' confidence in relation to teaching the mathematics topics and their beliefs about numeracy. The authors of this paper extended the questionnaire to include items related to the teaching of statistics. Printed questionnaires were distributed to the mathematics teachers, who were asked to rate their level of confidence in teaching 17 mathematics and statistics concepts on a 5-point Likert-type scale (0–4), with response options from very low to very high. Table 2 presents the results in terms of percentage per category with 0 being the least confident and 4 being most confident. There were no missing data.

Table 2. Confidence items: Category response percentages (N = 75)

	Very Low	Low	Moderate	High	Very High
Fractions	1.3	6.7	24.0	29.3	38.7
Decimals	1.3	8.0	24.0	33.3	33.3
Percentages	1.3	9.3	20.0	24.0	45.3
Ratios and proportion	4.0	13.3	33.3	21.3	28.0
Measurement	1.3	12.0	29.3	30.7	26.7
Presenting mathematics in an expository style (detailed explanation)	2.7	16.0	36.0	25.3	20.0
Patterns and algebra	2.7	12.0	26.7	28.0	30.7
Mental computations	2.7	18.7	38.7	22.7	17.3
Connecting mathematics to other key learning areas	6.7	20.0	36.0	21.3	16.0
Critical debate on the use of statistics in the media	10.7	26.7	36.0	20.0	6.7
Pie graphs and histograms	2.7	16.0	20.0	28.0	33.3
Simple probabilities	4.0	13.3	30.7	29.3	22.7
Range and variation	4.0	22.7	34.7	17.3	21.3
Inference and prediction	8.0	26.7	38.7	18.7	8.0
Connecting statistics to other key learning areas	5.3	21.3	30.7	22.7	20.0
Ideas of sampling and data collection	2.7	18.7	34.7	29.3	14.7
Using statistics in out-of-the classroom situations	5.3	20.0	36.0	21.3	17.3

For the Rasch analysis, there are tests of fit, which give information about the difference between the observed and the expected response (Tennant & Conaghan, 2007). In *RUMM2030* (Andrich et al., 2009), item fit residuals are chi-square statistics, calculated as standardized sums of all the differences between the expected and observed responses summed over all persons (Tennant & Conaghan, 2007). Items with fit residuals outside of the range –2.5 and +2.5 are considered as misfitting. The Person-Separation-Index (PSI) is an indicator of the internal consistency reliability of the scale and is equivalent to Cronbach's alpha, only using the logit value as opposed to the raw score in the same formula (Tennant & Conaghan, 2007). The PSI provides an indication of the power of the measure to distinguish amongst respondents with different levels of the trait being measured. In Rasch analysis, if the PSI is high (> 0.8), it suggests the instrument has been able to discriminate well between the persons' measures of the trait (Tennant & Conaghan, 2007). The properties of Rasch measurement apply only to the extent to which the data will fit the model's demanding requirement. When the data do not fit well with the model, Rasch models are useful in trying to understand the data by helping diagnose where the data are different from what was expected (Andrich, 2012). Briggs and Wilson (2003) advise that when performance on an instrument can be interpreted in a multidimensional way, these dimensions should be modelled separately first before the measure can be properly constructed. We used factor and a subtest analysis (Rasch analysis) to investigate the dimensionality. Principal components analysis, a factor analysis technique for identifying clusters of variables, was used in this study to extract factors. There are two categories of rotation techniques: orthogonal rotation and oblique rotation. Orthogonal rotation (e.g., Varimax and Quartimax) involves uncorrelated factors whereas oblique rotation (e.g., Direct Oblimin and Promax) involves correlated factors. The interpretation of factor analysis is based on rotated factor loadings, rotated eigenvalues, and scree test (Yong & Pearce, 2013).

We used a scree plot to assist in the decision making concerning the number of factors to use. Furthermore, the software RUMM 2030 provides a useful method using *t*-tests to check if two subsets have anything more in common besides the trait of interest. The *t*-tests are carried out on the two estimations of persons' locations on the two subtests of the main test. The two subsets of items are considered, and the person parameters on each subtest are estimated, generating two estimates for each person. The *t*-test is used to test if the estimate for each person location from the two subtests are statistically equivalent. A statistically significant t- test would indicate the level of the trait differs depending on which items are used for calibration (Smith, 2002). It is assumed that if the proportion of t- tests, for which the difference in the estimates falls outside the boundaries of acceptable significance, are greater than 5% then there may still be some degree of multidimensionality within the item set (Tennant & Conaghan, 2007).

We used a logistic regression to look for answers to the third research question about the relationship between teachers' levels of confidence and demographic factors of phase in which they taught, whether additional professional courses were taken, gender and teaching experience. Logistic regression is a predictive analysis which is effective for describing a dataset where there are one or more independent variables that determine an outcome (Peng et al., 2002) of a dependent or response variable that is measured as a dichotomous variable (two possible outcomes). In this study the binary response variable was teacher confidence (high or low confidence), and the independent variables were phase of teaching, gender, teaching experience and whether or not additional professional courses were completed.

5. RESULTS

5.1. DIMENSIONALITY OF THE INSTRUMENT

In carrying out the factor analysis to extract factors, we first produced a Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy, Bartlett's Test of Sphericity (Field, 2000), the results of which are presented in Tables 3 and 4. The KMO of Sampling Adequacy is a statistic that indicates the proportion of variance in the variables that might be caused by underlying factors. High values (close to 1.0) generally indicate that a factor analysis may be useful with the data. If the value is less than 0.50, the results of the factor analysis will probably not be very useful. Bartlett's test of sphericity tests the hypothesis that the correlation matrix is an identity matrix, which would indicate the variables are unrelated and therefore unsuitable for structure detection. Small values (less than the significance level of 0.05) of the significance level indicate that a factor analysis may be useful with the data. We found that the value of KMO of Sampling Adequacy is 0.895 and the Bartlett's test of sphericity is statistically significant at 0.05 level (0.000 < 0.05).

Table 3. KMO and Bartlett's Test for teachers' confidence

Tests	Value
Kaiser-Meyer-Olkin Measure of Sampling Adequacy	.895
Bartlett's Test of Sphericity Approx. Chi-Square	1043.101
Degree of freedom	136
Sig.	.000

This means the factor analysis is suitable for these current data. A principal components analysis (Table 4) was applied to extract components associated with teacher's confidence in teaching mathematics and statistics. We only found two factors whose eigenvalues were greater than 1 (Boudah, 2010), which together accounted for almost 65.667% of the variability in the original variables.

Table 4. Total explained variance

	Initial Eigenvalues			Extraction Sums of Squared Loadings		
Component	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	9.512	55.952	55.952	9.512	55.952	55.952
2	1.652	9.715	65.667	1.652	9.715	65.667
3	.920	5.415	71.082			
4	.866	5.097	76.178			
5	.693	4.076	80.254			

Figure 1 displays a scree plot of the principal components derived from the teachers' responses. The line is almost flat at factor three, which means only two factors account for much of the variance and can be retained for further analysis. The last big drop occurs between the second and third components, so using the first two components is suitable (Rea & Rea, 2016).

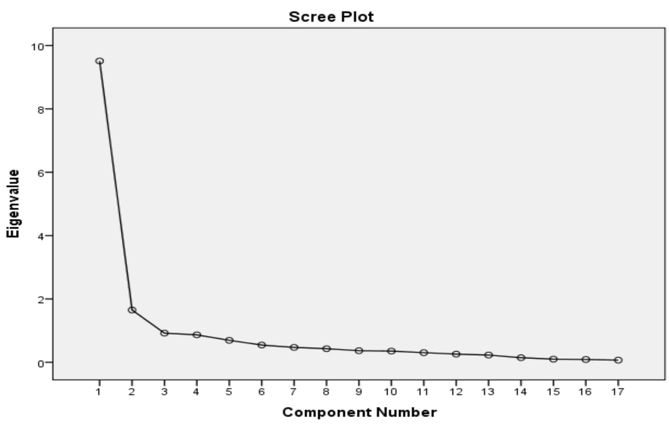

Figure 1. Scree plot

We used an oblique (promax) rotation to produce interpretable components (Baglin, 2014). When we used the rotation, the correlation between components 1 and 2 became 0.657 instead of being orthogonal (that is, instead of being independent, for the subsequent analysis). Table 5 displays two meaningful factors with factor loading coefficients sorted by size.

Table 5. Component matrix (extracted factors)

Teachers' confidence	Component 1	Component 2
Using statistics in out-of-the classroom situations	.975	
Connecting statistics to other key learning areas	.930	
Ideas of sampling and data collection	.826	
Range and variation	.797	
Pie graphs and histograms	.778	
Critical debate on the use of mathematic and statistics in the media	.737	
Connecting mathematics to other key learning areas	.618	
Simple probabilities	.461	
Inference and prediction	.454	
Presenting mathematics in an expository style (detailed explanation)		.971
Ratios and proportion		.954
Percentages		.920
Fractions		.828
Decimals		.770
Patterns and algebra		.664
Measurement		.583
Mental computations		.326

Extraction method: Principal Component Analysis
Rotation method: Promax with Kaiser Normalization
Rotation converged in three iterations

In trying to understand the implications further, we used the equating *t*-tests strategy in RUMM 2030 to explore the equivalence of test scores across the two subscales. The paired *t*-test provides comparisons of person location estimates derived from using the two different subsets of items taken from an initial scale made up of all the items. The two plots, shown in Figure 2, compare item subsets and scores over the two different sets of items.

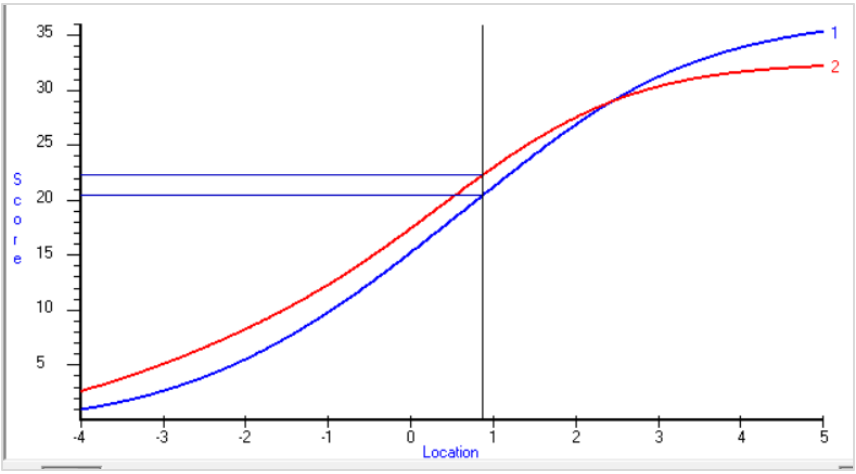

Figure 2. Graph of equating test

As seen from Figure 2, the curves of the two plots suggest there is a difference between the two subscales, even though the maximum score on Subtest 1 is 36 while it is 32 for Subtest 2. It is only at the higher end that people have higher scores on average in Subset 2 than in Subtest 1. Figure 2 also indicates on the one hand, a person with a location of 0.8699 has a predicted score of 22.273, that is, a mean of 2.784 on the items in Subtest 2. On the other hand, the person has a predicted score of 20.422, that is, a mean of 2.269 on the items in Subtest 1.

Using the paired *t*-tests, with 5% level of significance, we found there were 14 persons (18.6%) for whom the difference in estimates between the two subscales exceeded the 5% level of significance. These percentages are not negligible, suggesting the two subscales are significantly different from each other. This implies that the level of the confidence trait changes, depending on the set of items that are calibrated. These differences indicate multidimensionality as under Rasch model conditions, if the data fit the model, then analysis of any subset of items should produce equivalent person measures. Hence, this analysis confirms the teachers' confidence in these two subsets of concepts may be different.

Since the factor analysis suggested the presence of two dimensions, we consider the fit statistics for the analysis of the two subsets. The Rasch analysis was conducted with *RUMM2030* software (Andrich et al., 2009) with a focus of testing whether the response pattern observed in the data is close to the theoretical pattern predicted by the model. Table 6 and Table 7 present the fit statistics for the analysis for Subset 1 (confidence in teaching statistics concepts and those which require critical thinking skills and making connections across areas) and that for Subset 2 (confidence in teaching mathematics concepts).

Table 6. Summary statistics for Subset 1 of 9 items

	ITEMS [N = 9]		PERSONS [N = 75]	
	Location	Fit residual	Location	Fit residual
Mean	0.0000	0.2082	0.5611	-0.4644
SD	0.5273	1.1824	1.7292	1.5516
	Cronbach's alpha = 0.92150		Person separation index 0.90887	

Table 7. Summary statistics for Subset 2 of 8 items

	ITEMS [N = 8]		PERSONS [N = 75]	
	Location	Fit residual	Location	Fit residual
Mean	0.0000	0.3231	2.3325	-0.3087
SD	1.1785	1.6614	2.3921	1.3149
	Cronbach's alpha = 0.92881		Person separation index 0.92691	

In terms of reliability, in *RUMM2030* an estimate of the internal consistency reliability of the scale is the person separation index, which was found to be 0.90887 and 0.92691 for Subsets 1 and 2 respectively, all of which are higher than the minimum of 0.85 advised by Tennant and Conaghan (2007). The high value suggests that the instrument has been able to distinguish well between the persons in the study, showing that the estimation of the person's confidence locations is consistent across the model.

We also considered the item fit of the individual items to check whether they fit the model. It was found that all the items fit the model (all the probabilities are > 0.05 and the measures of their fit residuals are between –2.5 and 2.5 (Andrich & Marais, 2019), except Item 5 in Subset 2. This was the item measurement, with an FR of 3.023, which is slightly above the recommended limit.

5.2. ITEMS ABOUT WHICH THE TEACHERS WERE MOST (LEAST) CONFIDENT

We now consider in more detail the ordering of the items according to the teachers' endorsement. The application of a Rasch model allows one to represent the items hierarchically in order of difficulty, since with the analysis, the constituent items are hierarchically ordered in terms of difficulty level (Cavanagh & Waugh, 2011; Choi, 2014). As noted earlier, in this study the item location is interpreted in terms of whether teachers were confident in teaching the concept in the items. Since a lower location in the Rasch analysis represents an easier item, the interpretation for confidence items is that low difficulty means high confidence. That is, if an item A is located at a lower location than an item B it indicates that teachers were more confident about teaching item A than they were about item B.

Figure 3 and Figure 4 present the Wright person-item map representing our data on teachers' confidence in teaching concepts in mathematics and statistics. A Wright map is a commonly used Rasch figure for simultaneously plotting both the item and person estimates on the logit scale. The thresholds are the points of equal probability of adjacent categories, while item difficulty D_i of item *i*, is the point where the top and bottom categories are equally probable, that is the "balance point at which the highest and lowest categories are equally probable" (Bond & Fox, 2013, p. 120). There are multiple thresholds for each item (because the items are polytomous), so we use a summary location (the overall item location) instead of multiple thresholds for each item, in the Wright maps of Figure 3 and Figure 4. The plot is divided down the center by a line, with the left side displaying a horizontal histogram of the person estimates and the right side displaying the items according to locations arranged from lowest to highest. The left-hand column locates the person measures of confidence along the variable. The shape of the distributions indicates variability among the mathematics teachers in terms of their level of confidence to teach mathematics and statistics concepts.

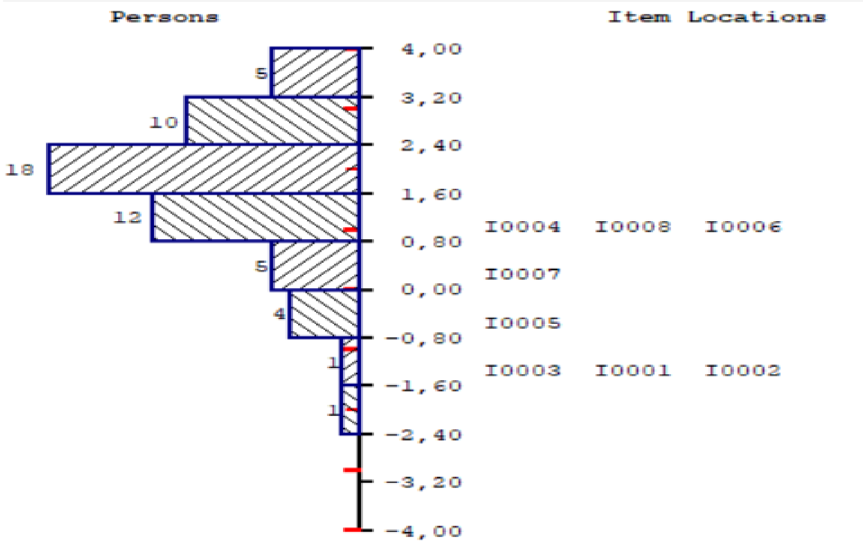

Figure 3. Item map for confidence in teaching mathematics concepts

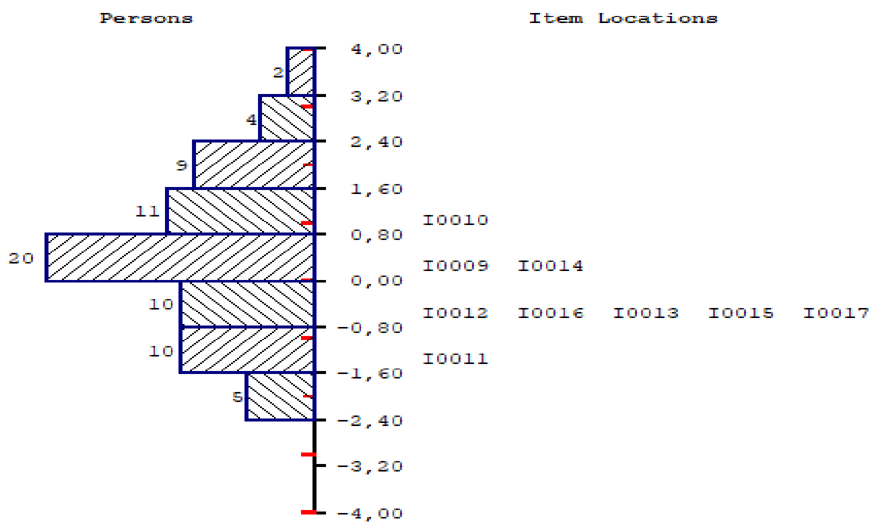

Figure 4. Item map for confidence in teaching statistics concepts and those which require connections

In terms of the mathematics topics, the concept with which the teachers had the least confidence was measurement, while they were most confident about teaching percentages. In terms of the items in Subset 1, which included statistics concepts as well as those requiring connections between areas, the concept the teachers were least confident about was about fostering critical debate on the use of mathematics and statistics in the media (Item 10). Noting the increased emphasis in curricula about fostering critical thinking as an explicit goal of teaching mathematics (DoE, 1997; Jablonka, 2014), the result that teachers were not confident about engaging their learners in critical debates, is important. It suggests that teachers need some support in this area. The item for which the teachers had the second lowest confidence was about making inferences and predictions (Item 14). The teachers' low confidence about dealing with these concepts may be understandable because it requires "reading beyond data" skills (Harrell-Williams et al., 2019, p. 197). As noted by Lovett and Lee (2018), the reasoning required for these skills is more abstract and sophisticated than the other levels of "reading data" and "reading between the data" (Harrell-Williams et al., 2019, p. 197).

The teachers were most confident about the teaching of pie graphs and histograms (Item 11). Harrell-Williams et al. (2015) similarly found that, overall, pre-service teachers expressed high confidence in their ability to teach graphs and tables. It is also important to note that these concepts have formed part of the core mathematics curriculum for many years supporting Begg and Edward's (1999) finding that teachers have greater confidence in the more familiar concepts.

It may not be possible to determine directly and with certainty the differences in confidence levels for the two subsets. Some inferences can be made, however, using the results in Table 7 and Table 8, which indicate the mean person location for Subset 2 is 2.3325, while it was 0.9849 for Subset 1. Note that Rasch models estimate the probability of the interaction between person B with an item *i*, as a logistic function of the difference between the person and item location (Andrich & Marais, 2019; Bond & Fox, 2013). Hence, it can be inferred that the probability of the teachers endorsing the items on average in Subset 2 is much higher than that for the items in Subset 1, because the mean of the person location for Subset 2 was higher than that of Subset 1. Since this is the same group of teachers who responded to the items in the two subsets, it can be inferred that the teachers displayed a higher confidence about teaching the mathematics concepts in Subset 2, compared to teaching the statistics concepts and those which require connections across other areas.

5.3. DIFFERENCES IN CONFIDENCE ACCORDING TO PHASE OF TEACHING, ADDITIONAL PROFESSIONAL COURSES, GENDER AND TEACHING EXPERIENCE

A logistic regression model was used to explore differences in confidence levels of teachers with respect to phase of teaching, additional professional courses taken, gender, and teaching experience. In order to perform a logistic regression, we computed a binary response variable using scores from the instrument used to measure teachers' confidence in teaching 17 mathematics and statistics concepts (see Table 2), hereafter mentioned simply as "confidence". The total score across the 17 items was used to classify the responses into two categories of high and low confidence. Since the maximum score was 68, teachers whose scores were 34 or lower were coded as displaying lower confidence while those who scored higher than 34 were coded as having higher confidence. Table 8 reports on the category frequencies for the binary response variable that will be used to model teachers' "confidence" and the effect of the demographic factors on the confidence (see Table 1). In this study, the binary response is High /Low levels of confidence, and we aim to see the effect of various demographic factors on the binary response variable. Overall results indicate that 52 (72%) teachers expressed a high level of confidence, whereas 21(28%) teachers in the study expressed a low level of confidence in teaching mathematics and statistics concepts.

Table 8. Teachers' confidence (response variable)

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Low confidence	21	28	28	28
	High confidence	54	72	72	100
	Total	75	100	100	

The *Omnibus Tests of Model Coefficients* is used to check that the new model (with explanatory variables included) displayed in Table 9, is a positive change over the null model. It computes chi-square tests to check if there is a significant difference between the -2LLs (log likelihoods) of the null model and the new model. If the new model has a significantly decreased -2LL compared to the baseline, it suggests the new model is describing more of the variance in the outcome and is an improvement. The null model without variables is not reported here. We are interested with the new model at step one, i.e., with variables which are significant so that it can be used for further analysis. In the current analysis, the chi-square test value is highly significant ($\chi^2 = 18.143, df = 8, p < 0.05$) so our new model is significantly better and can be used for further analysis.

Table 9. Omnibus test of model coefficients

		Chi-square	*df*	Sig.
Step 1	Step	18.143	8	.020
	Block	18.143	8	.020
	Model	18.143	8	.020

The Hosmer-Lemeshow test (Table 10) tests the null hypothesis that predictions made by the model fit very well with observed group memberships. A value which is greater than 0.05 indicate the model adequately fits the data (Guffey, 2012; Hosmer & Lemeshow, 2000). Since 0.950 is greater than 0.05 this means the goodness of fit is satisfied in the data.

Table 10. Hosmer-Lemeshow test

Step	Chi-square	*df*	Sig.
1	1.630	6	.950

Table 11 lists the results of the regression for the independent variables and the interaction effects. The Wald Chi-square statistic tests the unique contribution of each predictor. If the chi-square test is significant, it tells us that as a set, the variables contribute positively or negatively to the model.

Table 11. Parameter estimates for teachers' confidence and demographic factors

Variables	Code	*B*	*S.E.*	*Wald*	*df*	*Sig.*	*Exp(B)*
Gender	Male	Reference					
	Female	-2.143	1.572	1.859	1	.173	.117
Teaching experience	> 20 years (Reference)						
	0–10 years	.592	1.246	.226	1	.635	.553
	11–20 years	-3.204	1.547	4.288	1	.038	.041
Teaching experience *Gender	0–10 years *Female	.712	1.734	.169	1	.681	2,038
	11–20 years *Female	5.218	2.147	5.909	1	.015	184,519
	Gr 10–12 (Reference)						
Phases	Gr 4–9	-2.421	.813	8.858	1	.003	.089
Additional Professional Certification	Received it (Reference)						
	Not received	-2.412	1.019	5.597	1	.018	.090
*Additional Professional Certification	GET *Not received	2 882	1 588	3 296	1	. 069	17.849
	Constant	3 804	1 376	7 639		. 006	44.860

The findings show a significant interaction of teachers' confidence by teaching experience and gender, although on their own the factors of teaching experience and gender are not individually significant predictors of confidence. Table 12 indicates there is a significant difference in confidence by gender in the middle teaching years (11–20 years' experience), with female teachers being more confident at this stage of their career, than male teachers in the same stage. For those in the early years (0–10 years teaching experience), however, and those who are later in teaching careers (> 20 years teaching experience), the confidence level is similar between genders. This tells us that gender can play a role in confidence of teaching mathematics and statistics concepts during the mid-career years. This interactive relationship is depicted in Figure 5, which illustrates that female teachers in the middle years of teaching have higher levels of confidence than their male counterparts in the same stage of teaching.

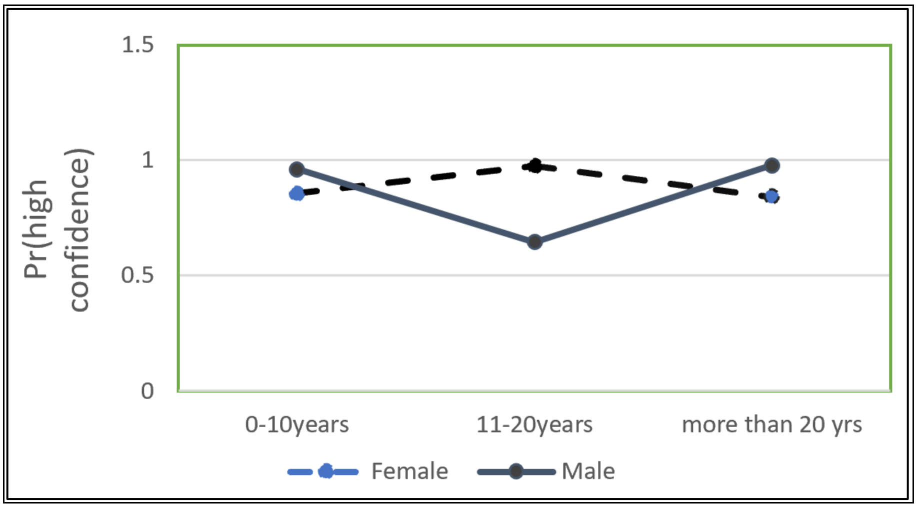

Figure 5. Predicted probabilities for high confidence by gender and age

Table 11 also provides evidence of an interaction between phases (GET and FET) that teachers teach and whether teachers completed additional professional learning courses. Figure 6 bears evidence that amongst those who have received additional professional certification, FET teachers have a higher confidence level in teaching statistics and mathematical concepts than is the case for GET teachers, though there is not much difference in levels of confidence amongst the two groups (GET and FET) if they have not received additional professional certification. This suggests that additional courses taken by teachers can contribute to successful effective teaching (Jackson et al., 2013; Umugiraneza et al., 2018).

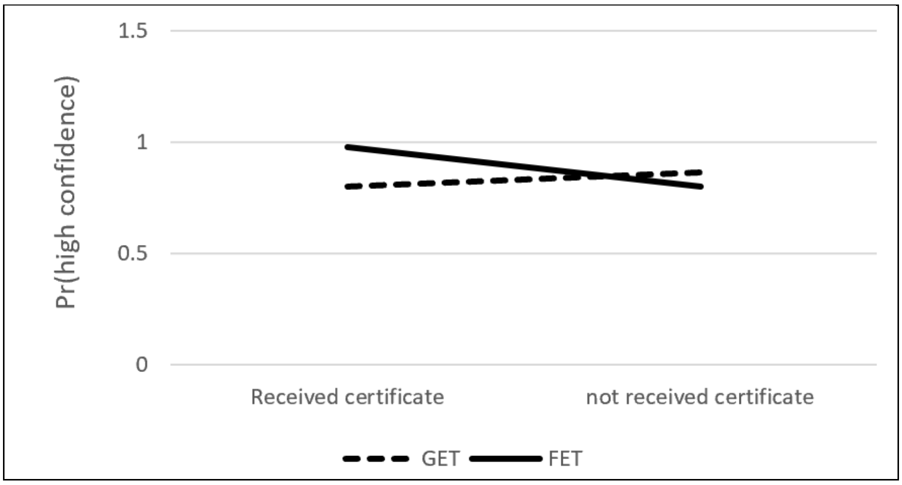

Figure 6. Predicted probabilities for high confidence by receiving certificate additional professional certification from PDGE, NPDE or honours.

6. DISCUSSION AND CONCLUSION

This study used various techniques to explore the differences between teachers' confidence in teaching mathematics and statistics concepts for a group of teachers from KwaZulu-Natal in South Africa. The findings showed that teachers had lower confidence about teaching concepts in statistics and those which require critical thinking skills and making connections across topics, as compared to the confidence in teaching mathematics concepts which have traditionally appeared in the curriculum for many decades. There have been many calls across the world for mathematics curricula to start reflecting more strongly the links across the curriculum and to apply lessons to real-life settings. In a previous study (Umugiraneza et al., 2018), it was found that many teachers within the group recognized the need for their learners to see real-life applications of mathematics to improve the participation and the engagement of their learners. We now have evidence, however, that teachers' recognition of the importance of teaching approaches did not necessarily mean they were confident about being able to implement this approach in the classroom. This study found that with respect to statistics concepts, teachers in this study are least confident about engaging their learners in critical debate about the use of statistics in the media, showing that these teachers are still struggling to deal with these curricular focuses, which have been introduced in many mathematics curricula across the world.

The findings in the previous study of the SETS instrument for high school teachers comprising 46 items support the presence of the three dimensions of teacher self-efficacy with respect to teaching statistics (Harrell-Williams et al., 2019). In this study, for which the instrument comprised 17 items, it was found that mathematics teachers' confidence in teaching mathematics and statistics concepts can be interpreted using multidimensional measures. The analysis in this study revealed two main principal components, suggesting that these two sets of items, respectively, have something in common that is different from all other items (Andrich, 2012). Hence, the teachers regard the teaching of the concepts from the first subset as having constructively different demands than the teaching of concepts comprising the second subset.

The items in Subset 2 comprise traditional mathematics topics that have been in the curriculum for a long time while those in Subset 1 are those related to statistics, which was only formally included as part of the South African curriculum starting in 2002 when data handling was specified as one of four outcomes (DoE, 2002). Subset 1 also includes concepts that require connections across other areas as well as critical thinking skills: concepts that were emphasized during the introduction of C2005 (DoE, 1997). Hence this study provides some support for the finding by Begg and Edwards (1999) that teachers displayed lower levels of confidence with the newer ideas in the statistics curriculum than the familiar topics they were accustomed to teach. More research is needed, however, to help us understand whether the main reason for the lack of confidence is because the topics are new, and they lack preparation or because of other reasons such as considering it to be of little importance.

Although many mathematics educators and researchers agree with the need for the changing focus to include more statistics education, little is known about the demands experienced by mathematics teachers as they try to implement these new ideas in their classrooms. In terms of Subset 1 items, teachers were least confident about concepts that require connections across other topics as well as those that require critical thinking. More qualitative research is required to help us understand why teachers were more confident with teaching certain concepts than others.

The study has also found a significant difference in confidence by gender during the middle teaching years (11–20 years' experience), with female teachers being more confident at this stage of their career than their male counterparts at the same stage. For those who were in the early or later years in teaching careers, however, the confidence level was similar between genders. The finding in our study for this group of teachers is that the relationship between teacher experience and confidence is not linear but there is an interaction between teacher confidence and gender during the middle years of teaching. Furthermore, the study found that amongst those who received additional professional certification, FET teachers have a significantly higher confidence level in teaching statistics and mathematics concepts than is the case for GET teachers, though there is not much difference in levels of confidence amongst the two if they have not received certification. This finding is of importance for South African education authorities where much effort has been expended on offering retraining and upgrading opportunities for teachers on a wide scale, although there are concerns about the effectiveness of such programmes (Bansilal, 2015).

REFERENCES

Adu, E. O., & Gosa, L. J. (2014). The teaching and learning of data-handling in primary schools: South African experience. *Mediterranean Journal of Social Sciences*, *5*(23), 814–819.

Andrich, D. (2012). *Advanced course in Rasch measurement theory* [course notes]. University of Western Australia.

Andrich, D., & Marais, I. (2019). A course in Rasch measurement theory. *Measuring in the educational, social and health sciences.* Springer.

Andrich, D., Sheridan, B. E. D., & Luo, G. (2009). *RUMM2030: Rasch unidimensional models for measurement.* RUMM Laboratory.

Avalos, B. (2011). Teacher professional development in teaching and teacher education over ten years. *Teaching and Teacher Education*, *27*, 10–20. https://doi.org/10.1016/j.tate.2010.08.007

Baglin, J. (2014). Improving your exploratory factor analysis for ordinal data: A demonstration using FACTOR. *Practical Assessment, Research, and Evaluation*, *19*(1), 1–15.

Bansilal, S. (2011). Assessment reform in South Africa: Opening up or closing spaces for teachers? *Educational Studies in Mathematics*, *78*(1), 91–107. https://www.jstor.org/stable/41485942

Bansilal, S. (2014). Using an APOS framework to understand teachers' responses to questions on the normal distribution. *Statistics Education Research Journal*, *13*(2), 42–57. https://doi.org/10.52041/serj.v13i2.279

Bansilal, S. (2015). Examining success rates of teacher students: An activity theory analysis. *Progressio*, *37*(1), 134–153.

Begg, A., & Edwards, R. (1999). Teachers' ideas about teaching statistics, *Proceedings of the 1999 Combined Conference of the Australian Association for Research in Education and the New Zealand Association for Research in Education*. Melbourne: AARE and NZARE. http://publications.aare.edu.au/99pap/beg99082.html

Ben-Zvi, D., & Garfield, J. B. (2004). Statistical literacy, reasoning and thinking: Goals, definitions, and challenges. In Ben-Zvi, D., & Garfield, J. B. (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 3–16). Kluwer Academic Publishers.

Beswick, K., Callingham, R., & Watson, J. (2012). The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher Education*, *15*(2), 131–157.

Beswick, K., Watson, J., & Brown, N. (2006). Teachers' confidence and beliefs and their students' attitudes to mathematics. In P. Grootenboer, R. Zevenbergen & M. Chinnappan (Eds.), *Identities, Cultures and Learning Spaces*. Proceedings of 29th annual conference of the Mathematics Education Research Group of Australasia, Canberra, July 1–5 (pp. 68–75). Sydney: MERGA.

Bond, T. G., & Fox, C. M. (2013). *Applying the Rasch model: Fundamental measurement in the human sciences*. Psychology Press.

Boudah, D. J. (2010). *Conducting educational research: Guide to completing a major project*. SAGE Publications.

Briggs, D.C., & Wilson, M. (2003). An introduction to multidimensional measurement using Rasch models. *Journal of Applied Measurement*, *4*(1), 87–100.

Carver, R., Everson, M., Gabrosek, J., Horton, N., Lock, R., Mocko, M., Rossman, A., Roswell, G. H., Velleman, P., Witmer, J., & Wood, B. (2016). *Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Report 2016.* https://commons.erau.edu/publication/1083

Cavanagh, R. F., & Waugh, R. F. (Eds.). (2011). *Applications of Rasch measurement in learning environments research* (Vol. 2). Sense Publishers.

Callingham, R., & Watson, J. (2014). Teachers' confidence in teaching statistical ideas. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in Statistics Education.* Proceedings of the 9th International Conference on the Teaching of Statistics, Flagstaff, Arizona, July 13–18. International Statistical Institute.

Choi, B. S. (2014). Keyform analysis of Rasch measurement accessible to clinicians in rehabilitation clinics. *Physical Therapy Korea*, *21*(2), 74–81.

Cobb, G. W., & Moore, D. S. (1997). Mathematics, statistics, and teaching. *The American Mathematical Monthly*, *104*(9), 801–823.

Chisholm, L., & Leyendecker, R. (2008). Curriculum reform in post-1990 sub-Saharan Africa. *International Journal of Educational Development*, *28*, 195–205. http://doi.org/10.1016/j.ijedudev.2007.04.003

Department of Education. (1997). *Curriculum 2005: Lifelong learning for the 21st century*. Pretoria National Department of Education.

Department of Education. (2002). *Revised National Curriculum Statement Grades R-9 (Schools): Mathematics*. Pretoria National Department of Education.

Donovan, N. J. (2018). Examining the item-level psychometric properties of the communicative effectiveness survey-revised for people with Parkinson's disease and Dysarthria. *Clinical Archives of Communication Disorders*, *3*(1), 42–51.

Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) report.* American Statistical Association. https://www.amstat.org/docs/default-source/amstat-documents/gaiseprek-12_full.pdf

Gattuso, L., & Ottaviani, M. G. (2011). Complementing mathematical thinking and statistical thinking in school mathematics. In C. Batanero, G. Burrill, C. Reading (Eds.). *Teaching statistics in school mathematics: Challenges for teaching and teacher education* (pp. 121–132). Springer.

Guffey, D. (2012). *Hosmer-Lemeshow goodness-of-fit test: Translations to the Cox Proportional Hazards Model*. [Masters thesis, University of Washington]. https://digital.lib.washington.edu/researchworks/bitstream/handle/1773/22648/Guffey_washington_0250O_11143.pdf;sequence=1

Handal, B., & Herrington, A. (2003). Mathematics teachers' beliefs and curriculum reform. *Mathematics Education Research Journal*, *15*(1), 59–69.

Harrell-Williams, L. M., Lovett, J. N., Lee, H. S., Pierce, R. L., Lesser, L. M., & Sorto, M. A. (2019). Validation of scores from the high school version of the self-efficacy to teach statistics instrument using preservice mathematics teachers. *Journal of Psycho-educational Assessment*, *37*(2), 194–208.

Harrell-Williams, L. M., Sorto, M. A., Pierce, R. L., Lesser, L. M., & Murphy, T. J. (2015). Identifying statistical concepts associated with high and low levels of self-efficacy to teach statistics in middle grades. *Journal of Statistics Education*, *23*(1). https://doi.org/10.1080/10691898.2015.11889724

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal*, *105*(1), 11–30.

Hosmer, D., & Lemeshow, S. (2000). *Applied logistic regression* (2nd ed). John Wiley & Sons.

Jablonka, E. (2014). Critical thinking in mathematics education. In S. Lerman (Eds.), *Encyclopedia of Mathematics Education*. Springer.

Jackson, C., Simoncini, K., & Davidson, M. (2013). Classroom profiling training: Increasing preservice teachers' confidence and knowledge of classroom management skills. *Australian Journal of Teacher Education*, *38*(8), 30–46. https://doi.org/10.14221/ajte.2013v38n8.2

Justice, N., Morris, S., Henry, V., & Fry, E. B. (2020). Paint-by-number or Picasso? A grounded theory phenomenographical study of students' conceptions of statistics. *Statistics Education Research Journal*, *19*(2), 76–102. https://doi.org/10.52041/serj.v19i2.111

Linacre, J. M. (2008). *Winsteps Rasch measurement* (Version 3.63.2). MESA Press.

Maoto, S., & Wallace, J. (2006). What does it mean to teach mathematics for understanding? When to tell and when to listen. *African Journal of Research in Mathematics, Science and Technology Education*, *10*(1), 59–70.

Masters, G. N. (1982). A Rasch model for partial credit scoring. *Psychometrika*, *47*, 149–174.

North, D., & Scheiber, J. (2008). Introducing statistics at school level in South Africa: The crucial role played by the national Statistics Office in training in-service teachers. *Proceedings of the 18th ICMI Study Conference and 2008 IASE Round Table Conference.* http://www.ugr.es/~icmi/iase_study/

North, D., & Zewotir, T. (2006). Introducing statistics at school level in South Africa. In A. Rossman & B. Chance (Eds.), *Working Cooperatively in Statistics Education.* Proceedings of the 7th International Conference on Teaching Statistics (ICOTS7), Salvador, Bahia, Brazil. International Statistical Institute. http://iase-web.org/documents/papers/icots7/2F1_NORT.pdf

North, D., Gal, I., & Zewotir, T. (2014). Building capacity for developing statistical literacy in a developing country: Lessons learned from an intervention. *Statistics Education Research Journal*, *13*(2),15–27. https://doi.org/10.52041/serj.v13i2.276

North, D., Scheiber, J., & Ottaviani, M. G. (2010). Training teachers to teach statistics in South Africa: Realities and attitudes. In C. Reading (Ed.), *Data and Context in Statistics Education: Towards an Evidence-based Society*. Proceedings of the 8th International Conference on Teaching Statistics (ICOTS8), Ljubljana, Slovenia. International Statistical Institute.

O'Sullivan, M. C. (2002). Action research and the transfer of reflective approaches to in-service education and training (INSET) for unqualified and underqualified primary teachers in Namibia. *Teaching and Teacher Education*, *18*(5), 523–539.

Peng, C.-Y. J., Lee, K. L., & Ingersoll, G. M. (2002). An introduction to logistic regression analysis and reporting. *The Journal of Educational Research*, *96*(1), 3–14.

Rae, A., & Rea, W. (2016). How many components should be retained from a multivariate time series PCA? *arXiv*, 1610.93588v2. https://doi.org/10.48550/arXiv.1610.03588

Tennant, A., & Conaghan, P. G. (2007). The Rasch measurement model in rheumatology: What is it and why use it? When should it be applied, and what should one look for in a Rasch paper? *Arthritis &Rheumatism*, *57*(8), 1358–1362.

Tolbert, D. (2008). Online mentoring and pre-service elementary teachers 'confidence to teach mathematics and science. *Journal of Student Scholarship*, *5*, 1–7.

Umugiraneza, O., Bansilal, S., & North, D. (2016). Teachers' confidence and beliefs in teaching mathematics and statistical concepts. *International Scientific Researches Journal*, *72*(09), 1–16. https://doi.org/10.21506/j.ponte.2016.9.34

Umugiraneza, O., Bansilal, S., & North, D. (2018). Examining teachers' perceptions about improving the teaching and learning of mathematics and statistics. *Statistics Education Research Journal*, *17*(2), 239-254. https://doi.org/10.52041/serj.v17i2.168

Venkat, H., & Spaull, N. (2015). What do we know about primary teachers' mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. *International Journal of Educational Development*, *41*, 121–130.

Wessels, H., & Nieuwoudt, H. (2011). Teachers' professional development needs in data handling and probability. *Pythagoras*, *32*(1), 1–9.

Wessels, H., & Nieuwoudt, H. (2013). Teachers' reasoning in a repeated sampling context. *Pythagoras*, *34*(1), 1–11. https://doi.org/10.4102/pythagoras.v34i1.169

Western Cape Education Department. (1998). *Guideline document: National examination 2001 Mathematics Paper 1 & 2. Higher grade and standard grade*. WCED.

Witt, M., Goode, M., & Ibett, C. (2013). What does it take to make a primary maths teacher? Two trainee teachers' successful mathematical journeys. *Teacher Education Advancement Network Journal (TEAN)*, *5*(1), 19–32.

Wright, B.D., & Masters, G. N. (1982). *Rating scale analysis: Rasch measurement*. Mesa Press.

Yim, H. Y. B., Abd-El-Fattah, S., & Lee, L. W. M. (2007). A Rasch analysis of the Teachers Music Confidence Scale. *International Education Journal*, *8*(2), 260–269.

Yong, A. G., & Pearce, S. (2013). A beginner's guide to factor analysis: Focusing on exploratory factor analysis. *Tutorials in Quantitative Methods for Psychology*, *9*(2), 79–94.

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