LINKING PROBABILITY AND STATISTICS IN YOUNG STUDENTS' REASONING WITH CHANCE

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ABSTRACT

This article reports on a study in which third-grade students (8–9 years) were given a degree of agency in conducting chance experiments and representing the outcomes. Students chose their own samples of 12 coloured counters, ensuring all colours were represented. They predicted the outcomes of item selection, tested their predictions, explained the outcomes, quantified their chances of colour selections, and created two representations displaying the probabilities. Children displayed awareness of randomness and variation, together with proportional reasoning, as evident in their identification of one or more colours as having a greater chance of being selected, or equal chances when proportions of colours were equal. Evidence of children's metarepresentational competence appeared in their creation of two representations to display their probabilistic outcomes, with bar and circle graphs, as well as stacked bars, created. The inclusion of their own forms of inscription revealed a range of probability and statistics understandings. In selecting and justifying their preferred representations for conveying their outcomes, students favoured both bar and circle graphs, with a focus on how accurately, effectively, and efficiently their representation displayed the data, with the importance of the inscriptions highlighted.

Keywords: *Statistics education research; Probability; Statistics; Representations; Early grades*

1. INTRODUCTION

With data on world events flooding the media, there is an escalating need for a basic understanding of probability and statistics, including the links between the two. Despite repeated recommendations for connecting children's learning of probability with that of statistics, connections between the two remain lacking in the primary grades. Even though probability and statistics are typically incorporated within the mathematics curriculum (English & Watson, 2016; Greer, 2001; Pfannkuch & Ziedins, 2014), their inclusion does not necessarily imply connection, however.

Given the apparent limited research on approaches to linking the two domains, the present study provided third-grade students some freedom in directing their own experiments involving samples of coloured counters. Students predicted the outcomes of item selection (cf. Sharma, 2015), tested their predictions, explained the outcomes, quantified their chances of colour selections, and created and compared two representations illustrating the probabilities.

2. FOUNDATIONS OF PROBABILITY AND STATISTICS

2.1. LINKING PROBABILITY AND STATISTICS

There are differing perspectives on whether probability should be treated as a discipline in its own right or whether it should be seen as a significant component of statistics. In an earlier article (English & Watson, 2016), I argued that statistical literacy encompasses statistics and probability, both of which must be addressed in developing young children's facility with chance situations. A similar stance was taken by Innabi et al. (2023), who viewed the learning and teaching of statistics as juxtaposing deterministic and stochastic (probabilistic) perspectives. Several other researchers have stressed the

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importance of linking probability and statistics, with recommendations for incorporating representations and modelling within early probability experiences (Greer, 2001; Langrall & Mooney, 2005; Pfannkuch & Ziedins, 2014; Steinbring, 1991). Likewise, Shaughnessy (2003) advocated the teaching of probability be connected with that of statistics—"statistics should motivate the probability questions" (p. 223).

2.2. ESTABLISHING FOUNDATIONS EARLY

The key foundations of probability and statistics need to begin in the early grades, yet many curriculum documents do not include probability until the late middle or early secondary grades (e.g., *Common Core State Standards for Mathematics*, 2012; Department for Education, 2021). It is difficult to determine why some nations have omitted probability and statistics from their primary curriculum, given that several studies have shown how young children can engage with these topics considerably earlier than recommended (e.g., English & Watson, 2016; Bryant et al., 2018; Groth et al., 2021; Langrall, 2018). As Groth et. al. (2021) warned, the increasing divergence between research in early probability learning and developments in curricula has "potentially negative consequences" (p. 242), leading to early erroneous conceptions about probability becoming firmly entrenched.

Experiencing two key features of probability—the variability of results and randomness (Savard, 2014)—are foundational to statistical reasoning. Further, as Lopes and Cox (2018) pointed out, statistical reasoning requires contributions from both combinatorial and probabilistic reasoning, which can bring young students closer to the ideas of randomness and variability. In contrast, probability is typically taught with deterministic reasoning being the dominant form in mathematics and science (Innabi et al., 2023).

2.3. REPORTED DIFFICULTIES IN CHILDREN'S REASONING WITH PROBABILITY

Establishing the early learning of chance and probability has been comparatively limited (Leavy & Hourigan, 2020), with mixed reports on what young children know and understand about the topic (e.g., Batanero, 2015; Falk et al., 1980; Piaget & Inhelder, 1975). While Piaget and Inhelder maintained that children below 7 years of age are not able to differentiate between randomness and non-randomness, other noted researchers such as Fischbein (1975) argued that young children are able to make intuitive probability judgements.

One of the discrepancies in claims about young children's reasoning with probability lies in the different task features, as indicated in Acredolo et al.'s (1989) study. Using tasks varying in the number of alternative outcomes as well as the number of target outcomes, Acredolo et al.'s findings showed that young children can estimate probabilities with reasonable accuracy but are unlikely to do so when presented with the standard choice paradigm—that is, when asking them to determine whether the likelihood of Event A is greater than, less than, or equal to, the likelihood of Event B.

Other reported difficulties include intuitive beliefs, such as throwing a six on a fair die is more difficult than a one, the role of luck, and the equiprobability bias, that is, the tendency to view any random trials of an experiment as an adequate indication of an "equally likely" outcome (English & Watson, 2016; Groth et al., 2021; Lecoutre, 1992; Sharma, 2015). Although difficult to overcome at times, even in the adult population (Fischbein, 1975), Groth et al. (2016) conjectured that some of these difficulties can provide a foundation for the emergence of normative thinking (i.e., accepted mathematical thinking). For example, visualizing random generators and how they operate, and recalling past experiences in playing games of chance can provide a basis for further learning. On the other hand, as Greer (2001) and Ritson (1998) pointed out, children's experiences in playing games with dice and other such games of chance do not readily lead to the abstraction of principles of probability.

Children's apparent difficulties were usually observed with adult-designed tasks with a focus mainly on procedural knowledge involving meaningless formula (Vásquez & Alsina, 2021). There appear few studies where students have some agency over the activity, in both conducting chance experiments and representing the outcomes (English, 2018; Gadanidis et al., 2017). Such agency can enhance students' understanding of core probability ideas (Abrahamson, 2012) and reveal untapped capabilities. Likewise, providing children the freedom to generate their own representations, in contrast to their usually narrow classroom experiences, can reveal their ability to create a range of data structures and representations (English, 2010; Hutchison et al., 2000; Lehrer & Schauble, 2007). Such opportunities have not been as prolific as desired, however (Cengiz & Grant, 2009; Lehrer & Schauble, 2007).

In providing third-grade students such agency in the present study, the following research questions (RQ) were addressed:

RQ. 1. How did students reason about (a) their predictions and outcomes? (b) hypothetical repeated trials with replacement?

RQ. 2. What forms of representation did the students create and how did they interpret these?

RQ. 3. Which representation did they consider more effectively conveyed their probability outcomes?

3. METHOD

3.1. PARTICIPANTS AND BACKGROUND

Participants were from one class of 27 third-grade students (aged 8–9 years) from an all-girls school in a middle-class suburb of Brisbane, Australia. Parental permission was not granted for data to be collected from three students but the students were still permitted to participate. The students were in the first year of a four-year longitudinal study investigating the development of STEM literacy through statistical modelling. The probability activities were implemented by the children's teacher across three classroom sessions, each of duration 1 hr 25 mins to 1 hr 50 mins, towards the end of the first year of the study. The teacher was highly experienced in primary education and was keen to implement the activities, especially since they enriched her current program.

The Australian curriculum requirements for probability included conducting chance experiments, identifying and describing possible outcomes, and recognising variation in results. The statistics component included collecting and organising data, creating displays using lists, tables, picture graphs and simple column graphs, and interpreting and comparing data displays (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2020).

3.2. PROCEDURES

Use of literature. As an introduction to the activities, the book, *Probably Pistachio* (Winborn, 2001) was discussed. The book helped the children appreciate that one cannot predict with certainty the outcomes of a probabilistic situation. A second story book, *It's Probably Penny* (Leedy, 2007) served as a post-activity to consolidate the ideas that had been explored.

Experiences with a random generator. A popular Bingo game was played by the class to experience the notions of randomness and variation in chance events within a familiar and fun context. To review children's knowledge of the chance ideas inherent in the random generator, they were asked questions such as, "Did everyone have an equal chance of winning?"; "Was it certain that someone would win?"; and "Was it possible for two people to win?"

Experimenting with chance. The activity was set within the context of a game manufacturer wishing to create a new children's game. The children were to help the company determine the chances of selecting various coloured counters from a mystery bag. A container of 36 counters comprising nine of each of four different colours (blue, red, green, yellow) was presented to each group of children (3-4 children per group). Each group was to select only 12 of the 36 coloured counters to place in their group's "mystery bag", using their choice of numbers of each colour but ensuring there was at least one of each colour in the bag. The numbers of each colour did not have to be the same. The remaining counters were returned to the container. Each group member recorded on her own data table the number of each coloured counter chosen for their mystery bag. Prior to counter selection, each child was to predict the coloured counter that she might draw if only one counter could be selected without looking. Once all students in the group had made their predictions, each student recorded the group predictions in their own data table.

Next, students took turns in selecting one counter without looking and recording the colour selected. The counter was to be returned to the bag before the next student selected a counter. The process was repeated until all group members had selected and recorded their outcome, as well as the outcomes of their group members.

Workbook questions. Each student was provided with a workbook in which she recorded and justified her own responses to each question asked, including: (1) Why did you predict you would select that colour? (2) Did you select the coloured counter that you predicted? Did any of your group members select what they predicted? (3) Can your predictions guarantee that a particular, coloured counter will be selected? (4) Describe any variation you can see in the outcomes on your table of data. (5) If you were to repeat the counter selection over and over (replacing the counter each time), how might the data in your table change? (6) Does each counter have an equal chance of being selected? (7) Is there a coloured counter that has the greatest chance of being selected? The least chance?

***Quantifying probabilities*.** While probabilities can be expressed as decimals, fractions, or ratios, Bryant and Nunes (2012) recommended ratios in preference to fractions for the elementary grades. For this study, however, it was decided to adopt a basic fraction notion (x items out of a total of y) as an extension of the students' introduction to fractions earlier in the year. Students explored an outcome as a fraction of all possible outcomes, reflecting one of the first definitions of probability, namely, the classical interpretation (Batanero et al., 2005; Nikiforidou, 2018).

For their particular mystery bag, the children recorded the probability of selecting each coloured counter, for example,

"There are .... chances out of 12 chances of selecting a green counter."

The term, *chance*, was used as it featured in the mathematics curriculum (ACARA, 2020) and was also familiar to the children from their everyday experiences. Next, the students were to imagine they could add four extra counters to their bag, choosing any colours (beyond the existing colours). The children recorded the colour of each additional counter followed by the new probabilities of selecting each colour.

Creating statistical representations of probability. The students were asked to represent the probabilities of selecting counters of each colour, indicate the type of representation they created, and explain how their representation displayed the probability of selecting each coloured counter. Students were provided with a choice of recording material including blank paper, lined paper, a circle, and 1.75cm^2 grid paper. It was not the intention to encourage a particular representational form, rather, to see the choices the children made, given the surprising preference for circle sheets displayed by younger children in previous studies (e.g., English, 2013a, 2013b). The students were not given any further directions on creating their representations.

Following their first representation, the students were invited to select a different recording sheet and again represent the chances of selecting each coloured counter. One of the aims here was to observe what diSessa (2004) termed "metarepresentational competence," which influences children's "native capacities" (diSessa, 2004, p. 306) to create and re-create their own forms of representation. On completion of their two representations, each student was to compare her two forms and explain which she thought conveyed the "chance story" more effectively, and why.

3.3. DATA ANALYSIS

Data sources included each student's written responses to the workbook questions, including her chance representations and interpretations. Transcripts of class and group discussions were also completed. Students without permission for their work to be included (n = 3) were excluded from the analysis. The workbook responses were coded using deductive coding (Miles et al., 2014) and then analysed using frequency distributions. The senior research assistant and I commenced the coding of the students' responses using content analysis to generate a number of codes. We then both undertook separate coding to refine the coding schemes. Any discrepancies were identified and further refinements were undertaken by the author.

Transcripts of the discussions of three groups of students were completed and served to illustrate in greater depth some of the students' responses and interactions in responding to the workbook questions. The transcripts were analyzed qualitatively, adopting the form of iterative refinement cycles for in-depth evidence of students' learning (Lesh & Lehrer, 2000).

4. SELECTED RESULTS

Not all results of the study can be reported in this article; rather, consideration is given to those aspects that directly address the three research questions.

4.1. RQ. 1: HOW DID STUDENTS REASON ABOUT (A) THEIR PREDICTIONS AND OUTCOMES? (B) HYPOTHETICAL REPEATED TRIALS WITH REPLACEMENT?

The first part of this question was coded as follows: (1) reference to randomness (2) application of proportional reasoning (3) use of subjective thinking and (4) application of positional imaging. For the workbook question on whether predictions guarantee outcomes, students' responses were coded yes/no and their reasons as (1) a detailed response displaying an advanced understanding of predictions and outcomes (2) an explicit reference to randomness and (3) reference to possible outcomes such as equal chances or chances of more than one colour being drawn. For the second component of the first research question, students' responses were coded according to whether they (1) recognised randomness and the possibility of repeated selection of the same-coloured counter, or (2) claimed that each outcome would always be different.

Of the 24 students, two included two reasons in justifying their predictions and two offered two reasons for their responses as to whether predictions guarantee outcomes. Three students referred to the notion of randomness when justifying their predictions, for example, "My prediction was yellow. I chose it because I randomly chose." The majority of students (65%) displayed proportional reasoning in justifying their predictions, such as, "I predicted green because there's more green than any other colour." Only a few students (19%) displayed subjective thinking (e.g., "Green looked like it should be picked.") and only one student used positional imaging, such as, "I predicted blue because when the counters were in a pile, blue was on top so when we were putting them in, it would still be on top."

In responding to the *question of whether predictions can guarantee outcomes*, all but two students responded, "No." Nineteen percent of students also offered a detailed response displaying an advanced understanding of predictions and outcomes. Examples included: "I guarantee I will get a colour, but I don't guarantee I'd get silver … I'm not certain the bag is full of the same colour"; "You can't be certain but it is not impossible unless there is none of that colour"; and "I guarantee I can pull out any colour but "I can't be certain that I will always pick a green counter, but it may be likely that I will pick a green or blue."

A further 19% of students made explicit reference to randomness in stating that predictions cannot guarantee outcomes, such as, "No, because chance is random and you can't guarantee to get a certain colour." The most common response (42% of students) included reference to equal chances or chances of more than one colour being drawn and thus, a prediction could not guarantee an outcome. Three students justified their response by stating, for example, "It is just luck; you can't really guarantee anything"; and "It's just your imagination."

In responding to the second component of Research Question 1 (*How did students reason about hypothetical repeated trials with replacement?*), the majority of students (70%, n = 16, one absent) claimed that the outcomes would be different, with no reference to the possibility of selecting a given colour more than once. Responses here included "The data would change because you wouldn't get the same colours each time"; "I'd get at least one of every colour"; "I think you wouldn't get the same colour because even if you predict a different colour, each colour will always change." One student explained that "We'd pick out different colours but we can't be certain which colour but we can predict it won't always be right."

Thirty percent of the students explicitly recognised the randomness of outcomes *and* the possibility of selecting a given colour more than once, including successively. Their responses included, "The selection is random; you could pick different colours or the same colour a couple of times;" and "It's a random chance so the same colour could go over and over or different colours each time could be pulled out.

4.2. RQ. 2: WHAT FORMS OF REPRESENTATION DID THE STUDENTS CREATE AND HOW DID THEY INTERPRET THESE?

For research question 2, consideration was given to the type of representation students created, whether inscriptions were included, and if so, the nature of the inscriptions. Even though the children had not been formally introduced to circle graphs, which are not taught until the later grades, these were a popular means of representing the chances of colour selection. Bar graphs were also favoured, which is not surprising given their focus in the curriculum (ACARA, 2020). For their first representations, 50% of students created bar graphs, with circle graphs also popular (38%). The three remaining representations were picture graphs. In contrast, children favoured circle graphs for their second representation (58%), with only 29% choosing bar graphs. Two students created picture graphs, and one student simply wrote a story text.

The majority of students (71%) added inscriptions to their representations, even though this was not a specific requirement of this component of the activity. Students' inscriptions for their first representation ranged from a sophisticated, explicit account of the probabilities displayed (only one student) through to simply indicating how many counters of each colour were represented. The former response was of the form:

> *Blue 4/12, 4 counters, most chance, 4/12 most likely.*
> *Yellow 2/12, 2 counters, least chance, 2/12 uncertain.*
> *Green 3/12, 3 counters, possible.*
> *Red 3/12, 3 counters, possible, equal chance [with green] 3/12.*

Three students included inscriptions that conveyed the chances of selecting each of the colours as a fraction notion, such as, "red has 3 chances out of 12, blue has 4 chances out of 12, etc." The most frequent form of annotation (42% of responses) indicated the chances of each colour selection or simply the frequencies of each colour, for example, "five green chances, five blue chances, one yellow chance etc." and "12 counters, 2 green. 2 yellow, etc."

There appeared to be a small increase in the number of children who explicitly linked chance notions to their second representation. Of the 20 students who included annotations for their second representation, six indicated the chances of each colour selection on their representation, with some offering more detailed annotations. For example, Jenny annotated her second representation noting:

> *Blue, yellow, red, green. This shows that there are 4 equal chances out of 12 so that means you had 4 out of 12 [chances] of getting a red, green, blue, or yellow.*

The remaining students annotated their second representation with an indication of the number of each coloured counter only, or the fraction of each coloured counter. One student, however, wished to create a third representation, a bar graph, with the accompanying inscription indicating both the probabilities of counter selection and the likelihood:

> *Most likely blue has the most chance 4/12, green/red are in the middle (possible 3/12 and possible 3/12), yellow has the least chance uncertain 2/12.*

Figures 1(a) and 1(b) provide an example of one student's two annotated representations. In Figure 1(a), the student chose to first represent the chances of selecting each coloured counter by drawing stacked bars. This was followed by a circle graph for her second representation (Figure 1(b)). It is interesting how this student annotated her first representation by referring to the difficulty in predicting outcomes due to randomness. She recorded the chances of selecting each coloured counter on her second representation.

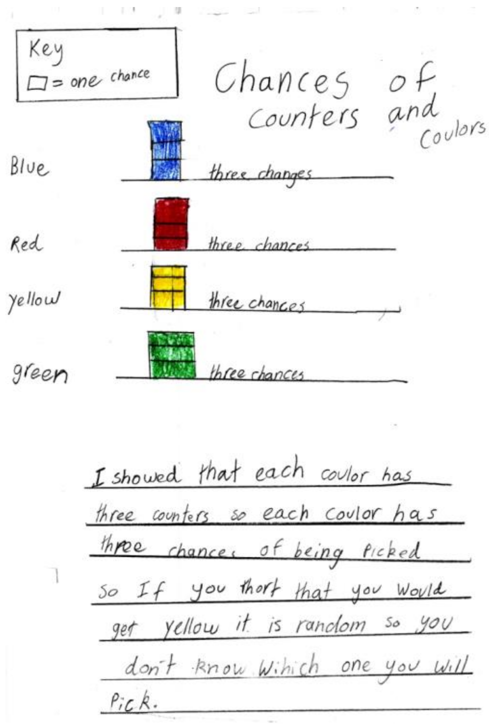

Figure 1(a). One student's first representation using stacked bars to display chances accompanied by detailed annotation.

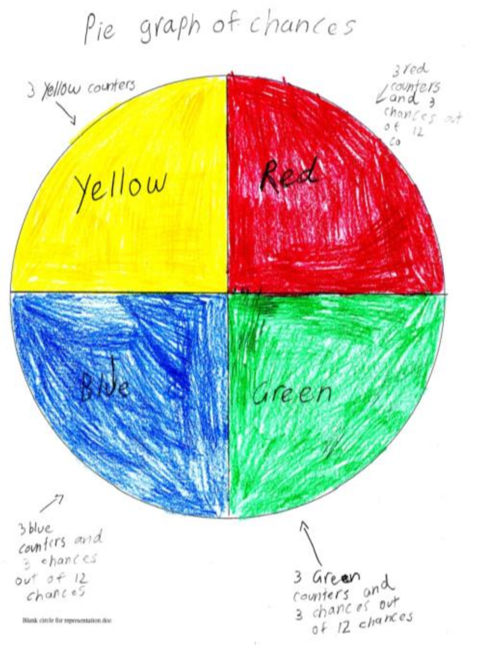

Figure 1(b). The student's second representation displaying a circle graph and detailed annotation.

Another interesting representation was created by three students, who chose to include an additional "total" bar on their bar graph, or an additional "stacked" bar displaying the chances of selecting each coloured counter (e.g., "Three chances of getting green is likely"; "The chance of getting red is possible"; "The chance of getting blue is possible;" "The chance of getting yellow is unlikely.") (Figure 2).

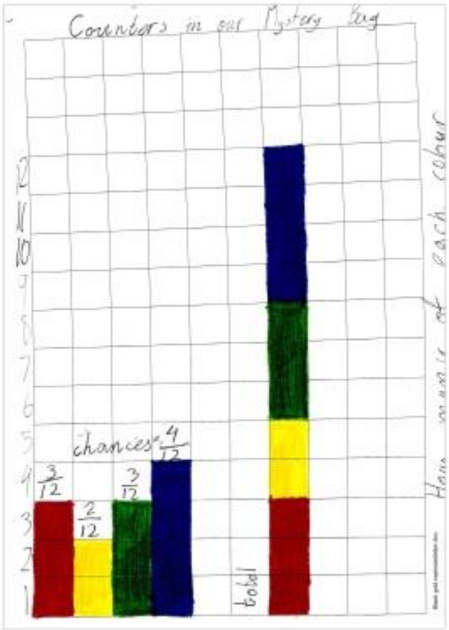

Figure 2. Bar graph displaying the chances of displaying the chances of selecting each colour, together with a "total" bar to indicate the number of counters in the sample.

4.3. RQ. 3: WHICH REPRESENTATION DID STUDENTS CONSIDER MORE EFFECTIVELY CONVEYED THEIR PROBABILITY OUTCOMES?

For Research Question 3, the type of representation students selected as conveying their probability outcomes more effectively, together with their justifications, were analysed. The latter included the extent of data and other information displayed, the ease of reading the representation, the accuracy of the representation and/or information displayed, the "efficiency" of the representation for displaying the data, and the representation's facility in displaying the "whole" or total number of counters in the sample.

Sixty percent of the students claimed that the bar graph was better, while the remainder preferred the circle graph. Reasons for selecting the bar graph included:

> *This is my first representation. It is a grid. Firstly, I think this one is the best of my two representations because it is easier to explain and understand and has more information like there are 5 green, 5 blue, 1 red, and lastly 1 yellow. So, 5 green out of 12, 5 blue out of 12 and 1 red out of 12 and 1 yellow out of 12. Red and yellow have the least chance of being picked out of the bag. But blue and green have the most likely chance of being picked. If I predicted I would predict green or blue because they are most likely of being picked.*

Students who preferred the circle graph offered reasons such as, "I think pie graph because it explains the chance in 2 ways—in fractions and in chance out of twelve." Another student found the circle graph "easier to read because it is a circle." Yet another student considered the circle graph was less effective in displaying the chance ideas, explaining:

> *Well, we did find that the pie graph didn't tell you a lot of information … if we didn't have the signs up* (referring to the annotation that she had included outside the circle graph), *such as "Green 4" you would have thought, well, how many counters would there be if we didn't say Green had 4 counters and Blue had 3 counters; but the only thing that you would know is that Blue and Red would be equal.*

5. DISCUSSION

This section examines the outcomes from each of the research questions and highlights areas in need of attention in designing classroom experiences.

5.1. STUDENTS' REASONING ABOUT THEIR PREDICTIONS, OUTCOMES, AND HYPOTHETICAL REPEATED TRIALS

Students' justifications for their predictions included limited reference to randomness. The majority displayed proportional reasoning in their explanations, that is, identifying one or more colours as having a greater chance of being selected, or equal chances when proportions of colours were equal. Such responses to predictions and outcomes reflect Pratt's (2000) argument that children reason with two different notions of randomness—a "local perception" related to the impossibility to predict the outcomes in each trial of an experiment, and a "global perception" involving children´s understanding of patterns appearing in multiple trials and in the distributions of outcomes.

Students' responses also reflect other research indicating that young children understand proportions as ratios even before they understand them as fractions (e.g., Bryant & Nunes, 2012). Although the students could reason proportionally in predicting probabilities of their counter selection, they were only dealing with simple, not compound events, which are naturally more difficult for both children and adults (Lecoutre, 1992). The role of proportional reasoning in the development of probability understanding has been the subject of several studies (e.g., Begolli et al., 2021; Fischbein & Gazit, 1984; Bryant & Nunes, 2012; Piaget & Inhelder, 1975). Yet, as Begolli et al. (2021) indicated, more research is needed in determining whether difficulties with proportional reasoning or with related mathematical concepts (e.g., part/part, part/whole, decimal, percentages) have a stronger influence on learning the concept of probability.

Students' awareness of variation in outcomes was evident in their recognition that predictions do not guarantee outcomes. Some students displayed a more advanced understanding of probability in their justifications, referring to notions of impossible, possible, certain, and randomness of chance events. With their focus on variation in outcomes, students overlooked the possibility of a particular outcome occurring more than once in succession, that is, their understanding of the independence of successive events revealed some difficulty. Only a small number of students recognised that randomness doesn't necessarily imply equal probabilities for any possible outcome. This equiprobability bias is often difficult to overcome in both children and adults and is frequently considered a deep-rooted misconception about randomness (Gauvrit & Morsanyi, 2014; Ingram, 2022). Yet, as Gauvrit and Morsanyi argued, a random variable has been defined traditionally as necessarily arising from uniformity, with this view remaining an implicit assumption in many classrooms. Repeating counter selection using a random generator might have facilitated a better understanding of randomness—a complex notion and a key statistics and probability concept that needs to be nurtured from a young age (Batanero, 2015).

5.2. FORMS OF REPRESENTATION AND INTERPRETATIONS

The students constructed a range of statistical representations incorporating inscriptions that revealed a linking of probability and statistics understandings. Children chose bar and circle graphs, as well as stacked bars, to represent their outcomes, even though the last two forms were not part of their third-grade curriculum (ACARA, 2020). Unexpected was the inclusion of a "total" bar, which a few students added to their bar graphs. This additional bar incorporated the total number of counters, the number of each colour, and in a couple of cases, the probability of each colour being selected. This inclusion suggests that the students were viewing the probabilities as fractions/parts of a whole, reflecting their learning during the activities. This total bar may also be viewed as a perceptual or "attentional anchor" (Abrahamson et al., 2021) for students in their representation and interpretation of their probabilities.

Students' inclusion of their own forms of inscription also revealed a range of probability and statistics connections, ranging from a focus on simply the number of each coloured counter represented, to indicating the chances of each colour being selected, to expressing chances as a fraction, and through to multiple forms of probability understandings. It could be that those students who just recorded the numbers of each coloured counter on their representations were viewing these in a proportional sense. However, the data did not enable us to determine if this were the case.

Students' responses in developing their own forms of representation provide insights into their meta-representational competence (diSessa, 2004; diSessa & Sherrin, 2000), that is, their capabilities in constructing and productively using a wide range of statistical displays, as well as critiquing and refining them. Fostering such competence in the early grades has been largely ignored, with instruction focused on a narrow set of standard curriculum-prescribed formats. Yet the present findings of this and other studies (e.g., diSessa, 2007) indicate the importance of providing more diverse learning opportunities in the classroom, where students have some agency in creating their representations.

5.3. REPRESENTATIONS CONSIDERED TO CONVEY PROBABILITY OUTCOMES MORE EFFECTIVELY

A facility with representations, including the ability to move readily from one form to another and to judge their adequacy for specific purposes, has been highlighted as enhancing students' reasoning with probability and statistics (diSessa, 2002; Greer, 2001; Meder & Gigerenzer, 2014; Nilsson et al., 2018; Steinbring, 1991). Students' explanations of their preferred representation for conveying their "chance story" suggest they were aware of the key features and purposes of statistical representations—in the present case for displaying data pertaining to probability. Students were focused on how accurately, effectively, and efficiently their representation displayed the data, with the importance of the inscriptions highlighted. This finding supports the previous claim that young students have untapped competencies that curricula need to capitalise on:

> Student[s] have powerful native ideas about MRC [meta-representational competence], but this fact has been all but invisible in the technical details of learning schooled representations … teaching representation as technique, without including other elements of MRC, likely contributes to a felt inauthenticity and brittleness of instruction. (diSessa, 2002, p. 105)

Students' preference for representations that were efficient and accurate, incorporated the "whole" or total number of counters, and were easy to read and interpret reflects aspects of what diSessa's (2002) terms "student critical capabilities." These capabilities include "quantitative precision" completeness, economy, and "conceptual clarity" (p. 109). Interestingly, diSessa (2002) hypothesised that "student criteria are design-linked" (p. 110), that is, they are more likely to be applied in activities that encourage representational creations, as was the present case. It is conjectured that had students not been given this agency, their full representational competence would not have been revealed.

6. LIMITATIONS

This study was limited to independent events involving one item being drawn with replacement. Further research could extend to a greater focus on sample space involving sample variation (Nilsson, 2014), two-item selection, and selection without replacement. Further use of a concrete random generator (Abrahamson, 2012) could yield greater insights into children's understanding of randomness and the outcomes of chance events. The extent to which the students' brief initial exposure to a random generator in the current study might have facilitated their subsequent responses was not determined. Likewise, any impact of the probability story book that was read as an introduction, was not ascertained.

7. CONCLUDING POINTS

This study has highlighted three main aspects of young students' learning in probability and statistics, namely, (a) they can deal with core probability ideas earlier than recommended by many curriculum documents (cf. Langrall & Mooney, 2005; Leavy & Hourigan, 2020); (b) elementary probability experiences can be linked effectively with statistical learning; and (c) activities in which students have some agency over their actions can reveal insights into their capabilities in the two domains.

The study differed from the traditional mechanistic textbook approaches applied in teaching probability and statistics (Vásquez & Alsina, 2021). The provision of agency enabled children to discern the variation in outcomes of their group members, even when there were equal chances of colour selection. Variation plays a central role in both probability and statistics, and clearly demands greater attention in the earlier grades. In line with Nilsson's (2021) argument, it is also conjectured that this task agency facilitated children's learning and understanding of experimental probability and provided insights into their probability and statistics reasoning.

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