CONTENT KNOWLEDGE AND ATTITUDES TOWARDS STOCHASTICS AND ITS TEACHING IN PRE-SERVICE CHILEAN MATHEMATICS TEACHERS

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ABSTRACT

Recognizing that mathematics teachers are facing increasing demands in the teaching of probability and statistics (stochastics) at school level, we are interested in analyzing current pre-service teachers' dispositions about content knowledge and attitudes towards the teaching and learning of stochastics. We implemented a quantitative study for a sample of 269 pre-service Chilean mathematics teachers to determine their understanding of stochastics content, their attitudes towards stochastics and its teaching, and whether these are related. We found weak associations overall, but stronger for some components. We conclude with recommendations based on these results to improve the Chilean teachers' preparation process (pre-service) and advice that could guide the professional development of teachers (in-service).

Keywords: *Statistics education research; Content knowledge; Attitudes; Pre-service teachers*

1. INTRODUCTION

Recently, stochastics knowledge has been considered a necessary part of the cultural heritage in order to function effectively in society (Batanero & Borovcnik, 2016; Ben-Zvi & Makar, 2016). We use the term *stochastics* to refer simultaneously to statistics and probability, and thus to emphasize the mutual dependence of knowledge and reasoning between these two domains (Batanero, 2019). The incorporation of stochastics education into the mathematics curricula of many countries has boosted its teaching at different levels of compulsory education (for example, Chile: Ministry of Education [MINEDUC], 2009, 2012, 2015; and U.S.: National Council of Teachers of Mathematics [NCTM], 2000). Such curricular modifications have established new demands and challenges for the teachers responsible for teaching statistics and probability. Adding stochastics to the middle and high school mathematics curricula requires different knowledge at a didactic and disciplinary level, as well as changes in teacher disposition and interest in teaching stochastics. The literature suggests, however, that, in general, teachers are not sufficiently prepared, and many do not feel competent in their abilities, to teach stochastics (Batanero at al., 2011; Groth & Meletiou-Mavrotheris, 2018).

Students, in this case pre-service teachers, react positively or negatively to a stochastics learning situation depending on their beliefs about themselves and about the content (Estrada, 2002). If the situation is repeated several times, producing the same type of emotional reaction (frustration, satisfaction, etc.), this could become an attitude (Gómez-Chacón, 2000). Negative attitudes could, in turn, impact teachers' willingness to address the study of statistical content outlined in the curriculum, and their willingness to improve their understanding of the content (Goldin et al., 2016). As Groth and Meletiou-Mavrotheris (2018) hightlighted, there are several attitude-related hypotheses awaiting strong empirical support. For example, in studies of the impact of attitudes on teachers' content knowledge, researchers have found moderate to low associations between these aspects (Estrada, 2002; Hannigan et al., 2013; Nasser, 2004; Zientek et al., 2011). Negative attitudes toward statistics appear to be clearly detrimental, but there seems to be a limit on the extent to which positive attitudes relate to increased knowledge (Hannigan et al., 2013). Groth and Meletiou-Mavrotheris (2018) claimed that one possible cause of limited evidence of correlations between attitudes and knowledge among pre-service and in-service teachers is the lack of instruments designed specifically for teachers.

Therefore, in this paper we are interested in exploring further the possible connections between stochastics content knowledge and attitudes towards stochastics and its teaching. We wanted to see whether a lack of association between knowledge and attitudes, as found in Hannigan et al. (2013), also applies to pre-service Chilean mathematics teachers. Following the recommendations of Groth and Meletiou-Mavrotheris (2018), we utilized scales designed specifically for mathematics teachers. In the following sections we discuss the theoretical foundations of our study (Section 2), our sample and the instruments used to assess pre-service teachers' content knowledge and attitudes towards stochastics and its teaching (Section 3), and our results (Section 4). We end with discussion (Section 5) of how these results compare to previous research, and how they guide our recommendations for Chilean mathematics teachers' training and professional development. We chose to conduct our study in Chile to contrast with previous research and to take advantage of the access we had to half the institutions that train mathematics teachers in that country, as a result of their interest in our research. Therefore, our results can provide an example of assessing the current state of pre-service teaching, as well as serve as a baseline for future comparisons as teacher preparation programs adapt new curricular demands similar to those placed on Chilean teachers.

2. FRAMEWORK

2.1. CONTENT KNOWLEDGE

To explore teachers' content knowledge, we used the teachers' Didactic-Mathematical Knowledge [DMK] model (Godino, 2009; Pino-Fan & Godino, 2015), which is based on the theoretical-methodological tools of Onto-Semiotic Approach (OSA; Godino et al., 2007), and whose application to statistics is developed in Godino et al. (2011). The DMK model characterizes the teachers' knowledge from three dimensions: (1) the Mathematical Dimension, related to content knowledge; (2) the Didactical Dimension, related to pedagogical content knowledge; and (3) the Meta Didactic-Mathematical Dimension, related to knowledge about norms, meta-norms, and reflection on practice. The Mathematical Dimension is built on the perspectives of Shulman (1986, 1987) and the Learning Mathematics for Teaching [LMT] project (Ball et al., 2008; Hill et al., 2008), and common frameworks in mathematics education (Ponte & Chapman, 2016).

In the case of statistics, the trend has been similar. For example, Callingham and Watson (2011) built their Teacher Knowledge for Teaching Statistics model using Shulman's perspective. Burgess (2011) and Groth (2007) recognized two types of content knowledge necessary for teaching statistics, *Common Knowledge* and *Specialized Knowledge.* Subsequently, Groth (2013) added a third component to the subject matter knowledge dimension of his model called Horizon Knowledge, from LMT perspective. Lee et al. (2014) proposed the Technological Pedagogical Statistical Knowledge model, where the *Statistical Knowledge* is referred to the understanding of why and how statistical investigations are carried out, from Wild and Pfannkuch's (1999) perspective. Nevertheless, in analyzing content knowledge, these proposals have some drawbacks. For example, Groth (2013) recognized the three types of subject matter knowledge in the LMT framework, but it can be difficult to tease apart specialized knowledge from aspects of Pedagogical Content Knowledge [PCK], like knowledge of content and students (Callingham & Watson, 2011). Similarly, distinguishing between specialized and common knowledge is not always trivial (Groth, 2013; Noll, 2011), and the perspective of Lee et al. (2014) is not specific regarding the development of statistical thinking.

In the DMK model, a clearer differentiation is established regarding the content knowledge necessary to teach statistics, as the theoretical perspective of this model assumes that knowledge for teaching is entirely specialized and unique to the teacher (didactical dimension), but it is sustained on the basis of solid content knowledge (mathematical dimension), and together enable teachers to reflect on their practice of teaching (meta didactic-mathematical dimension; Godino, 2009; Pino-Fan & Godino, 2015). The Mathematical Dimension considers two types of knowledge, common and extended, as illustrated in Table 1.

Table 1. Mathematical dimension categories of DMK model

Category	Description
Common Content Knowledge	Mathematical knowledge sufficient to solve problems and tasks covered in the Chilean mathematics curriculum and textbooks from grades 7 to 12 (MINEDUC, 2012, 2015, 2019)
Extended Content Knowledge	Knowledge of the mathematics content that comes later in the school curriculum, e.g., knowledge of stochastics beyond grade 12, and is included in content standards for teachers (MINEDUC & CREIP, 2012)

The application of the DMK model (Table 1) is based on the need to determine the knowledge that a mathematics teacher should possess on specific topics to be taught at some specific school grades (Pino-Fan et al., 2015). The *common* term is used in a different way than Ball et al. (2008), as it is a shared knowledge between the teacher and the students, so it is common between both of these constituents, not generally with everyone. At the same time, this interpretation is different from that conceptualized by Groth (2007) because it is referring to the knowledge of school statistics specific to the grade in which it is taught, and not to the skills developed in conventional courses in general. Regarding *extended* knowledge, our interpretation reformulates the LMT framework and Groth's perspective on Horizon Knowledge by differentiating based on educational level. For more details on this characterization, see Pino-Fan et al. (2018) in the context of calculus. In our case, when referring to the statistical training of teachers, we will differentiate between common and extended knowledge with respect to the current school curriculum.

2.2. ATTITUDES

We used McLeod's (1992) perspective on the affective domain, which "refers to a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition" (p. 576). We used the term affect in general and considered emotions, beliefs, and attitudes as their main descriptors, although our focus of interest was the latter. In addition, as there is no unanimity in the definition of attitude (Estrada, 2002; Estrada et al., 2011; Gómez-Chacón, 2000; McLeod, 1992; Philipp, 2007), we adopted the definition proposed by Philipp (2007):

> [Attitudes are] manners of acting, feeling, or thinking that show one's disposition or opinion. Attitudes change more slowly than emotions, but they change more quickly than beliefs. Attitudes, like emotions, may involve positive or negative feelings, and they are felt with less intensity than emotions. Attitudes are more cognitive than emotion but less cognitive than beliefs. (p. 259)

There is a large amount of research related to the assessment of students' attitudes to predict their academic performance in stochastics courses, and to monitor attitudinal changes before and after instruction (e.g., Carmona, 2004; Nolan et al., 2012; Veloo & Chairhany, 2013). Literature on teachers' attitudes, both pre- and in-service, however, is scarce (Estrada et al., 2011). Furthermore, Groth and Meletiou-Mavrotheris (2018) highlighted that the field has not yet made a strong distinction between "teachers' attitudes toward statistics" and "attitudes toward teaching statistics." Usually, those who analyze teachers' attitudes *towards statistics* follow some generic theoretical approach that can be applied in the general population (Estrada et al., 2011; Hannigan et al., 2013, Nasser, 2004; Zientek et al., 2011). While those who have been interested in teachers' attitudes *towards teaching statistics* have used frameworks that consider the specific characteristics of teachers' tasks when teaching statistics (Aparicio & Bazán, 2006; Estrada et al., 2004; Martins et al., 2015).

In our efforts to assess teachers' attitudes we followed the theoretical perspective of Estrada et al. (2018) who recognized that teachers' attitudes are a multidimensional construct that can be analyzed according to seven components of attitudes, organized in three dimensions. On this basis, Estrada et al. proposed the Attitudes' Scale Towards Probability and its Teaching (ASTP; Estrada & Batanero, 2015; Estrada et al., 2018). This scale is based on the earlier Scale of Attitudes towards Statistics for Teachers (EAEE; Estrada, 2002), but has better psychometric characteristics (Estrada & Batanero, 2020; Ruz et al., 2020d), and most importantly, the ASPT perspective separately assesses teachers' attitudes towards the content (dimension 1) and towards its teaching (dimension 2), as well as their interaction through the value assigned to both (dimension 3). Previous research using the ASTP has shown mostly positive attitudes in pre-service teachers' from elementary (Vásquez et al., 2019) and primary school (Estrada & Batanero, 2020; Estrada et al., 2018), and with pre- and in-service mathematics' teachers (Alvarado et al., 2018; Ruz et al, 2020d).

Finally, agreeing with Estrada et al. (2018), we considered it important to identify the attitudes of pre-service teachers about the topics they will teach. Therefore, following the way in which stochastics content is organized within the Chilean mathematics curriculum, and in most of stochastics courses for teachers, we differentiated between three content areas: descriptive statistics [D], probability [P], and statistical inference [I], to delve into whether current programs are more successful with some components than others. In order to do this, we expanded the instrument to address each content area separately by asking the same questions for each area, according to the components presented in Table 2.

Table 2. Description of the components of attitudes considered

Dimension	Component	Description/Example item
1. Attitudes toward Stochastics content	Affective [AS]	Feelings, positive or negative, towards stochastics content (e.g., I enjoy descriptive statistics [D]).
	Cognitive Competence [CCS]	Self-perception with regards to self-competence, knowledge, and intellectual skills when applied to stochastics content (e.g., Probability is easy [P]).
	Behavioural [BS]	Tendency to use stochastics tools when convenient (e.g., I use inferential statistics tools in everyday life [I]).
2. Attitudes towards teaching Stochastics content	Affective [AT]	Personal feelings, positive or negative, about teaching the content (e.g., I am sure I will enjoy teaching about descriptive statistics in the school [D]).
	Didactical Competence [DTS]	Teacher's perception of own ability to teach the content (e.g., It will be hard for me to teach probability [P] (reverse scored)).
	Behavioural [BTS]	Values the tendency to teach or have taught the content compared to other materials of the school curriculum (e.g., I will only teach inferential statistics [I] if there is time after teaching the other topics (reverse scored)).
3. Value of the content and its teaching	Value [VTS]	Appreciation of the usefulness, relevance, and importance of stochastics content and its teaching in personal and professional life (e.g., Probability [P] is only useful for games of chance (reverse scored)).

2.3. CONTENT KNOWLEDGE AND ATTITUDES

As Tishkovskaya and Lancaster (2012) pointed out, content knowledge and a positive attitude towards statistics are both critical for effective teaching. Most of the studies that have evaluated these content knowledge and attitudes simultaneously have done so in distinct ways. Teachers' content knowledge has been commonly assessed according to the grades earned in stochastics courses, or with knowledge tests such as the Statistical Reasoning Assessment (SRA; Garfield, 2003) or the Comprehensive Assessment of Outcomes in Statistics (CAOS; delMas et al., 2007). With attitudes, the trend has been to use some of the classic instruments, such as the Survey of Attitudes Toward Statistics (SATS-28; Schau et al., 1995; SATS-36, https://www.evaluationandstatistics.com/), or to propose a new instrument that also considers attitudes towards teaching, such as the scale EAEE for teachers (Estrada, 2002). In Table 3, we summarize the results from several key studies.

Table 3. Main research results on knowledge and attitudes towards statistics in teachers.

Reference	Participants	Knowledge	Attitudes	Correlations (attitude component)
Estrada (2002)	367 pre-service Spanish teachers	9 items of SRA test (mean of 12.1 out of 19 points)	SATS-28 (total mean score of 88.7 out of 140 points)	0.09 (difficulty) to 0.26 (cognitive), and 0.23 (total)
Nasser (2004)	162 pre-service Israelite teachers	10 open-ended questions (mean of 85.38 out of 100 points)	24 items of SATS-28 (weighted average of 4.66 out of 7 points)	0.11 (difficulty) to 0.28 (cognitive)
Aparicio and Bazán (2006)	87 Peruvian teachers	Performance in an introductory statistics course	EAEE (mean of 83.49 out of 125 points)	0.22 (total in post-test)
Zientek et al. (2011)	95 pre-service US teachers	Performance in a professional development program (87.03 out 100 points)	SATS-36 (weight mean of 3.29 out of 5 points)	0.14 (difficulty) to 0.49 (affective)
Hannigan et al. (2013)	104 pre-service Irish teachers	CAOS test (mean of 45 out of 100 points)	SATS-36 (weight mean of 4.94 out of 7 points)	-0.02 (effort) to 0.19 (cognitive)

For most of these studies, findings on the association between attitudes and content knowledge of teachers have been elusive. Estrada (2002) and Aparicio and Bazán (2006) reported low correlations (0.22 and 0.23, respectively) among participants with (on average) moderate responses in both scales. Nasser (2004) and Zientek et al. (2011) report higher mean achievement in content knowledge by their participants (mean of 85-87% of achievement) and average attitudes slightly higher than indifference's theoretical value, but the magnitude of the association between both content and attitudes differed between them. For example, in the affective component (feelings concerning statistics) shared by both SATS-28 and SATS-36 scales, Nasser reported a weak correlation of 0.17 whereas Zientek et al. reported a value of 0.49. This variation in results could be explained by their use of different measures of content knowledge. The pre-service teachers studied by Hannigan et al. (2013) had lower content knowledge (mean of 45% of achievement) but mostly positive attitudes, which translated into a weak association. In summary, previous research highlights that good content knowledge is strongly and positively related to positive attitudes towards statistics. The association loses intensity; however,when content knowledge is sufficient (slightly higher than the expected value) and the attitudes are more indifferent, or when knowledge is deficient, even when pre-service teachers attitudes are mostly positive.

In this way, stronger content knowledge seems to be a reasonable predictor of positive attitudes towards statistics, but not vice versa. Therefore, we were interested in exploring these same associations with a sample of pre-service Chilean mathematics teachers. Knowledge of whether/how these aspects are related can have implications for teachers' training and professional development. In particular, our research questions are:

1. What is the stochastics content knowledge of pre-service Chilean mathematics teachers, and does it differ for Common and Extended knowledge and/or content areas considered?

2. What is the nature of pre-service Chilean mathematics teachers' attitudes towards stochastics and its teaching? Are there attitudinal differences depending on the content area considered?
3. Is there a relationship between stochastics content knowledge and attitudes towards stochastics and its teaching? Do these associations differ within the three content areas and/or based on previous training?

3. METHOD

3.1. CONTEXT AND PARTICIPANTS

Chilean applicants for teaching positions must have a teaching qualification from a university or a professional institute appropriate for the level at which they plan to teach. The General Education Law defines teaching qualifications in terms of a licentiate degree (equivalent to a bachelor degree) in education, and a teaching entitlement. In most institutions, the requirements for the degree and the entitlement are completed simultaneously following a *concurrent model*, and this degree is called a Pedagogy in Mathematics degree. A smaller number of Chilean institutions follow the *consecutive model*, with one to two years continuing studies programs for graduate students (OECD, 2014). The teacher preparation process is regulated by the MINEDUC through the Center for Improvement, Experimentation, and Pedagogical Research (CPEIP). For example, for the 2019 academic year, a total of 30 Chilean universities offered the Pedagogy in Mathematics degree (www.cned.cl), where usually each institution formulates their own study plans, generating a varying set of courses and training opportunities. Nevertheless, since 2012, MINEDUC and CPEIP have established national standards on the fundamental knowledge and skills that must be promoted by the centers to train effective teachers for grades 7 to 12 (Franklin et al., 2015; MINEDUC & CPEIP, 2012). From this perspective, every year since 2016 they have organized the National Diagnostic Evaluation of Teachers' Initial Training (ENDFID) for all pre-service teachers who are twelve months away from graduation to diagnose their initial training.

The full sample for this study consisted of 269 Chilean pre-service mathematics teachers (150 men and 119 women, aged between 19 and 51, with an average age of 23.8 years), enrolled in 15 of the 30 universities in Chile that prepare pre-service mathematics teachers. Although the sampling was not probabilistic, we attempted to match the graduation rates of the geographical areas of the country, based on the announcement from the ENDFID (www.diagnosticafid.cl). In this sense, the 269 participants can be organized according to the geographical area: North = 18 (6.7%); Central = 177 (65.7%); South and Austral = 74 (27.5%). Regarding prior training, all the participants had taken all the required courses covering the stochastics content in their curricula, but the number of required courses differed across the universities, and not all of the courses taken had been approved (passed). In Table 4, we present the number of courses passed for the participants, according to the number of courses required in their respective universities.

Table 4. Number (and %) of participants according to the number of courses required.

Number of required courses	Passed courses 0	1	2	3
0 (n = 8)	0 (0)	2 (25.0)	2 (25.0)	4 (50.0)
1 (n = 36)	5 (13.9)	31 (86.1)	-	-
2 (n = 195)	10 (5.1)	23 (11.8)	162 (83.1)	-
3 (n = 30)	0 (0)	2 (6.7)	7 (23.3)	21 (70.0)

The first row of Table 4 applies to the one university in our sample following the *consecutive model*. This graduate program does not require stochastics training, so the students reported the number of courses they passed as undergraduates. The second row shows that among those who have one required course in their *concurrent* model, five (13.9%) have taken the course but have not yet passed it, and 31 (86.1%) did. In addition, for those who have at least one required course in their university (n = 261), approximately 82% have already passed all courses in the planned stochastics content.

We used this classification to organize the participants according to their previous training, but the interesting thing is the content of these courses. Thanks to the collaboration of the institutions that participated in this study, we had access to the details of their stochastic training programs. Participants with one required course were enrolled in universities from the north (50%) and center (50%) geographical zone of the country. This individual course focuses on exploratory data analysis, with slight coverage of classical probability, and culminates with a brief discussion of parameter estimation. The group with two required courses is the largest. Most of these participants were enrolled in a university from the central zone of the country (69.7%), while the rest were part of institutions from the south zone (30.3%). In these institutions, the first course is intended to introduce exploratory data analysis and classical probability theory, while the second focuses on the modeling of random phenomena through probability distributions and culminates with statistical inference elements such as sampling distributions and parameter estimation. Among those who had taken three courses on stochastics, half came from institutions in the south of Chile, and the rest from the center area. In these cases, the first course is intended to develop skills to explore and describe data, the second focuses on probability from a more mathematical perspective in the framework of measurement theory and finite mathematics, while the third focuses entirely on statistical inference. In this last course, in addition to reviewing the classical estimation methods, the course culminates with some non-parametric estimation techniques.

3.2. INSTRUMENTS

As motivated in Section 2, the instrument we developed consisted of two sections, the Scale of Common and Extended Stochastics Content Knowledge (CESK) and the Scale of Attitudes towards Stochastics Contents and its Teaching (ASCT).

CESK scale For the design of this instrument, we followed the recommendations of Batanero and Díaz (2005). Prior research about teachers' content knowledge has highlighted the need to promote understanding of key stochastics concepts (Pfannkuch & Ben-Zvi, 2011; Ruz et al., 2018). Much of this research has focused on the understanding of specific stochastics objects (Ben-Zvi & Makar, 2016b; Garfield & Ben-Zvi, 2008), where difficulties and deficiencies have been reported systematically, showing that this content can be difficult for novice students as well as pre-service teachers. This motivated us to design an instrument that considered various key topics of stochastics, such as found in the SRA (Garfield, 2003) and the CAOS test (delMas et al., 2007), but built from the recommendations on the content knowledge that should be promoted in the stochastics training of mathematics teacher.

We carried out a Content Analysis (Krippendorff, 1990) of the local and international requirements on the statistical education of teachers (Franklin et al., 2015; MINEDUC & CPEIP, 2012). Results were organized in the *Guide of Didactic Suitability of Instructional Processes in Statistical Didactics* (GDS-ISD; Ruz et al., 2019), where the Content Knowledge requirements were synthesized through 23 indicators included in the *Statistical content* component from the Epistemic dimension of the GDS-ISD. Some indicators were not considered in this assessment, such as teachers knowing and applying the statistical problem-solving process (Franklin et al., 2015), as these indicators were not consistent with the current Chilean training programs. Traditionally these programs have been based on accumulation of concepts and procedures, rather than a focus on problem solving and data wrangling. We obtained a total of 20 stochastics content learning objectives for mathematics teachers' education (Appendix 1).

With these 20 learning objectives, we used previous literature and instruments to build an item bank, from which we selected a set of three items for each learning objective, 60 in total. To analyze the degree to which the item effectively represents what is intended to be measured (evidence of content validity), we contacted ten experts in Statistics Education (all with PhDs and more than seven years of research experience in this area) to evaluate, on a five-step Likert scale (1-*very inappropriate* to 5-*very appropriate*), the relationship between an item and the desired learning objective, and to verify that the answers were correct. Among the three items for each learning goal, we selected the one with the highest mean agreement score and lowest variability among the experts' ratings using the Fleiss Kappa index of 0.605, *moderate-substantial* agreement (Landis & Koch, 1977). This gave us the 20 items for the first version of the CESK scale.

Subsequently, we applied this version of the scale to a pilot sample of 126 pre-service math teachers from Chile (21 men and 21 women) and Spain (49 men and 35 women) (Ruz et al., 2020a). Items ranged from easy to difficult (percentage of correct answers between 10% and 90%) and showed adequate discrimination (biserial correlation coefficient higher than 0.2) and sufficient reliability (Cronbach's alpha = 0.564). From these results, we adjusted aspects of the implementation, such as the time allowed, as well as the instrument. In particular, we modified some response options to make sure only one was correct for each item and also altered some unpopular distractors.

Finally, each item was classified according to the stochastic content area (descriptive statistics [D], probability [P], and statistical inference [I]), and to the DMK content knowledge category (Common or Extended). For example, the first item assesses the objective “Match different representations to explore, summarize, and describe patterns in univariate data,” and should be solvable by grade 8 students, where “evaluate the way the data is presented, comparing the information from the same data represented in different representations" (MINEDUC, 2015, p. 115) is promoted – common knowledge. In contrast, the second item assesses the ability to “estimate the correlation coefficient between two variables based on their graphical representation,” which is not part of the school curriculum, but is promoted in teacher training standards such as "calculate and interpret the correlation coefficient in linear regression and understand the meaning of influential point and atypical point" (MINEDUC & CPEIP, 2012, p. 124)–extended knowledge. The final version of CESK scale is included in Appendix 2.

ASCT scale The ASCT scale consists of 28 items, four items in each of the seven components of the ASTP, each offered on a five-step Likert scale (from 1-*strongly disagree* to 5-*strongly agree*). Each item was written separately for each of the three stochastics content areas (Appendix 3), and respondents selected their choices for each area on a separate answer sheet (for example see Figure 1).

ITEM	DESC. STAT.					PROBABILITY					INFERENCE				
I enjoy the lesson in which Descriptive/ Probability/ Inferences contents is explained	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5

Figure 1. Fragment of ASCT scale' answers sheet

Fourteen items are expressed affirmatively (e.g., “I enjoy the lesson in which statistics/probability/inference contents is explained” that is part of the AS component) and the other 14 are presented negatively (e.g., “If I could remove some content from the math curriculum it would be statistics/probability/inference” from the BTS component). Scores on the negative items were reversed before analysis, so higher scores always correspond to more positive attitudes. In the pilot study, the scales had considerable reliability (Pedhazur & Pedhazur, 1991; Santiesteban, 2009) for each content area (Cronbach' alpha of 0.899 [D], 0.879 [P], and 0.856 [I]; Ruz et al., 2020b).

4. RESULTS

4.1. CONTENT KNOWLEDGE

Overall, the scores (Figure 2) obtained in the questionnaire ranged from 0 to 13 points (out of 20 possible), with a mean of 5.29 points and a standard deviation of 2.72 points. Cronbach's alpha, 0.571, was slightly higher than in the pilot study and still sufficient for this type of evaluation (Pedhazur & Pedhazur, 1991; Santiesteban, 2009).

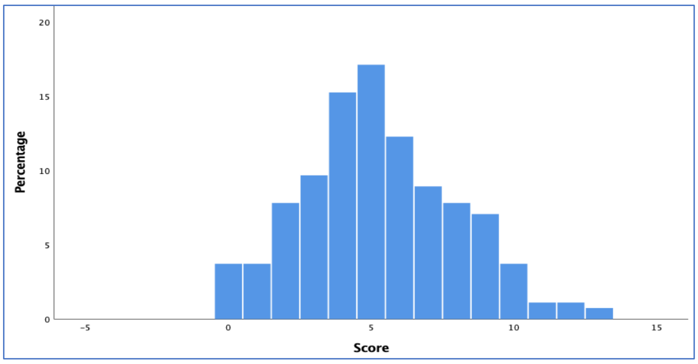

Figure 2. Histogram of the scores obtained in the CESK scale.

The proportion of correct answers for each item ranged from 0.05 for item 15, which evaluated an application of Bayes' theorem, to 0.59 for item 11, which evaluated the ability to compare the distribution of data between two groups. Next, we explore the proportion of correct responses out of the 6-7 items for each content area (Figure 3a), and out of 10 items for each DMK content knowledge categories (Figure 3b).

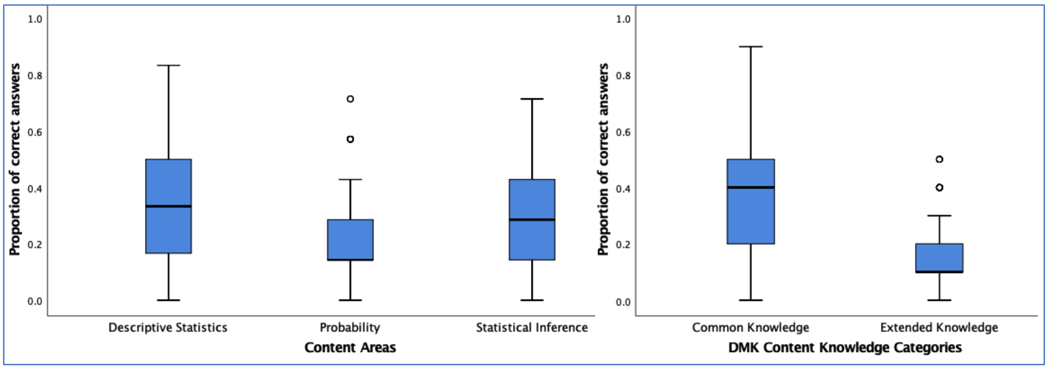

Figure 3. Boxplots for the proportion of correct answers according to (a) Content Areas, and (b) DMK categories

According to the content areas (Figure 3a) we note a higher median score (0.333) for the items on descriptive statistics, and the lowest median score for the probability content (0.142). As we expected, results were higher for common knowledge with a median of four correct questions out of 10, and lower for the extended knowledge, with a median of just one correct item (Figure 3b). We also found lower variability for the probability content and the extended knowledge category respectively; that is, in general, our participants demonstrated consistently low performance across these items. In Table 5, we compare our results on the 10 items of *common content knowledge* with the original source, reporting the number (and percentage) of subjects who answered the question correctly on each case.

Table 5. Number (%) correct for Common knowledge items according to previous and our studies

Item	Area	Reference and item position in previous research	Previous results	Our results (n = 269)
1	D	delMas et al. (2007), item 33	303 (41.5%)	112 (41.6%)
3	P	Wroughton & Cole (2013), item 1, 3, and 9	36 (28.6%)*	65 (24.2%)
4	P	Gómez et al. (2014, p. 214)	66 (82.5%)	119 (44.2%)
5	I	Allen (2006, p. 418), item 11	71 (42.0%)	135 (50.2%)
6	D	Ruz et al. (2020a), ítem 5	75 (59.5%)*	137 (50.9%)

9	I	Olivo (2008, p. 174), item 4	108 (42.9%)	90 (33.5%)
10	I	delMas et al. (2007), item 35	318 (44.2%)	56 (20.8%)
11	D	delMas et al. (2007), item 11, 12, and 13	642 (85.3%)	158 (58.7%)
13	P	Contreras (2011, p. 179), item 2	63 (32.1%)	55 (20.4%)
20	P	Tauber (2001, p. 181), item 12	77 (77.8%)	60 (22.3%)

*Data from our pilot study. Previous source did not report empirical results.

Our pre-service teachers struggled the most with an item asking to distinguish dependent, independent, and mutually exclusive events (item 13, probability), with 20.4% of subjects responding correctly. These results were lower than those reported by Contreras (2011) with 196 Spanish primary pre-service teachers (32.1% correct). The modal response, with 27.5%, corresponded to the misconception of confusing exclusive and independent events. Our subjects, however, showed better performance when comparing the variability between groups through an exploratory data analysis (item 11, descriptive statistics), although the results were also lower than those obtained in the post-instruction application of the CAOS test with 753 American university students (item 12, 85.8% correct). One fifth of our subjects demonstrated the misconception that groups cannot be compared when the sample sizes are not equal (20.4% of responses).

The greatest variation between the previous study and our results occurred on the item evaluating the ability to recognize the characteristic shape of the Normal curve (item 20, probability). Our subjects (22.3%) showed worse performance than the 99 Spanish university students of Human and Social sciences analyzed by Tauber (2001; 77.8% responding correctly). Most of our pre-service teachers (26.0%) chose the option of a wrong interpretation of the empirical rule (three-sigma rule) citing that for a normal distribution 50% of the observations will fall within one standard deviation of the mean, instead of 68.3 %. In Table 6, we compare our results with those previously studies for the 10 items evaluating *Extended Content Knowledge*.

Table 6. Number (%) correct for Extended knowledge items according previous and our studies

Item	Area	Reference and item position in previous research	Previous results	Our results (n = 269)
2	D	Allen (2006, p. 420), item 15	161 (52.5%)	56 (20.8%)
7	I	Vallecillos (1994, p. 181), item 3	298 (68.6%)	86 (32.0%)
8	P	Contreras (2011, p. 183), item 6	46 (23.5%)	34 (12.6%)
12	I	Own elaboration	-	24 (8.9%)
14	D	delMas et al. (2007), item 14	256 (34.3%)	74 (27.5%)
15	P	Contreras (2011, p. 178), item 1	15 (7.7%)	13 (4.8%)
16	I	delMas et al. (2007), item 25, 26, and 27	388 (54.5%)	40 (14.9%)
17	D	delMas et al. (2007), item 22	391 (52.6%)	47 (17.5%)
18	P	Cruz-Ramirez et al. (2014, p. 1127)	24 (19.1%)*	42 (15.6%)
19	I	delMas et al. (2007), item 28, 29, and 30	535 (74.3%)	22 (8.2%)

*Data from our pilot study. Previous source did not report empirical results.

For these 10 items, our participants showed the lowest performance on the item about Bayes's theorem (item 15, probability) with only 4.8% answering correctly. These results were slightly lower that those reported by Contreras (2011) with Spanish primary prospective teachers (7.7%). Our subjects mostly showed the misconception of confusing conditional and joint probability (selected by 52.0% of the full sample). Better performance was shown when subjects were asked to establish the statistical hypothesis from a research question (item 7, statistical inference) with 32% answering correctly. This result was lower that those reported by Vallecillos (1994) with 436 Spanish university students (68.6% correct). In this case, 33.4% of participants make the mistake of setting hypothesis in the wrong way (33.4% of selections). Item 12 was designed by the authors, but it underwent changes after its pilot implementation, so we do not have prior information on its use.

In addition, the item with the greatest variation between the prior study and our results corresponds to the item differentiating between correct and incorrect interpretations of confidence level (item 19, statistical inference). Our participants (8.2%) showed worse performance than the 720 American university students analyzed by delMas et al. (2007). It is interesting to note that in the original CAOS instrument, 74.3% of students could recognize a correct interpretation as valid, but had more difficulty recognizing incorrect interpretations as invalid. When forced to select only one version as correct (as in our instrument), participants struggled substantially. Many of our participants misinterpreted level of confidence as the percentage of the sample between the confident limits (32.3% of selections).

4.2. ATTITUDES

First, as shown in Table 7, we examined the total attitude score (across the 28 5-point Likert items) for each content area (max possible 140). Then we averaged the three content areas for a global measure of attitudes towards stochastics and its teaching (again max possible 140). We only consider cases where no response was omitted, which explains the different values of *n* in Table 7.

Table 7. Distribution of ASCT scale according to content area results

Content	*n*	Median	Mean	*SD*	Percentile 20
Descriptive Statistics	242	103.50	104.61	13.87	93.00
Probability	237	107.00	105.57	12.88	94.00
Statistical Inference	232	99.00	99.00	12.48	89.00
Global (Mean)	230	102.17	103.34	11.36	93.33

The global measure of attitudes towards stochastics and its teaching (last row of Table 7) ranged from 75 to 132 points on average, with a mean of 103.34 points and a standard deviation of 11.36 points. From this we observe that most participants declare positive attitudes towards each of the content areas, with more than 80% of our prospective teachers giving total scores that exceed the theoretical value of indifference (84 points, for answering 3 for all 28 items). We also note that the median and mean scores are higher for probability and lower for statistical inference, similar to the pilot study results (statistical inference mean 96.7 out 140 points, compared to 102.5 and 104.4 for descriptive and probability respectively). For reliability, we obtained a Cronbach’s alpha of 0.864, which is considered more than adequate (Pedhazur & Pedhazur, 1991; Santiesteban, 2009).

Next, we grouped the results according to the different attitude components (Table 2), using each student’s mean score obtained for the four items of each component (for a maximum response of 20 points) as is shown in Figure 4.

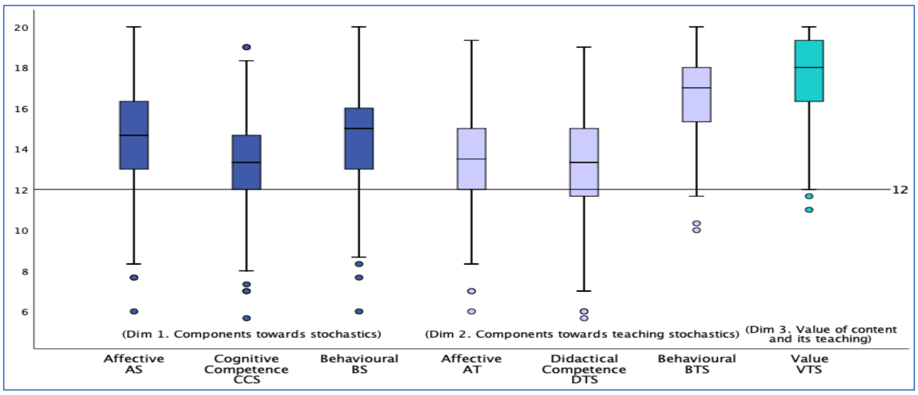

Figure 4. Boxplots on the average scores in each component of the ASCT scale

Summary statistics are presented in Table 8, accompanied by the number of valid responses in each case.

Table 8. Summary statistics of the components and the total ASCT scale scores.

Dimension	Attitudes' components	*n*	Median	*SD*	Percentile 30
1. Towards stochastics content	Affective [AS]	258	14.50	2.77	13.00
	Cognitive Competence [CCS]	260	13.00	2.52	12.00
	Behavioural [BS]	260	14.67	2.54	13.00
2. Towards teaching stochastics	Affective [AT]	263	13.33	2.37	12.00
	Didactical Competence [DTS]	256	13.00	2.68	12.00
	Behavioural [BTS]	257	17.00	1.98	16.00
3. Value of content and its teaching	Value [VTS]	256	18.00	2.05	16.70

From these results, we note that the attitudes of these participants were mostly positive throughout the seven components. Not only are the median scores above the indifference position, so are the 30th percentiles, indicating that at least 70% of the participants assigned scores of at least 12 points across the four questions for all seven components. The highest medians correspond to value towards stochastics content and its teaching (VTS) and behaviour towards teaching the content (BTS), as these pre-service teachers appeared to value the stochastics content in the curriculum and plan to teach it. On the other hand, the lowest medians correspond to the cognitive competence (CCS) towards the stochastics content and didactic competence (DTS) towards its teaching. These prospective teachers considered their intellectual mastery of stochastics content less positively, as well as their didactic ability to carry out the teaching.

4.3. CONTENT KNOWLEDGE AND ATTITUDES

Next, we analysed the possible relationships between knowledge and attitudes toward stochastics content and its teaching (averaged across the three content areas). As we can see in Figure 5, a generally positive association is observed: when the overall ASCT score is below average, the performance on the CESK scale was generally also lower.

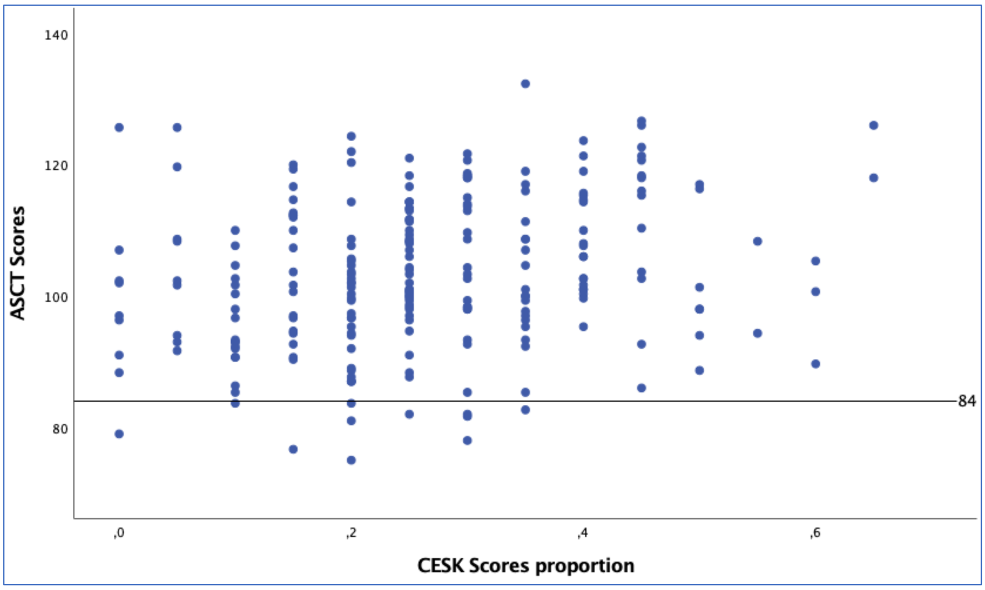

Figure 5. Scatterplot of ASCT and CESK proportions correct (with line at indifference attitude value)

To further the analysis, in Table 9 we present the Pearson correlation coefficients between the different attitude components, the DMK content knowledge categories, and the total scores of each scale.

Table 9. Correlation coefficients between the ASCT components and the CESK categories

ASCT Components	DMK content knowledge categories		
	Common [CKSC]	Extended [EKSC]	Total
Affective [AS]	0.155	0.143	0.181
Cognitive Competence [CCS]	0.133	0.133	0.161
Behavioural [BS]	0.238	0.081	0.213
Affective [AT]	0.133	0.072	0.131
Didactical Competence [DTS]	0.142	0.100	0.152
Behavioural [BTS]	0.075	0.043	0.075
Value [VTS]	0.127	-0.024	0.081
Total	0.222	0.164	0.239

Considering absolute correlations between 0.2 and 0.4 to be moderate, between 0.1 and 0.2 to be low, and below 0.1 to be inconclusive, we note that the overall score on the ASCT scale is moderately correlated with common knowledge and weakly correlated with extended knowledge (last row of Table 9). Also, a slightly positive linear relationship was obtained between the overall results of the CESK scale and each of the individual attitude components (last column of Table 9), although the intensity of these correlations was low in five of them (AT, DTS, CCS, AS, and BS), and inconclusive for BTS and VTS which, as shown in Table 8 were those with a larger average score and less dispersion. In other words, participants tended to rate the value and behavior of stochastics teaching highly, regardless of their performance in the content knowledge questions. For the other items, with less consistency in subjects' attitudes, there were weak but positive correlations with their knowledge.

When considering the correlations between the Common Knowledge (CKSC) category and the different components of attitudes (second column of Table 9), we noted positive low to moderate results for six of the attitude components, with a higher correlation for the BS component (those who better understand the content are more likely to apply the content in everyday situations). In other words, the participants with stronger common content knowledge tended to have more positive attitudes. The seventh inconclusive correlation refers to the behavioural component towards teaching the content (BTS). Subjects tended to give higher scores for their commitment in teaching the content, regardless of their knowledge of school stochastics.

When considering the results for the Extended Knowledge (EKSC) category and the components of the ASCT scale (third column of Table 9), the correlations were all lower for EKSC compared to CKSC, and conclusive for just two of them (AS and CCS). In other cases, even though we previously saw mostly positive results in the use of stochastics (BS), affective (AT), didactic competence (DTS), behaviour (BTS), and value (VTS) towards teaching the content, these attitudes do not appear strongly related to the participants' level of knowledge about the stochastics that go beyond the school curriculum (which was generally low). In this sense, given that the scores were so low for EKSC, the lack of correlations with ASCT components could have been due to a floor effect.

To delve into how or whether these associations differ within each content area, we explored the relationship between knowledge about the different content areas and the three dimensions of attitudes' components considered in Table 2 (towards stochastics contents, towards teaching stochastics, and value of content and its teaching). For each person, we looked at the total score of the six (descriptive statistics) or seven (probability and statistical inference) questions in that content area, and the corresponding attitude dimension's score towards each of the three content area (Figure 6).

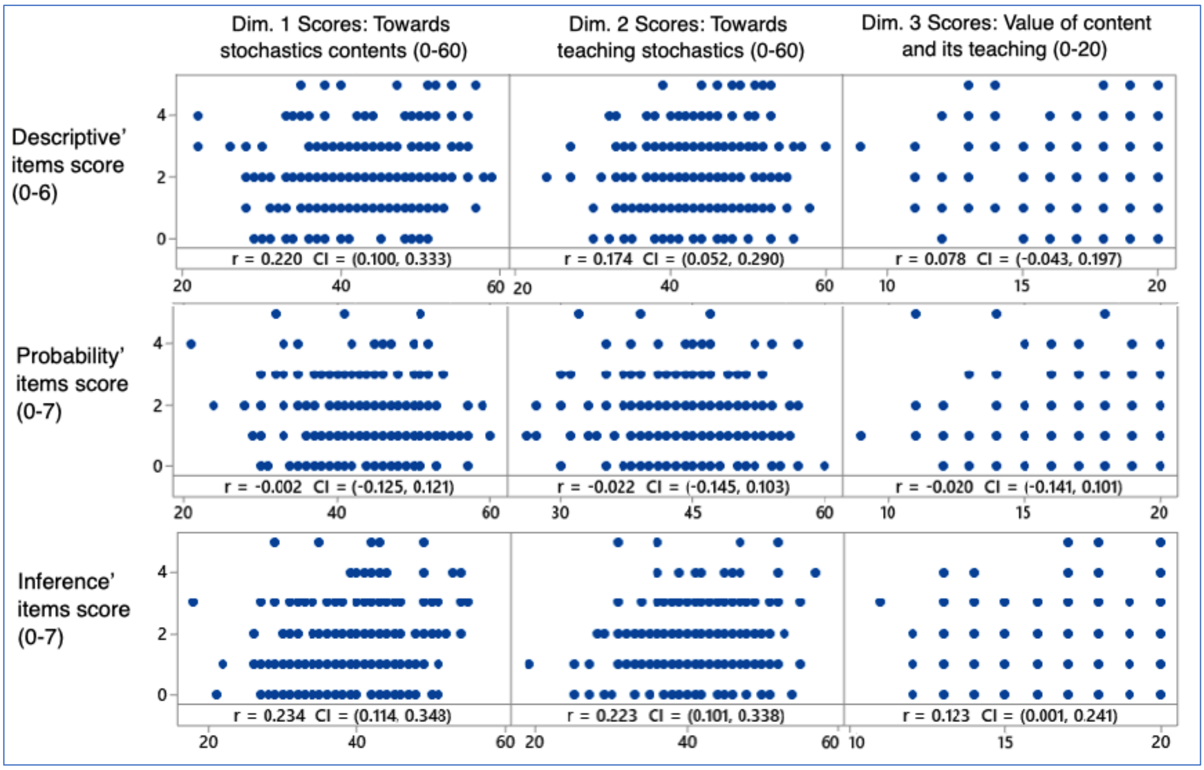

Figure 6. Scatterplots of CESK content area and ASCT scale dimensions scores

There was a moderate positive correlation (0.220) between the attitudes *towards* Stochastics-Descriptive Statistics content (total across 60 points in 12 items of first dimension) and the performance on the six Descriptive Statistics questions. This correlation was slightly lower with regard to their attitudes towards *teaching* Descriptive Statistics content (0.174), and inconclusive with regard to the *value* of Descriptive Statistics content and its teaching (0.078). In other words, participants who performed better on the Descriptive Statistics items tended to be more likely to learn that content and to teach it, but were not more likely to use it in their own lives (most students tended to value that content regardless of their own performance). In the probability area, the situation was totally different, where none of the correlations were conclusive, and actually show slightly negative coefficients. The first two attitude dimensions' patterns were different compared to those obtained in the descriptive statistics area but showed a similar trend in the value dimension. In this case, the subjects highly favored the *value* of probability to face everyday situations regardless of the knowledge demonstrated on the seven items in this content area. Participants tended to show more positive attitudes for probability (Table 7), but their performance on those items were generally low (Figure 3a). As with the results on EKSC, the lack of correlations could have been due to a floor effect. For the statistical inference question, we found a moderate positive correlation between the performance on the seven Statistical Inference questions and the attitudes t*oward* this content area (0.234), and the attitudes towards *teaching* Stochastics-Inference content (0.223). Participants' attitudes towards learning and teaching inference content tended to be higher for those who performed stronger on the content questions. In the value dimension, the behavior was similar to the descriptive statistics questions, although in this case the correlation was slightly higher. That is, our subjects valued inference content and its teaching regardless of their performance in the knowledge items on this topic. Finally, in Figure 7 we explored the relationship between knowledge and attitudes towards stochastics according to the previous training of the participants (Table 4).

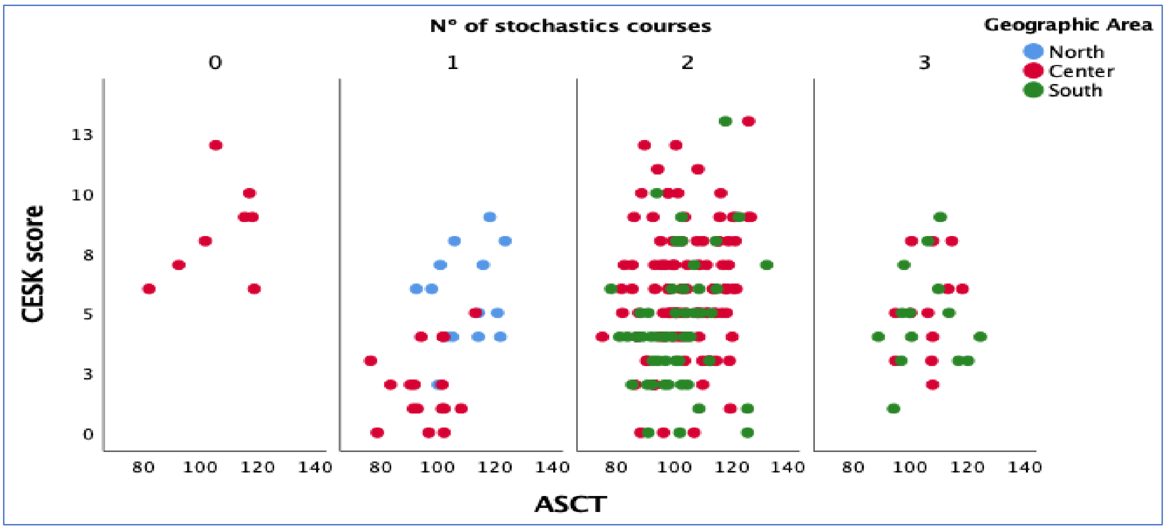

Figure 7. Scatterplots of CESK and ASCT scores according to number of required courses

In Table 10 we present some summary statistics about the total score in CESK scale, the global measure of attitude from the ASCT scale, and the relationship between them, separated according to the number of stochastics courses required in the initial training of the participants.

Table 10. Summary statistics of CESK and ASCT scores, according to previous training

Required	CESK scale (0-20)			ASCT scale (0-140)			Relationship	
courses	*n*	Mean	*SD*	*n*	Mean	*SD*	*n*	Pearson corr.
0	8	8.38	2.07	8	106.29	13.60	8	0.387
1	36	3.94	2.73	31	102.19	11.98	31	0.545
2	195	5.46	2.69	165	103.06	11.47	165	0.184
3	30	5.00	2.12	26	105.60	9.12	26	0.151

The group of students with zero required courses in stochastic content correspond to the participants who followed the *consecutive* model of initial training, belonging to a university in the central zone of Chile. These subjects had higher knowledge scores and higher attitude scores, and the correlation between them was stronger than previously reported (Table 9). That is, most of the participants (except for a couple of cases, see Figure 7) with better performance on the knowledge scale expressed more positive attitudes towards stochastic and its teaching, or vice versa; but this is for just eight subjects. Participants with one required course showed the lowest mean scores on the CESK and ASCT scale, but the highest correlation of those reported in this study (0.545). We can observe a more defined linear upward trend for these subjects (Figure 7), reflecting that most of those with worse performance on the knowledge scale declared lower positive attitudes on the ASCT scale (and vice versa). The largest group is the one with two required courses. These participants obtained the highest mean score on the CESK scale, and the mean score on the ASCT scale was slightly higher than for those who complete only one stochastics course. The association between the CESK and ASCT scales for this subgroup did not reflect a clear linear trend (Figure 7), which is reinforced by the low but positive correlation obtained (0.184). The last group included the subjects who had taken three courses on stochastics. The results (Table 10) show that the mean score on the knowledge scale was slightly lower than the previous group, but in terms of attitudes, these were the highest among those who follow the concurrent training model. The correlation coefficient was the lowest, reflecting that participants who declared the more positive attitudes towards stochastic and its teaching did not necessarily correspond with those with the best performance on the knowledge scale.

5. DISCUSSION

5.1. ADDRESSING THE RESEARCH QUESTIONS

What is the stochastics content knowledge of pre-service Chilean mathematics teachers, and does it differ for Common and Extended knowledge and/or content areas considered? Teachers need a solid understanding of the content they will teach (Ball et al., 2008; Godino et al., 2017), so their weaknesses do not have negative implications for classroom instruction and student learning. For the first research question, we explored the performance showed by participants using the *Common and Extended Stochastics Knowledge* (CESK) scale (Appendix 1). We found the content knowledge of these pre-service teachers was generally poor, with slight differences according to the content areas in which we organized stochastics (highest mean score was in the descriptive statistics content area, and lowest was in the probability area), but in all cases insufficient for the local requirements of these professionals (Figure 3a). Although more than 80% of the participants had already passed all their required stochastics courses (Table 4), the majority had not yet mastered the topics they will be teaching (Table 5) or the extended knowledge of school stochastics (Table 6). These findings support recent ENDFIP studies (CPEIP, 2018, 2019) that reported worse results in stochastics and geometry, and better in calculus and algebra. While we expected lower performance in extended knowledge, the results indicate these teachers will have difficulty broadening their perspective on the content they are teaching, reflecting on and enhancing their teaching, and helping students connect topics. Furthermore, the Chilean school curriculum (Comunicaciones CIAE, 2018; MINEDUC, 2019) appears to be moving towards more *extended knowledge* content expectations (e.g., big data techniques are planned to be included in the last levels of compulsory education), highlighting the importance of keeping up-to-date the readiness of pre-service teachers for teaching the school curricula. It appears that the stochastics courses currently taken by prospective teachers in Chile need to be reformulated to improve the disciplinary training of these education professionals.

What is the nature of pre-service Chilean mathematics teachers' attitudes towards stochastics and its teaching? Are there attitudinal differences depending on the content area considered? For the second research question, we used the *Attitude towards Stochastics Contents and its Teaching* (ASCT) scale (detailed in Table 2). Overall results (Table 7) showed that participants tended to have positive attitudes (higher than the indifference score), following patterns seen in previous research (Aparicio & Bazán, 2006; Alvarado et al., 2018; Estrada, 2002; Estrada et al., 2004; Estrada et al., 2018; Hannigan et al., 2013; Martins et al., 2015; Ruz et al., 2020d; Vásquez et al., 2019; Zientek et al., 2011). For the seven individual attitude components (Table 8), we also saw positive attitudes with the lowest median score (13 points) belonging to Cognitive Competence towards the stochastics content and Didactic Competence towards its teaching, and the highest median score (18 points) to the Value towards stochastics content and its teaching. These pre-service teachers agreed the content is important, but also did not feel very comfortable in their content knowledge and skills to teach it.

Regarding the results in each content area (Table 7), our participants assigned higher mean attitude scores towards probability and descriptive statistics, but lower towards statistical inference (although greater than indifference position, in all cases). Further research will explore how these differences in attitudes by content area manifest across the different components of the ASCT scale (Figure 4) and include qualitative analysis of reasons behind these different perceptions.

Consequently, our results are most similar to those of Hannigan et al. (2013) with a sample of Irish pre-service mathematics teachers—weak content knowledge and mostly positive attitudes toward statistics. Hannigan et al. mainly attributed their results to the fact that in Ireland stochastic training for teachers has a strong emphasis on mathematics. They concluded that their participants' scores may be reflecting positive attitudes toward mathematics rather than statistics, and because the two disciplines are different, it is not surprising to find the absence of a relationship between attitudes towards mathematics and performance on statistics items. The mathematical emphasis is similar in Chile and we may also be seeing teachers' attitudes towards mathematics instead of stochastics. This would also explain the more positive attitudes towards probability content in our sample compared to the more statistical content areas. Moreover, even though Estrada and Batanero (2008) concluded that the main reason for negative attitudes by prospective teachers was due to a lack of preparation or knowledge about stochastics content, our results were the opposite. It seems that a more positive attitude is not a strong predictor of better performance.

Is there a relationship between stochastics content knowledge and attitudes towards stochastics and its teaching? Do these associations differ within the content areas and/or based on previous training? For the third research question, we used the results on both scales, CESK and ASCT. Overall, the correlation between the total scores of both instruments was 0.239, positive and conclusive for this sample (Table 9 and Figure 5). This correlation was as low as in previous research (Aparicio & Bazán, 2006; Estrada, 2002; Hannigan et al., 2013; Nasser, 2004; Zientek et al., 2011), but it reflected behavior again similar to Hannigan et al. (2013). Furthermore, considering the relationship between the total score on the ASCT scale and the results for the *common* and *extended knowledge* variables, participants with stronger common content knowledge tended to have more positive attitudes, but for the extended knowledge category, attitudes did not appear strongly related to the subjects' level of knowledge about the stochastics that go beyond the school curriculum (which was generally low). The results on the total score in the CESK scale (content knowledge) correlated positively but weakly with all the components of attitudes considered, but was less conclusive with the behavioural towards teaching stochastics (BTS) and the value of content and its teaching (VTS) components. It seems that the participants tended to rate the value and behavior of stochastics teaching highly, regardless of their performance in the content knowledge questions.

Regarding the relationship between knowledge about the different content areas and the attitudinal dimensions towards the contents, its teaching, and the value given to both (Figure 6), again the correlations were low. Results on the association in the *descriptive statistics* content area were positive but weak. Participants had positive attitudes towards this content even with low knowledge, but their knowledge was less useful in predicting their (high) attitudes towards teaching the material. For the *probability* content area, correlations were very weak and even negative. Some students with higher probability content knowledge scores had below average (but positive) attitudes. In this sense, given the high positivity of the attitudes declared towards this area (Table 7) and the low performance in the items on this knowledge (Figure 3a), the lack of correlations could be due to a floor effect. In the *statistical inference* area, the correlations between content knowledge and attitude scores were back to being generally positive but weak, as in the descriptive statistics area. Participants who performed better on knowledge items tended to be more willing to learn and teach this content (Figure 6). This situation may be due to the fact that attitudes towards inference were less positive than towards descriptive statistics, as was the knowledge demonstrated in the items on this topic. Lastly, results in the *value* towards stochastics and its teaching' dimension the association was inconclusive in all three content areas considered, where most of our pre-service teachers' tended to value learning and teaching stochastics, even with weaker understanding of the material.

Regarding previous training, we can see that the performance and attitudes of the participants varied according to their degree program (Figure 7). The association between demonstrated knowledge and declared attitudes was stronger for those who had taken only one course on stochastics, and it becomes less intense for those who took two or three courses respectively. Therefore, a recommendation for future study is to review the assessment results with the participating pre- or in-service teachers, to see how they judge their own knowledge, and to explore why they did not do better on some of the content knowledge questions from a qualitative perspective. It would also be interesting to know why some of those who seem to understand the material better had below average attitudes. Maybe they better understand the limitations and subtleties of the methods, but maybe they also think many of the more subtle nuances will be difficult, less appealing topics to teach.

Finally, we posit that the weak association between attitudes and conceptual knowledge presents an opportunity for improving content knowledge; with generally positive attitudes these teachers should be motivated and open to further their understanding. And we propose to begin with those components and content areas where their disposition was less positive, such as in the cognitive (content knowledge) and didactic (knowledge for teaching) components, and the probability and inference areas. Therefore, in the following section we propose some recommendations for the stochastics education of Chilean mathematics teachers to strengthen their content knowledge, and at the same time continue to promote a positive attitude to the content and its teaching.

5.2. RECOMMENDATIONS

As future teachers, the participants of this study represent a substantial number of professionals who will soon teach stochastics in Chileans schools. And most of them will not take additional university level courses in this content, so we cannot expect teachers' content knowledge and attitudes to improve without systematic improvements to teacher preparation programs and ongoing support for these professionals. Based on the main results of this study, where subjects tended to have positive attitudes regardless of their content knowledge, we provide the following suggestions.

- *Reorganize the stochastics training of teachers* for the content that teachers must teach in secondary school (common knowledge), as well as the extensions of this content (extended knowledge). Teachers training institutions include 1 to 3 stochastics courses, dedicating at most 6% (3/50) of the entire pre-service teachers' curriculum to this content (www.cned.cl). Furthermore, these courses usually promote traditional teaching (based on a hierarchical accumulation of concepts, which are learnt in a linear sequence), which tends to be more mathematical in nature, often losing sight of the importance of aspects such as context and variability when facing stochastics problems. As Batanero and Díaz (2012) state "stochastics knowledge is more complex and systemic, so stochastics problems must include much more interpretative activities than other areas of mathematics" (p. 7). Statistical thinking is different from mathematical thinking, and a strong background in mathematics does not necessarily translate to an adequate statistical thinking, as Hannigan et al. (2003) concluded with pre-service Ireland secondary mathematics teachers.
- *Promote content knowledge and content knowledge for teaching simultaneously.* Currently, most programs in Chile require at least one course on stochastics, but only one-third of these universities (10 out of 30) require a course about teaching stochastics (Ruz et al., 2020c). This could explain the lack of confidence for teaching stochastics demonstrated by our participants. Teachers' training in stochastic content should model the effective pedagogical practices these teachers should be using in their future classes. In particular, priority should be given to the use of real data to develop the understanding of the central ideas of stochastics; to detecting the main characteristics that make stochastics different from mathematics; to reflecting on good practices for teaching stochastics, including identifying the reasoning and common misconceptions of students; to developing new methodologies to assess this content, such as the use of projects; and to promoting the use and mastery of various technological resources to introduce simulation and streamline efforts to compute procedures of interest.
- *Utilize modern approaches to develop stochastic reasoning.* We propose that in order to improve this situation, training institutions should consider some modern approaches to develop adequate stochastics reasoning. Examples of approaches include, organize the teaching of statistics through the problem-solving process from the GAISE recommendations (Franklin et al., 2005) or statistical investigation method (PPDAC enquiry cycle: Problem, Plan, Data, Analysis, Conclusion; Wild & Pfannkuch, 1999) or Pfannkuch et al. (2016) SWAMTU Probability Modeling Cycle (Situation (Problem), Want-to-know (motivation), Assumptions (ideas about system dynamics), Model (applying structure), Test (does the model make sense?), and Use (apply the model)). In this sense, we propose that in order to develop students' reasoning of probability and the modeling of random phenomena, it is essential that teacher education promote a balanced view of probability from both mathematical and empirical perspectives, the joint use of symbolic structure and theory with simulation, provide insights through visualizations of randomness and random processes, and promote the ability to study more complex systems for which there are currently no theoretical results. Thus, core ideas such as randomness and conditioning (items with lower performance by our participants) can be improved. Another option could be a simulation-based introduction to statistical inference approach, where simulations are used to introduce sampling variation and the reasoning of statistical estimation intuitively and informally, without requiring students to first learn the more detailed mathematical underpinnings. The main idea of this approach can be synthesized by Cobb's three Rs: *Randomize* data production; *Repeat* by simulation to see what is typical; and *Reject* any model that puts the data in its tail (Cobb, 2007). Some examples that have systematized these ideas into full curricula are Lock et al. (2017) and Tintle et al. (2016). In particular, these approaches allow consideration of statistical inference earlier in teacher training. This allows

learners to focus on the entire statistical process, rather than differentiating and separating descriptive statistics, probability, and statistical inference.

In summary, we specifically suggest that Chilean universities update their stochastics curricula for initial training to include more recent recommendations to better prepare teachers, such as those mentioned above. In those updated approaches, the content covered would be very similar to a traditional course, but the content would be introduced in context through genuine applications, where changes in pedagogy focus on active learning based on students' explorations. From our perspective, this update should ideally stop differentiating between courses focusing on stochastics or teaching stochastics for teachers, but such discussions should be integrated and promoted simultaneously. Courses should also focus on affective aspects that may be developed by learning from teaching practice, deep exploration of stochastics content, and use of technological environments for teaching (Groth & Meletiou-Mavrotheris, 2018). For this purpose, we highlight as valuable the instruments developed in this research, which may be useful to evaluate the effectiveness of potential changes introduced to the stochastics education for teachers. In addition, training institutions, the CPEIP, or schools themselves, should provide professional development opportunities for in-service teachers free of charge, especially in stochastics, during at least the first two years of teaching practice. Providing this professional development after teachers have had the opportunity to experience first-hand students' questions and misconceptions can potentially be more effective. We believe that with such steps, in the near future, mathematics teachers can respond satisfactorily to the demands of stochastics in their professional field.

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APPENDIX 1.
LEARNING OBJECTIVES OF THE CESK SCALE ITEMS ACCORDING TO THE CONTENT AREA AND DMK CONTENT KNOWLEDGE CATEGORY THEY EVALUATE

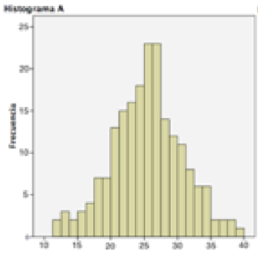
N	Content Area	DMK category	Learning Objective
1	D	CCK	Match different representations to explore, summarize, and describe patterns in univariate data.
2	D	ECK	Estimate the correlation coefficient between two variables based on their graphical representation.
3	P	CCK	Differentiate between application of different discrete probability distributions.
4	P	CCK	Evaluate likely amount of random variation of a probability experiment
5	I	CCK	Apply the Law of Large Numbers in problem solving.
6	D	CCK	Deduce the behavior of a data distribution from summary statistics.
7	I	ECK	Establish the statistical hypothesis from a research question.
8	P	ECK	Calculate a conditional probability
9	I	CCK	Recognize factors that influence the width of a confidence interval.
10	I	CCK	Recognize the effect of sample size on the variability of the sampling distribution.
11	D	CCK	Compare the variability between groups through exploratory data analysis.
12	I	ECK	Interpret and apply the logic of statistical hypothesis statements.
13	P	CCK	Distinguish dependent, independent, and mutually exclusive events.
14	D	ECK	Estimate and compare standard deviations for different data distribution.
15	P	ECK	Use Bayes' theorem in problem solving.
16	I	ECK	Recognize correct and incorrect interpretations of p-value.
17	D	ECK	Distinguish between correlation and causality.
18	P	ECK	Apply the main properties of Poisson' distribution in problem solving.
19	I	ECK	Identify errors in the interpretation of confidence level.
20	P	CCK	Recognize the characteristic shape of the Normal curve.

APPENDIX 2. CESK SCALE

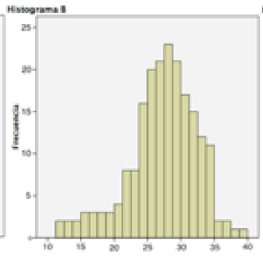
1) Un estudio examinó la longitud de una cierta especie de pez en un lago. El plan era tomar 200 peces y examinar los resultados. En la siguiente tabla se muestran algunos estadísticos de resumen sobre la longitud de los peces, en milímetros (mm).

Promedio	Mediana	Desv. Estándar	Mínimo	Máximo
27,3 mm	28,6 mm	5,1 mm	12,1 mm	38,0 mm

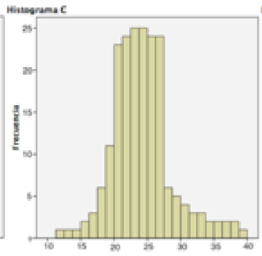
¿Cuál de los siguientes histogramas corresponde a los datos mostrados?



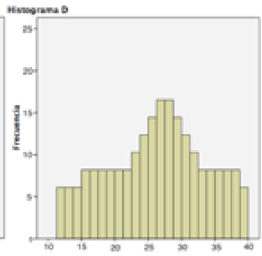
A. Histograma A



B. Histograma B*

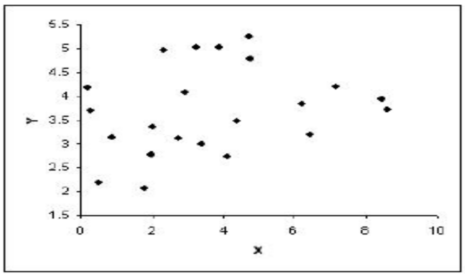


C. Histograma C



D. Histograma D

2) El siguiente gráfico de dispersión muestra la relación entre las puntuaciones de una escala de ansiedad (X) y las calificaciones de un examen de ciencias (Y). En base al gráfico, deduzca el coeficiente de correlación entre X e Y.



A. 0
B. 0,3*
C. 0,7
D. 1,3

3) ¿En cuál de las siguientes situaciones se puede modelar la variable aleatoria de interés por medio de una distribución binomial?
A. En una empresa de fabricación, aproximadamente el 5% de los productos son defectuosos. Se pretende calcular la probabilidad de que el tercer artículo defectuoso sea el vigésimo producto seleccionado.
B. Al lanzar un par de dados usuales (no trucados y de seis caras cada uno) queremos saber el número de lanzamientos necesarios antes de obtener por primera vez una suma de 7.
C. Un policía ha descubierto que aproximadamente el 15% de los vehículos que detiene son de fuera de la región. Estamos interesados en conocer la cantidad de vehículos de otra región en los próximos 50 que detiene.*
D. En un juego de mesa, se desea apostar a la carta con mayor probabilidad de ser extraída de una baraja inglesa (52 naipes).

4) Un profesor vació sobre la mesa un paquete de 100 chinches metálicos obteniendo los siguientes resultados: 68 caen con la punta para arriba y 32 caen hacia abajo. Luego, el profesor pide a 4 estudiantes (Alba, Bernardo, Carmen y David) repetir el experimento, lanzando cada uno las 100 chinchetas. Entre las siguientes opciones, ¿cuál colección de resultados piensas que es más probable?

A.

Resultado	Alba	Bernardo	Carmen	David
Punta arriba:	32	70	35	65
Punta abajo:	68	30	65	35

C.

Resultado	Alba	Bernardo	Carmen	David
Punta arriba:	69	68	67	68
Punta abajo:	31	32	33	32

*B

Resultado	Alba	Bernardo	Carmen	David
Punta arriba:	63	70	64	71
Punta abajo:	37	30	36	29

D.

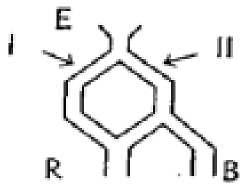
Resultado	Alba	Bernardo	Carmen	David
Punta arriba:	50	51	48	53
Punta abajo:	50	49	52	47

5) La estatura media de los universitarios es de 178 centímetros (cm), con una desviación estándar de 8 cm. Mientras que la altura promedio de las universitarias es de 165 cm, con una desviación estándar de 10 cm. Usted realiza un experimento en su universidad midiendo la altura de 100 hombres y 100 mujeres. ¿Qué resultado le sorprendería más?
A. Un hombre con una estatura de 200 cm.
B. Una mujer con estatura de 188 cm.
C. Que la estatura promedio de las mujeres en su universidad sea menor de 163 cm.
*D. Que la estatura promedio de los hombres en su universidad sea mayor de 185 cm.

6) La puntuación obtenida por Ana en un examen corresponde al percentil 35 respecto a sus compañeros de clase, mientras que el resultado de Elena en la misma prueba corresponde al percentil 70. ¿Cuál de las siguientes afirmaciones es correcta?
A. Ana obtuvo mejor o igual puntuación que 35 compañeros de su clase.
B. Ana responde bien aproximadamente una tercera parte del examen.
C. Elena obtiene el doble de puntuación que Ana en el examen.
D. El 30% de los compañeros de su clase obtuvieron mejor o igual puntuación que Elena.*

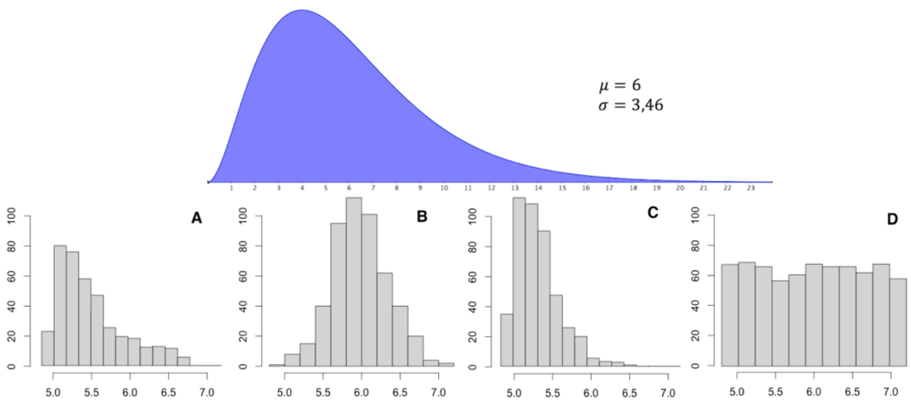
7) En una encuesta electoral se desea averiguar si hay más ciudadanos a favor de la política económica del presidente que en contra de la misma. Suponiendo que p representa la probabilidad asociada a que los habitantes estén de acuerdo con dicha política económica y que q = 1 – p. ¿Cuál de las siguientes hipótesis elegiría como hipótesis nula?
A. $p > q$ B. $p = q = \frac{1}{2}$ * C. $p \neq q \neq \frac{1}{2}$ D. $p < q$

8) Una bola se suelta en E. Si sale por R, ¿cuál es la probabilidad de que haya pasado por el canal I?
A. 0,50
B. 0,33
C. 0,66*
D. No se puede calcular



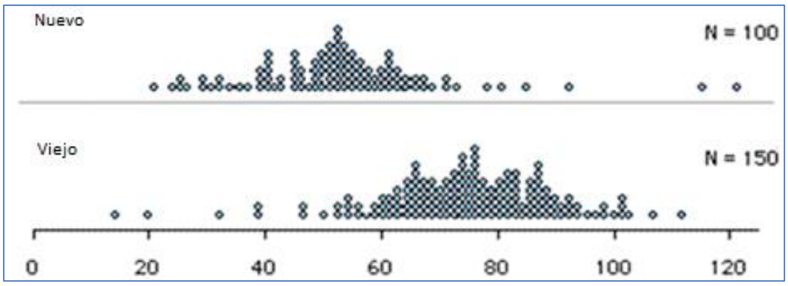
9) Sobre algunos factores que influyen en la amplitud de un intervalo de confianza (IC) para la media poblacional, ¿cuál de las siguientes afirmaciones es verdadera?
A. El IC será más ancho cuando se utilice una muestra grande.
B. Si la desviación estándar de la población disminuye, la anchura del IC disminuye*
C. La elección del nivel de confianza no influye en la amplitud del IC.
D. Al aumentar la media muestral aumenta la amplitud del IC.

10) A continuación se presentan cinco gráficos. El de la parte superior corresponde a la distribución de una población respecto a los resultados de una prueba. A partir de estos datos, ¿qué gráfica (A, B, C o D) crees que representa una distribución de 500 medias muestrales a partir de muestras aleatorias de tamaño 100 cada una?



A. Gráfico A B. Gráfico B* C. Gráfico C D. Gráfico D

11) Una compañía de telefonía desarrolló una nueva fórmula de fibra óptica para mejorar la velocidad de navegación en internet de sus usuarios. Para probar la efectividad de esta nueva fórmula, se desarrolló un análisis exploratorio de datos en un pueblo de 250 habitantes. Entre ellos, 100 recibieron esta nueva conexión, mientras que los otros 150 permanecieron con conexión por cable. Posteriormente, se le solicitó a cada participante visitar una página de prueba, registrándose el tiempo (en segundos) de demora en tener acceso a ella. Los resultados de este procedimiento se muestran en los siguientes gráficos:

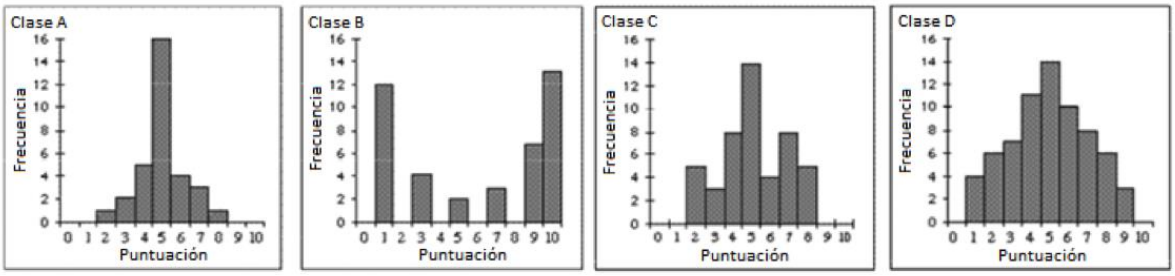


Las próximas declaraciones fueron realizadas por estudiantes de estadística diferentes. ¿Cuál de ellas es correcta?
A. La antigua conexión funciona mejor. Dos personas tuvieron acceso en menos de 20 segundos. Además, la mayor demora - dos minutos - fue con la nueva fórmula.
B. La nueva conexión funciona mejor. El tiempo promedio de la nueva fórmula de acceso a internet es menor que el de la antigua conexión, tarda unos 20 segundos menos.*
C. Ambas conexiones alcanzan igual velocidad de navegación, ya que la diferencia entre los resultados obtenidos no es estadísticamente significativa.
D. No concluiría nada de estos datos. El número de habitantes en los dos grupos no es lo mismo, así que no hay una manera justa de comparar las dos fórmulas.

12) Suponiendo que la variable aleatoria esperanza de vida (X) sigue una distribución normal, se desea evaluar la factibilidad de aumentar la edad de jubilación de un país, que actualmente es de 70 años. Considere un contraste de hipótesis con $H_0 \colon \mu_X = 70$ v/s $H_1 \colon \mu_X > 70$ y un nivel de significación de $\alpha = 0{,}05$ ($Z_{0{,}95} = 1{,}645$). Se ha tomado una muestra aleatoria de 500 ciudadanos, obteniendo un estadístico muestral igual a 2,5. Entonces:
A. No se debe rechazar H_0, por tanto, la hipótesis alternativa ha de ser falsa.
B. Se debe rechazar H_0, ya que la probabilidad de que sea cierta es 0,05.
C. No se debe rechazar H_0, ya que el estadístico de prueba no cae en la región de rechazo.
D. Se debe rechazar H_0, aunque no sabemos si hemos tomado la decisión correcta.*

13) Se extrae una carta al azar de una baraja española (40 cartas diferentes, con números del 1 al 7, sota, caballo y rey; y cuatro palos diferentes: oros, copas, espadas y bastos). Considere los sucesos: A: “Se extrae una carta de oros” y B: “Se extrae un rey”. ¿Los sucesos A y B son independientes?
A. No son independientes porque en la baraja hay un rey de oros.
B. Sólo si sacamos primero una carta para ver si es rey y se vuelve a colocar en la baraja y luego sacamos una segunda carta para mirar si es un oro.
C. No, porque P(Rey de oro) = P(Rey)·P(oro)
D. Si, en todos los casos.*

14)) A continuación se presentan los resultados de una prueba, en escala de 0 a 10 puntos, para cinco clases de estadística diferentes. ¿En cuál de ellas esperarías tener la dispersión más baja, y por qué?



A. Clase A, porque tiene la mayoría de los valores cercanos a la media.*
B. Clase B, porque tiene el menor número de puntuaciones distintas.
C. Clase C, es de las con menor rango y puntuaciones más homogéneas.
D. Clase D, porque se la forma de su distribución se asemeja más a la curva normal.

15) Un taxi se vio implicado en un accidente nocturno con choque y huida posterior. Hay dos compañías de taxis en la ciudad, una con vehículos de color verde y otra de color azul. El 85% de los taxis de la ciudad son verdes y los demás azules. Un testigo identificó el taxi como azul. El tribunal comprobó la fiabilidad del testigo bajo las mismas circunstancias que había la noche del accidente y llegó a la conclusión de que el testigo identificaba correctamente cada uno de los colores en el 80% de las ocasiones y fallaba en el 20%. ¿Cuál es la probabilidad de que el taxi implicado en el accidente fuese realmente azul?
A. 0.80 B. 0.15 C. 0.15·0.80 D. 0.41*

16) Un artículo de investigación informa de la efectividad de un nuevo medicamento para disminuir la pérdida de visión en personas con un tipo de degeneración ocular. El artículo indica un p-valor de 0,04 en la sección donde describen el análisis realizado. A partir de esta información, ¿cuál de las siguientes interpretaciones sobre el p-valor es correcta?
A. La probabilidad de obtener resultados tan extremos, o más extremos que los de este estudio, si el medicamento en realidad no es efectivo.*
B. La probabilidad de obtener resultados tan extremos, o más extremos que los de este estudio, si el medicamento es efectivo.
C. La probabilidad de que el medicamento sea efectivo.
D. La probabilidad de que, si el medicamento no es efectivo, el resultado se deba al azar.

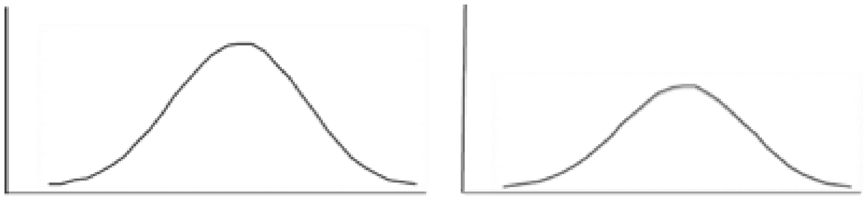
17) En tu vecindario, mediante un análisis exploratorio de datos se determinó un coeficiente de correlación de 0,9 entre el nivel de ingresos y la cantidad de plástico reciclado en una semana. ¿Qué indica este valor?
A. En este vecindario ganar más dinero provoca que las personas reciclen más.
B. Los vecinos que menos reciclan son los que ganan más dinero.
C. Aunque los vecinos que menos dinero ganan son los que menos reciclan, esto no necesariamente significa que ganar menos sea la causa de reciclar menos plástico.*
D. Las personas que más ganan son las que más plástico reciclan.

18) El número de aviones que aterriza a un aeropuerto es un factor importante para coordinar la asignación de pistas de aterrizaje. Suponga que el índice de llegadas de aviones se distribuye según una distribución Poisson con una tasa de 6 llegadas por hora. ¿Cuál de las siguientes afirmaciones es verdadera?
A. Se espera que lleguen 3 aviones en media hora.*
B. Es igualmente improbable que lleguen 4 aviones en una hora a que lo hagan 8 en el mismo período.
C. La probabilidad de que aterricen 6 aviones en una hora es casi 1.
D. Es imposible que lleguen 9 aviones en una hora.

19) Los estudiantes de una escuela secundaria quieren estimar el número medio de chips de chocolate que contienen las galletas de una marca X. Para ello, recopilan una muestra aleatoria de galletas,

cuentan los chips en cada una y calculan un intervalo de confianza del 95% para el número promedio de chips por galleta [18,6 – 21,3]. ¿Cuál de las siguientes interpretaciones de los resultados es correcta?
A. Tenemos el 95% de certeza de que cada galleta producida por esta marca tiene aproximadamente entre 18,6 y 21,3 chips de chocolate.
B. Esperamos que el 95% de las galletas tenga entre 18,6 y 21,3 chips de chocolate.
C. Esperamos que aproximadamente el 95% de todas las medias muestrales posibles de esta población oscilarán entre 18,6 y 21,3 chips de chocolate.
D. Si tomaramos varias muestras del mismo tamaño, en el 95% de los intervalos calculados estará contenido el verdadero número medio de chips por galleta.*

20) ¿Cuál de las siguientes afirmaciones sobre las curvas que se muestran a continuación es cierta?



A. No se puede asegurar que las curvas son normales, pero si que sus medias y desviaciones típicas no son iguales.
B. Las dos curvas son normales, pero las desviaciones típicas son diferentes.*
C. Las dos curvas son normales, pero las medias son distintas.
D. En ambas curvas, el 50 % de los datos está comprendido en el intervalo $(\mu - \sigma; \mu + \sigma)$

APPENDIX 3. ASCT

1) Me divierto en las clases en las que se explican los tópicos de estadística/probabilidad/inferencia.
2) Utilizo información que incluyan elementos de estadística/probabilidad/inferencia a la hora de tomar decisiones.
3) Será difícil para mi enseñar los tópicos de estadística/probabilidad/inferencia.
4) Los contenidos de estadística/probabilidad/inferencia ayudan a entender el mundo de hoy.
5) Me gusta la estadística/probabilidad/inferencia, es un tema que siempre me ha interesado.
6) El tópico de estadística/probabilidad/inferencia es fácil.
7) Nunca he usado los contenidos de estadística/probabilidad/inferencia fuera de un contexto científico.
8) Domino los principales contenidos de estadística/probabilidad/inferencia.
9) Estoy seguro que me gustará enseñar los contenidos de estadística/probabilidad/inferencia en la escuela.
10) Creo que sabré detectar y corregir errores y dificultades de los alumnos con los temas de estadística/probabilidad/inferencia.
11) Solo enseñaré estadística/probabilidad/inferencia si me queda tiempo después de los otros temas.
12) El tópico de estadística/probabilidad/inferencia no sirve para nada.
13) El tópico de estadística/probabilidad/inferencia no tiene tanto valor como otras ramas de la matemática.
14) Me resultará fácil diseñar actividades de evaluación sobre los contenidos de estadística/probabilidad/inferencia.
15) Uso los contenidos de estadística/probabilidad/inferencia en la vida cotidiana.
16) Me siento intimidado ante los contenidos de estadística/probabilidad/inferencia.
17) El tópico de estadística/probabilidad/inferencia lo entiende solo la gente del área científica.
18) Evito leer informaciones donde aparecen términos sobre estadística/probabilidad/inferencia.
19) Los conocimientos sobre estadística/probabilidad/inferencia ayudan a los alumnos a razonar críticamente.
20) Se debería dedicar más tiempo a enseñar los temas de estadística/probabilidad/inferencia en los primeros niveles de educación.
21) Me preocupa saber responder preguntas sobre el tópico de estadística/probabilidad/inferencia de mis alumnos.
22) No me siento preparado para resolver cualquier problema sobre estadística/probabilidad/inferencia.
23) Pienso que no seré capaz de preparar recursos didácticos apropiados para las clases donde deba enseñar el tópico de estadística/ probabilidad/inferencia.
24) Cuando sea pertinente, utilizaré los contenidos de estadística/probabilidad/inferencia en los otros ejes del currículo de matemática que enseñe.
25) Si pudiera eliminar alguna materia del currículo de matemática sería el tópico de estadística/probabilidad/inferencia.
26) No tengo mucho interés en enseñar el tema de estadística/probabilidad/inferencia aunque aparezcan en el currículo.
27) No me agrada resolver problemas de estadística/probabilidad/inferencia.
28) Como futuro profesor, creo que me sentiré cómodo al enseñar el tópico de estadística/probabilidad/inferencia.