

## The role of dotplots in statistics education practice

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*Statistical graphs play a crucial role in data literacy, yet pupils frequently misinterpret histograms. In education, textbooks typically progress from case-value plots to histograms but research suggests that dotplots and their variants (e.g., histodots) can support this transition. The study asked: How and how often are dotplots and their variants used in Dutch textbooks? An analysis of 19 Dutch textbooks (Grades 7–10) showed that dotplots are rare and not used to support transitions to histograms or boxplots. Variants like hatplots and histodots were absent. Hence, a gap is apparent between research on learning progressions in statistics education and the Dutch curriculum as implemented in textbooks, which teachers often closely follow. Although the reasons for this limited uptake of research remain speculative, possible explanations include a lack of exemplary large-scale or quasi-experimental studies and a shortage of exemplary, classroom-tested materials to support implementation.*

### INTRODUCTION AND THEORETICAL BACKGROUND

Many students misinterpret histograms—for example, by confusing them with case-value plots—and interpreting each bar as one measurement and the height as the measured value (Boels et al., 2025), see Figure 1. Despite careful analysis and interventions, misinterpretations remain widespread (e.g., Kaplan et al., 2014; Whitaker & Jacobbe, 2017). Dotplots and their variants can support students' reasoning as dotplots show all the data, with the positions of the dots on the number line representing the measured value (Boels, 2023a), similar to the positions of bars in histograms. For example, although the graphs in Figure 1 look very similar, their respective dotplots (Figure 2) immediately clarify that the number of measured values and their distribution are very different.

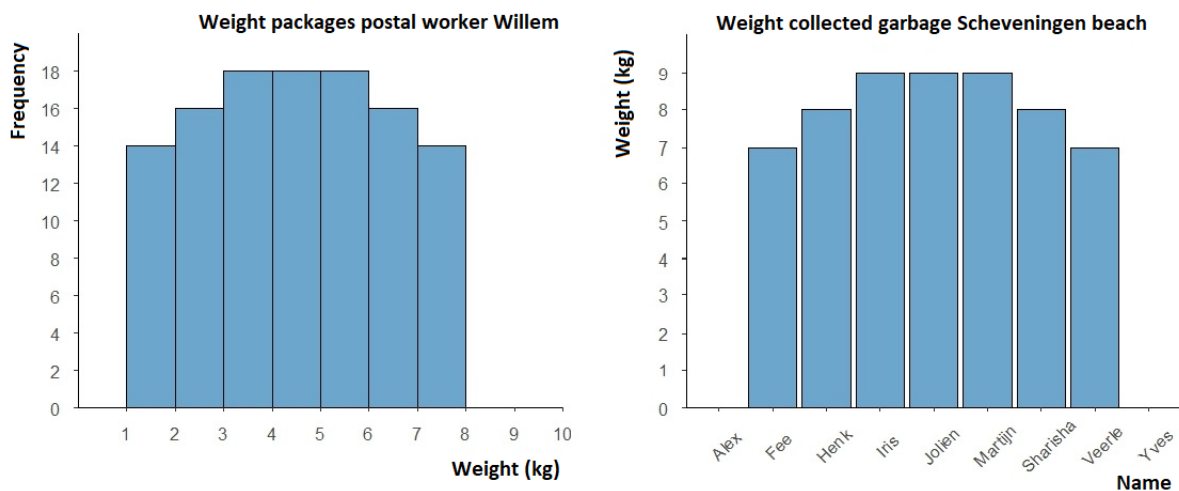


Figure 1. Two graphs with weight data of postal packages (left) and collected garbage (right).

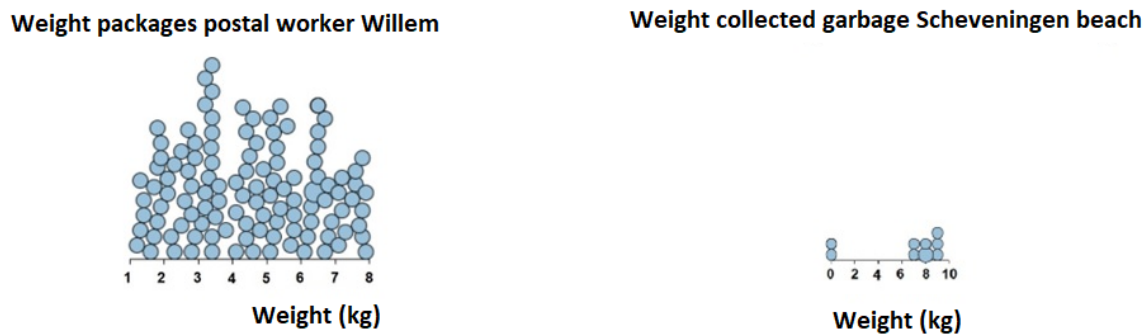


Figure 2. Dotplots using the same data as plotted in Figure 1.

### *Importance of different graph types for data analysis and interpretation*

Garfield and Ben-Zvi (2008) note that students should understand the statistical analysis process including, “how to graph the data as a first step in analyzing data, and how to know when that’s enough to answer the question of interest” (p. 15). They also emphasize “that different representations of the same data set may reveal different aspects of the distribution” and “that visually examining distributions is an important and necessary part of data analysis” (p. 49). Several examples exist where graphing the same data in a different format reveals different aspects of the distribution (e.g., Figure 3). Pastore et al. (2017) demonstrate that conclusions drawn in psychological research can vary depending on the type of graph used: bar charts showing only the mean or the mean with standard deviation, boxplots, or histograms. Boxplots and histograms may suggest a difference between conditions that is not apparent in bar charts, or the reverse, especially when the difference is due to outliers that disappear once those outliers are removed. They state that in the two examples given, “the use of bar charts did not allow to properly detect false negative (example 1) and false positive (example 2) research findings” (p. 2). They show with a third example that boxplots may also lead to false conclusions and conclude that “one graph does not fit all”.

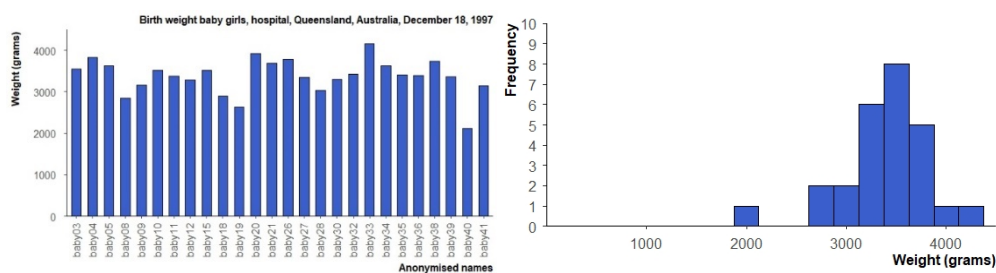


Figure 3. Case-value plot (left) and histogram (right) with the same data and distribution but a very different shape (Boels et al., 2019b).

### *Insights from eye-tracking research*

Recent advances in eye-tracking research have provided new insights into how students interpret statistical graphs (Boels, 2023b) such as case-value plots and histograms (Boels et al., 2025), dotplots (Boels et al., 2023; Boels & Van Dooren, 2023; Schreiter & Vogel, 2023) and boxplots (Abt et al., 2024). A key finding from eye-tracking studies is that students and teachers often interpret histograms as case-value plots, despite paying attention to the axis labels (e.g., Boels et al., 2019a). Another is that students in Grades 10–12 can easily grasp single dotplots (Boels, 2023a). However, both primary and secondary students struggle more when comparing dotplots (Boels et al., 2023; Schreiter & Vogel, 2023). Comparing histograms (Boels et al., 2025) and boxplots (Abt et al., 2024) turn out to be even more difficult. Both graph types represent aggregated data. These studies provide further support for empirical studies that already showed the importance of using dotplots (Frischemeier, 2023) and their variants—such as histodots (Bakker, 2004; Boels, 2023a) and hatplots (Bakker, 2004; Fielding-Wells,

2018; Frischemeier, 2019)—to gradually transition from case-value plots to histograms and boxplots (Figure 4). This sequence of graphs is based on logical-historical analysis (e.g., Bakker, 2004; Boels, Alberto & Shvarts, 2023) and the findings of eye-tracking research.

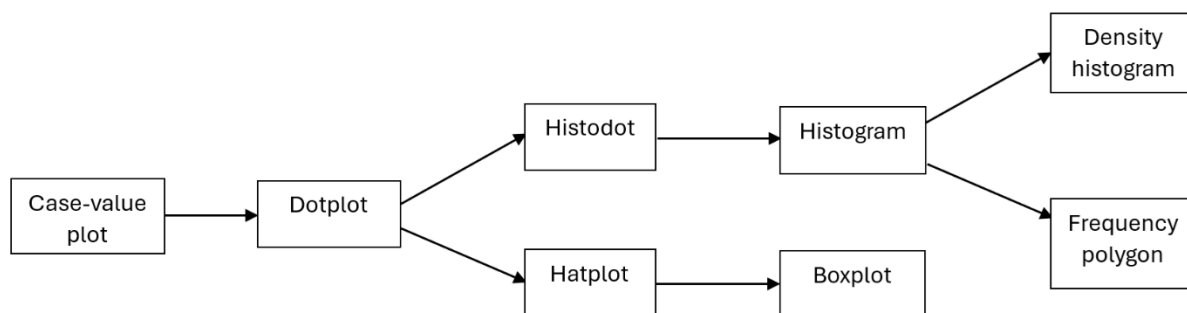


Figure 4. Simplified sequence of graphs as part of the local instruction theory (Boels, 2025).

Research has shown the importance of dotplots and their variants as intermediate graphs. Our interest is whether these results are apparent in Dutch textbooks. Therefore, the research question is: *How and how often are dotplots and their variants used in Dutch textbooks?*

## METHOD

The textbooks of *Moderne Wiskunde* [Modern Math] (13<sup>th</sup> edition) were chosen to be analysed because past editions had used at least a few dotplots (unpublished research of Boels) and for reasons of availability. We analysed all textbooks for lower secondary education (Grades 7–9; for 12–15 years old students) on their use of dotplots and their variants such as histodots and hatplots. In total 29 textbooks for lower secondary education were included in the study. As the pre-vocational education track (vmbo) ends at Grade 10, the seven textbooks for this Grade-track-combination were also analysed. Some have already been analysed (see Table 1), while others will be analysed later this year. Furthermore, some other data were collected, such as Grade level, whether this Grade has a chapter on statistics, univariate graph types found, and counts of dotplots. Graphs were counted if displayed, but also if students were required to draw the graph.

## RESULTS

As data collection and analysis are still ongoing, this paper reports preliminary results based on 19 textbooks from vocational education Grades (7–10). Although we expected that dotplots would be scarce, we had hoped to find at least a few examples, in line with, for example, GAISE-II-guidelines (Bargagliotti et al., 2020). However, no dotplots were found, and only one infographic featured dots arranged in a square shape (Table 1). The infographic was not used to support a transition to boxplots or histograms. A similar situation appears to apply to other textbooks, including the most widely used Dutch mathematics textbook, *Getal & Ruimte* [Number & Space], according to unpublished research by Boels.

Table 1. Overview of results per textbook.

Textbook	Grade	Statistics chapter	Univariate graph types	Count of dotplots	Approx. total number of graphs*
Vmbo-b, part A	7	No	NA	NA	NA
Vmbo-b, part B	7	No	NA	NA	NA
Vmbo-b, part C	7	No	NA	NA	NA
Vmbo-b, part A	8	No	NA	NA	NA
Vmbo-b, part B	8	No	NA	NA	NA

Textbook	Grade	Statistics chapter	Univariate graph types	Count of dotplots	Approx. total number of graphs*
Vmbo-b, part C	8	Yes	Pictorial distribution bar graph, circle diagram, distribution bar graph, histogram	0**	42
Vmbo-b, part A	9	Yes	Pictorial distribution bar graph, circle diagram, distribution bar graph, histogram	0	25
Vmbo-b, part B	9	No	NA	NA	NA
Vmbo-b, part C	9	No	NA	NA	NA
Vmbo-b, part A	10	Yes	Pictorial distribution bar graph, circle diagram, distribution bar graph, histogram	0	27
Vmbo-b, part B	10	No	NA	NA	NA
Vmbo-kgt, part A	7	No	NA	NA	NA
Vmbo-kgt, part B	7	No	NA	NA	NA
Vmbo-kgt, part A	8	No	NA	0	1
Vmbo-kgt, part B	8	Yes	Pictorial distribution bar graph, circle diagram, distribution bar graph, histogram, stem-and-leaf plot	0	68
Vmbo-k, part A	9	Yes	Pictorial distribution bar graph, circle diagram, distribution bar graph, histogram, stem-and-leaf-plot	0	54
Vmbo-k, part B	9	No	NA	NA	NA
Vmbo-k, part A	10	Yes	Pictorial distribution bar graph, boxplot, circle diagram, distribution bar graph, histogram, open-circle diagram, misleading pictogram, stem-and-leaf-plot, whisker-quartiles-median-plot	0	75
Vmbo-k, part B	10	No	Circle diagram	0	1

Vmbo is pre-vocational education. b, k, g, t are four educational tracks in Dutch pre-vocational education, starting with basic. NA means not applicable. \*Counts include uni-, bi- and multivariate graphs. \*\*One graph with dots in a square shape was found.

Although most graphs were univariate, we also found some bivariate graphs, such as time-plots (e.g., depicting pocket money per week; time along the horizontal axis), case-value plots (e.g., depicting highest temperature for several weather stations on a certain day), and line graphs (time along horizontal axis). We also found multivariate graphs, such as a time/line plot with two vertical axes (one for the bars, the other for the line). For the line graphs and time plots, it is not always obvious whether these are univariate or bivariate (see Cooper & Shore, 2010 for a detailed discussion on a similar issue for distribution bar graphs). Furthermore, the textbook *Moderne Wiskunde* [Modern Math] consequently uses the word bar graph for all graphs with bars, including histograms. As the choice of graph type depends on the measurement level of the data and whether the data are univariate, bivariate or

multivariate, it is important to distinguish between different types of graphs by giving them different names. While such terminology exists in English (as shown in Table 1 and the accompanying paragraph), we have started to introduce similar terms in Dutch (Boels, 2019). However, their use is not yet widespread in the Netherlands. Distinguishing graph types is also important as the mean (Boels, 2023a) and variability (Cooper & Shore, 2010) are assessed differently in different graph types.

## CONCLUSION AND DISCUSSION

Our research question is: *How and how often are dotplots and their variants used in Dutch textbooks?* The findings showed that the analysed textbooks contained no dotplots, addressing the "how often" aspect of the research question. Consequently, dotplots were not used to support transitions to histograms or boxplots, addressing the "how" aspect of the question. Furthermore, hatplots and histodots were absent, hence, also not used for this transition. Even though the analyses of all textbooks are ongoing, it becomes already clear that a gap exists between findings from statistics education research and the curriculum as implemented in the textbooks. In the Netherlands, many teachers follow the textbooks (Van Amerongen, 2012). The lack of explicit instructional transitions between case-value plots and histograms—through dotplots and their variants—may possibly contribute to students' persistent misinterpretations, highlighting the need for structured learning progressions (Boels, 2025).

The results of this study suggest that the influence of research findings on educational practice may still be limited. Given that the influence of research on educational practice was not investigated in the present study, statements regarding possible reasons for this limited influence should be considered speculative. One possible explanation is the small number of large-scale statistics education studies involving multiple classes and students in the Netherlands (e.g., Bakker, 2004). In addition, quasi-experimental research is often lacking, which leads to the omission, in many meta-analyses and research-based recommendations, of the results of research on improvements in statistics education. Another limiting factor may be the lack of concrete, exemplary materials to guide teachers in translating research findings into practice.

On the positive side is that the Dutch curriculum is currently undergoing a change (Brons & Tolboom, 2024; Selten & Vliegthart, 2024; SLO, 2025) with increased emphasis on statistics education. From previous curriculum reforms, we have learned that tested sample materials (e.g., cTWO, 2007) often find their way into textbooks. There is, therefore, an urgent need for materials that have been thoroughly tested in different classrooms—for example, through a design-based implementation research approach—so that they are well adapted to the specific conditions of the education in the country where the research has been done.

## NOTE

<sup>i</sup>The very different shape for the same distribution and the many misinterpretations about shape related to distribution stress that we need to rethink whether shape is a relevant characteristic of a distribution.

## ACKNOWLEDGEMENTS

The authors thank Kees Hoogland<sup>†</sup> for his comments on the draft version of this paper.

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