DESCRIBING THE DIDACTIC-STOCHASTIC KNOWLEDGE OF PRE-SERVICE MATHEMATICS TEACHERS: THE CASE OF CHILE

Felipe Ruz, Francisca M. Ubilla, and Valentina Giaconi
Pontificia Universidad Católica de Valparaíso, Chile
Universidad de O'Higgins, Chile
felipe.ruz.a@pucv.cl

Stochastic education of teachers has become an important research topic as mathematics teachers are usually responsible for statistics and probability teaching in schools. However, the emergence of new theoretical approaches has highlighted the problem of organizing and describing the professional knowledge needed to teach stochastics. The aim of this paper is to characterize the didactic-stochastic knowledge needed for pre-service mathematics teachers, considering Chile as a case study. Following a qualitative approach through a content analysis of the Chilean Standards for Pre-service Teacher Education, we obtained, as a result, a set of 37 indicators organized according to the Didactic-Mathematical Knowledge Model, and validated by the judgment of eight experts. The indicators consider disciplinary aspects (stochastics content); students' knowledge and their learning (cognitive content) and teacher interests (affective content); and the instructional processes (interactional and mediational content) and their link with other areas of knowledge (ecological content). We hope that the identified indicators become a useful tool to organize and evaluate stochastic education programs for teachers.

INTRODUCTION

Stochastic knowledge (Probability and Statistics) has become a necessary cultural component for every citizen to be able to function effectively in the information age (Ruz et al., 2020). This has motivated a reform in mathematics curricula, integrating elements of stochastics within the compulsory school trajectory in numerous countries, including Chile (MINEDUC, 2009; 2021). Consequently, researchers have shown an increased interest in teachers' Stochastics education in recent years (Salcedo & Díaz-Levicoy, 2022; Tauber & Pinto, 2021), as professional development programs examine how to best prepare mathematics teachers to teach stochastics according to these current demands and needs. This leads to a need to consistently and validly characterize the professional knowledge that a future mathematics teacher must develop to effectively teach stochastics, with the aim of identifying key aspects to reinforce and/or reform in this learning process of future teachers. Mathematics teacher education is a complex process, which involves the mastery of the mathematics that will be taught, being competent in teaching mathematical thinking, and learning from practice (Strutchens et al., 2017). Similarly in statistics education, teacher knowledge has been described in terms of content knowledge and knowledge of teaching (Burgess, 2012; Callingham et al., 2016; Callingham & Watson, 2011; Groth, 2007; 2013; Watson, 2001). In addition, the most recent proposals include a focus on the role of teaching with technology for stochastics (Huerta, 2018; Lee et al., 2016; Lee & Hollebrands, 2008; Wassong & Biehler, 2010, 2014).

Recent efforts have been made across the globe to establish a pre-service teaching curriculum for Stochastics (e.g. US (Franklin et al., 2015) and Chile (MINEDUC & CPEIP, 2021), which usually translates into guidelines or standards that become official documents defining which aspects must be mastered by a pre-service teacher in regard to the teaching of stochastics. Such standards are an essential source of information to understand teacher development; however, these guidelines are not always tied to explicit theoretical frameworks. In this paper, we describe a process for mapping an existing set of standards to a theoretical framework for teaching, focusing on teachers' didactic (or pedagogical) content knowledge, which in turn allows for assessment and organization of teacher education programs, as well as comparisons across different research perspectives (Groth & Meletiou-Mavrotheris, 2018).

THE CHILEAN PERSPECTIVE

In Chile, the main compulsory curricular document for teacher education is called Standards for Pre-service Teacher Education (SPTE) (MINEDUC & CPEIP, 2021). This document guides the content of the pre-service teacher programs promoted by Chilean universities, and at the end of the process, those graduating from these programs must take the National Diagnostic Test for Pre-service Teachers

In: EM Jones (Ed.), *Fostering Learning of Statistics and Data Science*
Proceedings of the Satellite conference of the International Association for Statistical Education (IASE),
July 2023, Toronto, Canada. http://iase-web.org

(www.diagnosticafid.cl/). Presently, these assessments are formative and the results are only used as reference. However, taking the test is compulsory for obtaining a teaching degree. The SPTE guidelines could be considered both a way to improve the teaching profession and a way to control teachers' practices (Flores, 2016). For improving teacher education programs and for guiding research in teachers' pedagogical content knowledge for teaching statistics, it is important to develop a list of "indicators" for meeting the standards that is actionable, manageable, and consistent with the desirable characteristics for whom will teach stochastics in Chilean schools.

TEACHER'S DIDACTIC-MATHEMATICAL KNOWLEDGE MODEL

Teacher's Didactic-Mathematical Knowledge Model (referred to here as CDM from Spanish, Modelo de Conocimiento Didáctico-Matemático) (Godino, 2009; Pino-Fan et al., 2018) incorporates the conceptualization of pedagogical content knowledge (Shulman, 1987), mathematical knowledge for teaching (Ball et al., 2008), and the notion of proficiency in the teaching of mathematics (Schoenfeld & Kilpatrick, 2008). From this perspective, a teacher's CDM consists of the following components: (1) *Mathematical knowledge*, related to the knowledge of content, which allows the teacher to solve mathematical problems which will be implemented in the classroom, linking them to those mathematical ideas arising in subsequent grades; (2) *Didactic (or pedagogical) knowledge*, which reinterprets the mathematical knowledge for teaching of Ball et al.'s model (2008), using six facets about the teacher's specialized knowledge to teach mathematics. That is, given a mathematics task, the teacher must be able to integrate the diverse meanings and concepts at play (epistemic knowledge) and must be also able to solve the task using different didactic resources (mediational knowledge), and considering all the interactions between the participants of the lesson (interactional knowledge), or adapt the task to the knowledge (cognitive content) and interest (affective content) of students, in a specific context or discipline (contextual content) (Godino et al., 2017); (3) *Meta-didactic knowledge*, including the knowledge required for teachers to reflect about their own practice, with the aim of evaluating and detecting possible improvements in the mathematics' teaching process.

As the CDM model refers to mathematics education, Ruz et al. (2019) adapted it to the teaching of probability and statistics, in particular to reflect the differences between teaching mathematics and statistics (Rossman et al., 2006). The resulting set of indicators was lengthy, with many interactions across the dimensions, and specific to the previous version of the Chilean standards (MINEDUC & CPEIP, 2012). Therefore, the main goal of this manuscript is to characterize the didactic-stochastic knowledge of future Chilean mathematics teachers based on the CDM model, that is, create a "knowledge guide." In this paper, we outline our methodology, present our resulting list of indicators of teaching knowledge for teaching stochastics, and discuss possible applications.

METHODOLOGY

To develop the set of indicators, we follow a methodological process of four stages:

Stage 1. Selecting the document and standards: The analyzed document was the Chilean Pedagogical and Disciplinary SPTE (MINEDUC & CPEIP, 2021). This document is used as a reference for the accreditation of teacher education programs in Chile, and it is the basis for the National Diagnostic Test for Pre-service Teachers (ENDFID). Applying these standards and being accredited is a legal obligation for all the teaching programs in Chile.

Stage 2. Development of indicators according to the CDM model. This stage consists of three phases. The first phase consisted of classifying the descriptions of different content areas in the standards according to the six facets of didactic knowledge of the CDM; the second phase consisted of writing an initial list of indicators based on the phase one, while the third phase consisted of contrasting the current version with those 90 indicators developed by Ruz et al. (2019), and based on this comparison, was generated a first version of 64 indicators that was sent to the experts.

Stage 3. Experts' judgment analysis. To validate the list of indicators, an analysis was carried out by eight experts. All the participating experts obtained a Doctorate degree in Mathematics or Statistics Education, or a related field, between 5 and 21 years ago, and work in stochastic pre- and in-service teacher education. They were asked to consider four criteria in their assessment: For the first three criteria (*clarity, coherence*, and *relevance*) a four-step Likert scale was used, starting from 1 ("Not met") to 4 ("Met fully"), and the fourth, *sufficiency*, was assessed qualitatively in the experts' justifications. To quantify the degree of agreement between the experts' assigned scores, we used Aiken's V index (Aiken, 1980), where we consider values of the V point above 0.7 as acceptable (Aiken, 1985).

Stage 4. Filtering and creating the final proposal of indicators. Finally, based on the results of the analysis described in Stage 3, we reviewed those indicators that obtained V point values below 0.7, and reformulated them to take into account the experts' suggestions. For example, we reduced the number of indicators to minimize redundancies in the final list. We also reviewed, and if needed, rewrote, merged, or eliminated indicators so each indicator corresponded to a single component of the CDM model, generating our final version.

To develop the knowledge guide, we applied a qualitative approach, analyzing our data through content analysis. This technique, implies coding, categorization, comparison of pre-existing categories, and the creation of links between the generated categories. This is done to finally be able to draw theoretical conclusions from the analyzed text (Cohen et al., 2007). We applied Content Analysis technique in the four previous stages, where it was necessary to perform qualitative analysis of text fragments (phase 1), infer indicators from the text according to the CDM model (phase 2), consider the qualitative observations of the sufficiency criterion in the expert judgment (phase 3), and reduce the final indicators (phase 4). It is important to note that this paper presents the list of indicators as a result, while a more detailed description of the entire analysis process can be found in Ruz et al. (in press).

RESULTS

The final set of indicators is presented below, organized by the CDM didactic content areas.

Epistemic Content

- Understands the characteristics of the statistical models describing data variability in their context.
- Understands stochastics' principles and historical-epistemological meanings.
- Critically evaluates the use of descriptive and inferential procedures to solve problems in different knowledge areas.
- Links descriptive and inferential statistics using data as evidence and expresses conclusions with a certain degree of uncertainty.
- Communicates stochastic ideas consistently and effectively using oral or written language.
- Articulates different data representations, being able to build them both manually and with technology.
- Critically evaluates the validity of conclusions emerging from a stochastic analysis process.
- Links the process of stochastic problem solving with stages associated with empirical research.

Cognitive Content

- Understands theories of human learning and their relation to the teaching of stochastics.
- Builds, selects, and adapts assessments that are coherent with the stochastic learning methodologies used.
- Considers the difficulties and erroneous conceptions of all students to (re)organize the learning experiences.
- Understands the value of digital tools in the stochastic learning processes.
- Sequences learning objectives of stochastics, coherent with the curriculum, and the students' previous knowledge and skills.
- Applies gradual approximations, from informal to formal, to introduce the understanding of stochastic topics of greater difficulty.

Affective Content

- Applies motivation theories to promote engagement, persistence, and self-efficacy of students in the learning of stochastics.
- Considers contexts and situations of interest for students in the modeling of stochastic phenomena.
- Promotes positive attitudes towards stochastics and its own skills such as research, communication, and critical thinking.

- Promotes willingness and commitment of all students towards the learning of stochastics.
- Promotes the development of socioemotional competencies for decision-making and awareness of context in the learning of stochastics.
- Promotes students' self-esteem and academic self-efficacy when learning stochastics.
- Generates strategies for an equitable and active participation of all students, valuing diversity in all its expressions.

Interactional Content

- Guides their students to move from guided work to autonomous, reinforcing students' metacognitive skills in the learning of stochastics.
- Analyzes data and evidence contributed by assessments to improve the teaching techniques used.
- Monitors the students' level of stochastic understanding before, during, and/or after the class.
- Establishes respectful and inclusive interaction rules, coherent with the dynamics of a stochastics class.
- Promotes an interactive teaching style, centered in real problems where the stochastic research process is valued.
- Generates instances between students that allow them to model and reason stochastically to make decisions about a problem.
- Promotes collaborative work between peer teachers in the critical evaluation of didactic strategies used in the teaching of stochastics

Mediational Content

- Promotes virtual and in-person learning opportunities fostering stochastic competencies.
- Promotes a welcoming and stimulating class environment during the stochastic teaching and learning process.
- Integrates digital environments in different formats to solve stochastic problems.
- Articulates different didactic material, and digital resources included in the curriculum, and suggested by research in the teaching of stochastics.
- Optimizes lecture time in mathematics lessons to address the teaching of stochastics.

Ecological Content

- Renews their teaching strategies based on educational research in the field of stochastic education and the curricular updates.
- Promotes the link of stochastics with other disciplines in which data intervene and uncertainty exists.
- Promotes the use of stochastics in decision-making based on data present in modern democracies.
- Considers the determining factors and restrictions of their students' social environment in the processes of teaching and learning of stochastics.

DISCUSSION

In this work, we have addressed the issue of characterizing the professional knowledge that a future mathematics teacher must develop to effectively teach stochastics, considering the Chilean reality as a case study. As a result, we have obtained a final set of 37 indicators, organized according to Professor Godino's Didactic-Mathematical Knowledge Model (CDM) (Godino, 2009). In this vein, we described the method used to obtain an updated, more synthesized, and consistent version of the desirable characteristics for those who will teach stochastics in Chilean schools. In comparison to the first approach proposed by Ruz et al. (2019), the interactions between theoretical contents in the CDM are avoided, and there is no differentiation between statistics and probability, including both under the umbrella of stochastics to highlight the unbreakable link between both disciplines.

Consequently, we consider the initially established goals as achieved, as we obtained a set of indicators reviewed by content experts. Thus, we can position the resulting indicators within the family of set indicators of that nature whose use depends on the educational stakeholder who utilizes it. For example, in this paper, the final indicators cover the didactic-stochastic knowledge of future teachers in relation to their students, while in the case of teacher trainers, these facets will enable them to guide, adapt, and develop study processes of the teaching of statistics in teacher education programs. Moreover, this work presents a methodology that enables the articulation of theoretical models, such as CDM, with other types of documents or content organization through the validation of experts.

In addition, from a theoretical perspective, this set of indicators establishes guidelines for a future didactic-stochastic knowledge model for pre- and in-service teachers. The list enables characterization of the knowledge types at play during the stochastic teaching and learning processes in different scenarios. Moreover, the list could guide the reflection on which knowledge types are more needs to reinforce the teaching of stochastics. Finally, a constant update of teacher education programs is needed, as the school curriculum continually incorporates new education perspectives—even more so in the current era, where data and uncertainty play a fundamental role in citizens' daily activities. Thus, this knowledge guide hopes to be a contribution and a foundation on which to work on when thinking and reflecting on teacher stochastic education.

ACKNOWLEDGEMENTS

This research was carried out thanks to funding from ANID-Chile grants: FONDECYT 3220122, FOVI 220056, the Basal Funding for centers of excellence (CMM) FB210005, and Millenium Scientific Initiative NCS2021_014.

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